SKM 3413 - DRILLING ENGINEERING

Chapter 3 - Drilling Hydraulics

Assoc. Prof. Abdul Razak Ismail

Petroleum Engineering Dept.
Faculty of Petroleum & Renewable Energy Eng.
Universiti Teknologi Malaysia

Contents

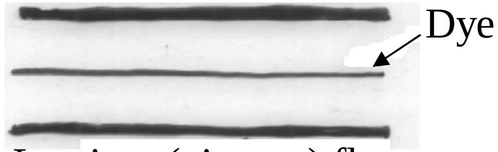
- Review of flow in pipes *(Fluid Mechanics)*
- Drilling mud flow (circulating) system
- Newtonian fluid flow calculations
- Bingham plastic fluid flow calculations
- Δp across bit nozzles
- Δp calculation for typical system

Review of flow In Pipes

- Real fluid flow is much complex compare to perfect fluid flow.
- Shear stress
  - Between fluid particles
  - Between fluid particles and pipe's wall

  } μ fluid
- Energy equilibrium principles are used to solve the problems.
- Partial differential equation (Euler's equation) has no general solution to solve problems.
- Results from experiment (*analytical*) and semi-empirical method needs to be used to solve flow problems.
- There are 2 types of steady flow of real fluid exists:
  - Laminar flow
  - Turbulent flow

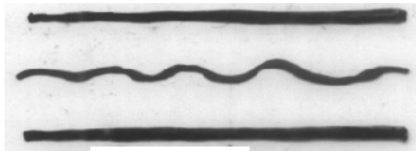
This one?



Laminar (viscous) flow

or

this one?



Transition flow

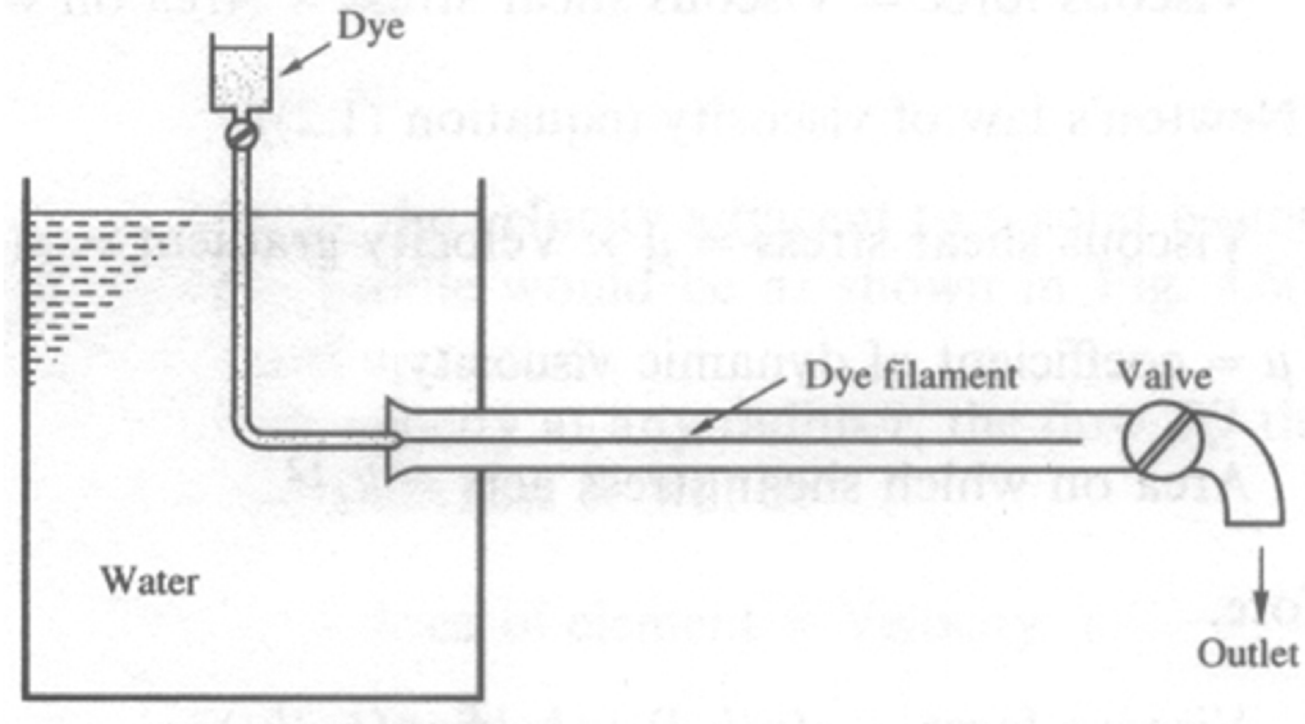
or

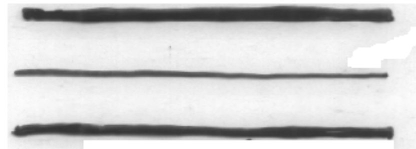
this one?

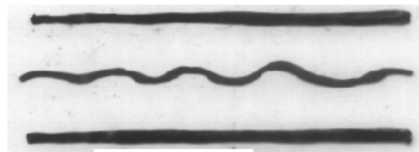
Turbulent flow

- All three types of flow actually do occurred in real fluid flow.
  - Laminar flow $\Rightarrow v\downarrow$
  - Turbulent flow $\Rightarrow v\uparrow$
- The problem is: what is $v\uparrow$ and $v\downarrow$ .Why we need to know?

- This phenomenon was first investigated in 1883 by ***<u>Osborne Reynolds</u>*** in an experiment which has a classic in fluid mechanic.




Laminar flow


Transition flow


Turbulent flow

- After a few experiments, he found out a mathematical relationship:

$$\frac{\rho v d}{\mu}$$

- This mathematical relationship can be used to determine the types of flow.
  - $\frac{\rho v d}{\mu} < 2000$ laminar flow
  - $2000 < \frac{\rho v d}{\mu} < 4000$ transition flow
  - $\frac{\rho v d}{\mu} > 4000$ turbulent flow
- Subsequently until now, this mathematical relationship is known as ***Reynolds number, Re (or N_{Re})***.

$$\mathrm{Re} = \frac{\rho v d}{\mu} \Rightarrow \text{dimensionless}$$

- laminar flow : $\mathrm{Re} < 2000$
- transition flow : $2000 < \mathrm{Re} < 4000$
- turbulent flow : $\mathrm{Re} > 4000$

$$\frac{\rho v d}{\mu}$$

where:

ρ = fluid density
v = fluid average velocity
d = pipe inside diameter
μ = fluid absolute viscosity

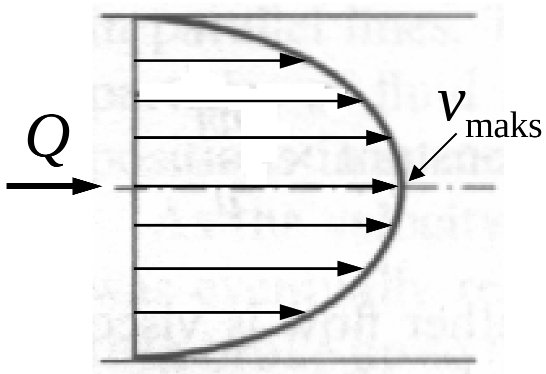
- If kinematic viscosity, υ, is inserted in the equation:

$$\upsilon = \frac{\mu}{\rho}$$

$$\mathrm{Re} = \frac{vd}{\upsilon}$$

- Fluid velocity profile in a pipe:

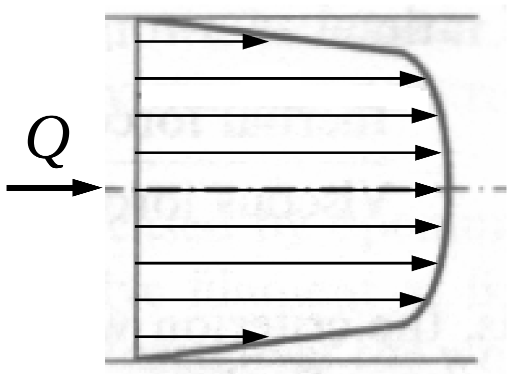
Laminar flow



$$\bar{v} = v_{avg} = \tfrac{1}{2} v_{maks}$$

$$\bar{v} = v_{avg} = \frac{Q}{A}$$

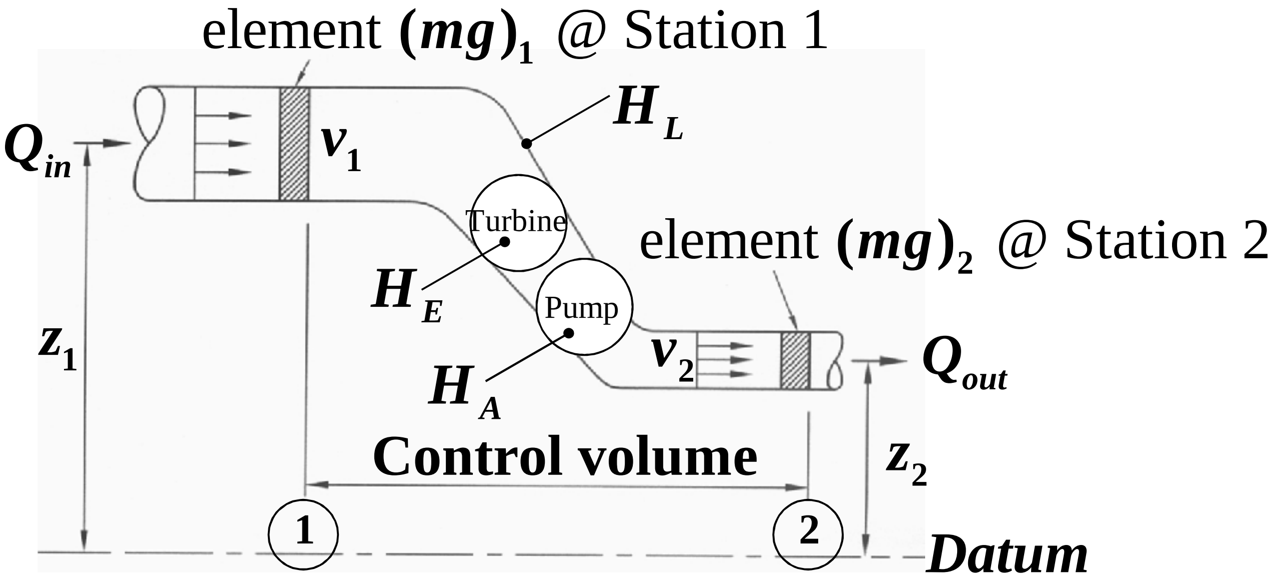
Turbulent flow



$$\bar{v} = v_{avg} = \frac{Q}{A}$$

Mechanical Energy of a Flowing Fluid

- Consider the situation below:



- The energy possessed by a flowing fluid consists of internal energy and energies due to pressure, velocity, and position

$$\begin{matrix}\text{energy at}\\\text{section 1}\end{matrix} + \begin{matrix}\text{energy}\\\text{added}\end{matrix} - \begin{matrix}\text{energy}\\\text{lost}\end{matrix} - \begin{matrix}\text{energy}\\\text{extracted}\end{matrix} = \begin{matrix}\text{energy at}\\\text{section 2}\end{matrix}$$

- This equation, for steady flow of incompressible fluids in which the change in internal energy is negligible, simplifies to

$$\left(\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1\right)+H_A-H_L-H_E=\left(\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2\right)$$

Energy Losses In Pipe

- ***Def***.: Any energy losses in closed conduits due to friction, H_L.
- This types of losses can be divided into 2 main categories:
  - Major losses, $H_{L\text{-}major}$, and
  - Minor losses, $H_{L\text{-}minor}$.
- From Bernoulli's equation:

$$\left(\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1\right)+H_A-H_L-H_E=\left(\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2\right)$$

- Energy added to the system, H_A, is frequently due to pump fluid head, H_P, energy extracted, H_E, is frequently due to turbine fluid head, H_T, Bernoulli's equation can be simplify as:

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1+H_P=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2+H_T+H_{L-\text{major}}+H_{L-\text{minor}}$$

Major Losses In Pipe

- ***Def***.: The head loss due to friction in long, straight sections of pipe.
- The losses do happen in pipe, either in laminar or turbulent flow.

a. Laminar flow

- Problem solved analytically → derived purely from mathematical relationship
- Hagen-Porseuille equation

$$\Delta p_f = \frac{32\mu v L}{d^2}$$

in the forms of head loss, H_L

$$H_L = \frac{32\mu v L}{\gamma d^2}$$

- Darcy-Weisbach equation

by replacing $\text{Re} = \frac{\rho v d}{\mu}$ into Hagen-Porseuille equation

$$H_L = \frac{64}{\text{Re}} \frac{L}{d} \frac{v^2}{2g}$$

b. Turbulent flow

- From Darcy-Weisbach equation for laminar flow

$$H_L = \frac{64}{\text{Re}} \frac{L}{d} \frac{v^2}{2g}$$

$$H_L = f \frac{L}{d} \frac{v^2}{2g}$$

- Where, for laminar flow, $f = \frac{64}{\text{Re}}$ } a simple mathematical relationship.
- For turbulent flow, f has to be solved empirically → experiment need to be done.
- In laminar and turbulent flow, f is known as friction coefficient or friction factor.

Friction Factor

a. Laminar flow

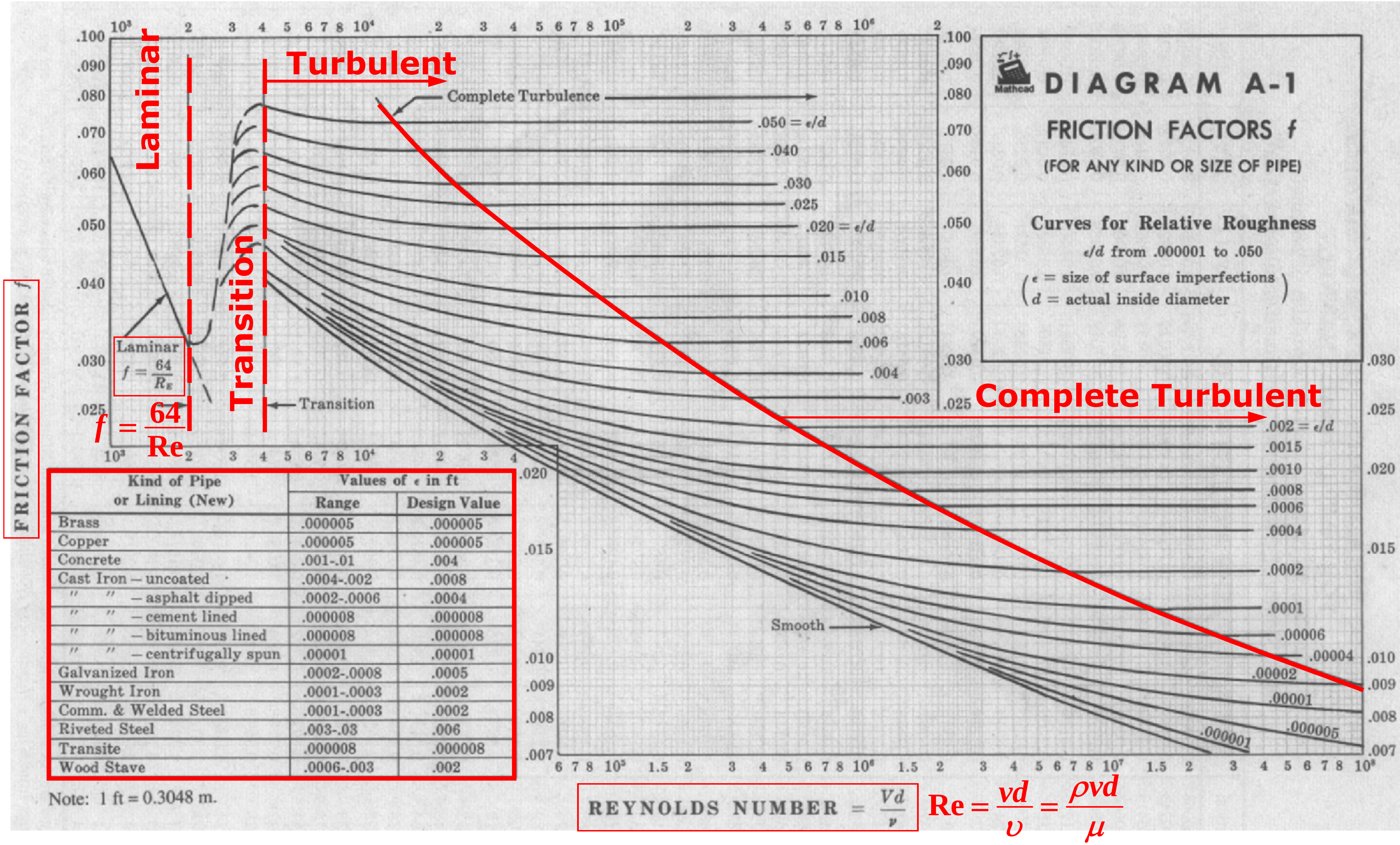
- Darcy-Weisbach equation

$$H_L = f\frac{L}{d}\frac{v^2}{2g} \quad \text{where} \quad f = \frac{64}{\text{Re}}$$

b. Turbulent flow

- In the literature (from 1900's – current date), there are many studies that have been conceded by various researchers.
  - Blasius's equation (1913)
  - von Karman's equation modified by Prandtl
  - Nikuradse's equation (for smooth and rough pipes)
  - Colebrook-White equation (1940's)
  - Moody
  - Barr's equation (1975)

Friction Factor .... *(cont. 2)*



Kind of Pipe or Lining (New)	Values of ε in ft — Range	Values of ε in ft — Design Value
Brass	.000005	.000005
Copper	.000005	.000005
Concrete	.001-.01	.004
Cast Iron — uncoated	.0004-.002	.0008
" " — asphalt dipped	.0002-.0006	.0004
" " — cement lined	.000008	.000008
" " — bituminous lined	.000008	.000008
" " — centrifugally spun	.00001	.00001
Galvanized Iron	.0002-.0008	.0005
Wrought Iron	.0001-.0003	.0002
Comm. & Welded Steel	.0001-.0003	.0002
Riveted Steel	.003-.03	.006
Transite	.000008	.000008
Wood Stave	.0006-.003	.002

Note: 1 ft = 0.3048 m.

Moody Chart

Normal practice in determination of *f*

1. Calculate Re to determine the types of flow.

2. H_L calculation: used Darcy-Weisbach equation.

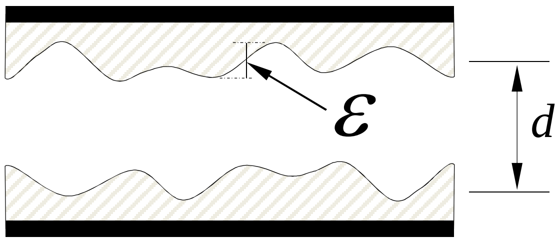
$$H_L = f \frac{L}{d}\frac{v^2}{2g}$$

3. For laminar flow: $f = \frac{64}{\text{Re}}$

4. For turbulent flow:

   a. Determine pipe relative roughness, $\frac{\varepsilon}{d}$

   Where:
   ε – pipes absolute roughness
   d – pipe internal diameter



e is depend on pipe's material, normally is given in tabular forms.

Material (new)	Absolute roughness, ε	
	ft	mm
Riverted steel	0.003 - 0.03	0.9 - 9.0
Concrete	0.001- 0.01	0.3 - 3.0
Wood stave	0.0006 - 0.003	0.18 - 0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Glass	0.0 (smooth)	0.0 (smooth)

   b. Obtain f from Moody chart, @ Re, $\frac{\varepsilon}{d}$

Attention

1. In this subject, SKM1043, the f that we are using, is the American friction factor, $f_{American}$.
2. The value of $f_{American}$ is different to the one that used by the British

$$f_{American} = 4 f_{British}$$

$$\frac{64}{\text{Re}} \qquad \frac{16}{\text{Re}}$$

needs to refer different Moody Chart

3. Sometimes: $\lambda = f_{American} = 4 f_{British}$

- Since the mud enters the drill string and leaves the annulus at essentially the same elevation, the only ***pressure required*** is to overcome the ***frictional losses in the system***.
- Hence, the discharge pressure at the pump is defined by:

$$\Delta p_t = \Delta p_s + \Delta p_p + \Delta p_c + \Delta p_b + \Delta p_{ac} + \Delta p_{ap} \quad .... (3.1)$$

where:

Δp_t = pump discharge pressure
Δp_s = pressure loss in surface piping, standpipe, and mud hose
Δp_p = pressure loss inside drill pipe
Δp_c = pressure loss inside drill collars
Δp_b = pressure loss across bit nozzles
Δp_{ac} = pressure loss in annulus around drill collars
Δp_{ap} = pressure loss in annulus around drill pipe

- The solution of Eq. (3.1) is rather tedious; separate calculations are needed for each section
- There are 4 different types of model used to calculate frictional pressure losses in mud circulating system:
  - Newtonian
  - Bingham plastic
  - Power-law
  - API Power-law
- Due to the limitation of the syllabus, Power-Law and API Power-Law models will not be discussed in this subject.
- All calculations will be focused on Newtonian and plastic fluid models.

Newtonian Fluid Flow Calculations

- Similar to generalized flow system approach, calculation of Δp for pipe flow requires a knowledge of which flow pattern pertains to the specific case, since different equations apply for each situation.

- Definition of the existing flow pattern is given by a dimensionless quantity known as the Reynolds number (N_{Re}):

$$N_{\mathrm{Re}} = \frac{928 \rho \bar{v} d}{\mu} \qquad \text{.......... (3.2)}$$

where:

N_{Re} = Reynolds's number

$\bar{v}$ = average velocity of flow, ft/sec

ρ = fluid density, ppg

d = pipe inside diameter, in.

μ = fluid viscosity, cp

q = circulating volume, gal/min

- Similar to generalized flow system approach, that if
  - laminar flow : $N_{\mathrm{Re}} < 2000$
  - transition flow : $2000 < N_{\mathrm{Re}} < 4000$
  - turbulent flow : $N_{\mathrm{Re}} > 4000$
- The Δp in laminar flow is given by the Hagan-Poiseuille law; this, in practical units, is

$$\Delta p = \frac{\mu L \bar{v}}{1{,}500 d^2} \qquad \text{.......... (3.3)}$$

where:

Δp = laminar flow Δp, lb/in^2

L = length of pipe, ft

- For turbulent flow, Fanning's equation applies:

turbulent flow

$$\Delta p = \frac{f \rho L \bar{v}^2}{25.8 d} \qquad \text{.......... (3.4)}$$

where:

Δp = turbulent flow Δp, lb/in^2

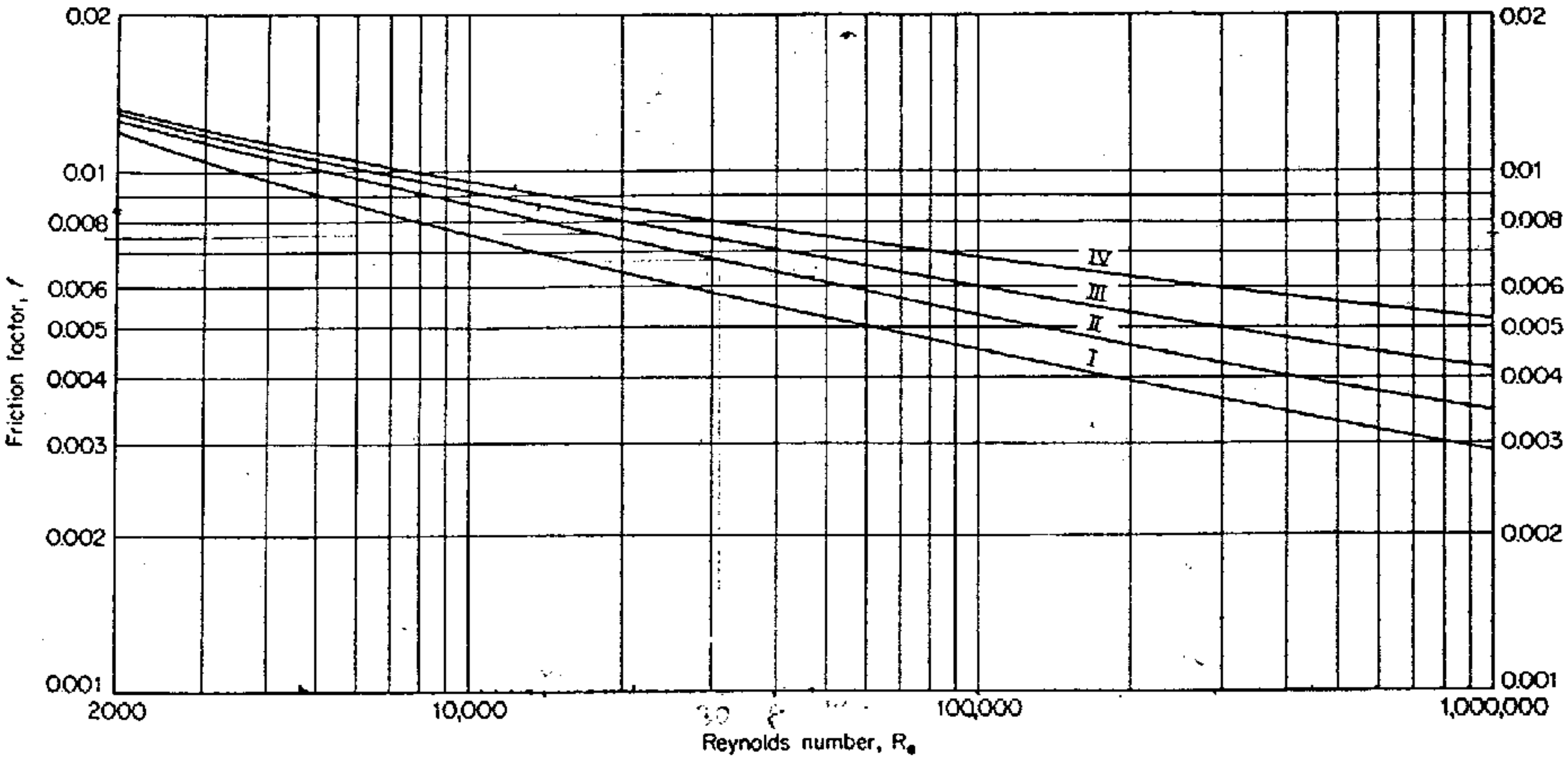
f = Fanning friction factor

- The friction factor *f* is a function of and pipe roughness, and has been evaluated experimentally for numerous materials (see Fig. 7.1)
- Δp calculation for Newtonian fluid flow systems in the following manner:

a. Calculate N_{Re} from Equation (3.2).

b. If $N_{Re} < 2000$, use Equation (3.3) to calculate the pressure drop.

c. If $N_{Re} > 2000$, use Equation (3.4). In this case the friction factor *f* is obtained from Figure 7.1 or its equivalent.



I Lowest values for drawn brass or glass tubing (Walker, Lewis, & McAdams)
II For clean internal-flush tubular goods (Walker, Lewis, & McAdams)
III For full-hole drill pipe or annuli in cased hole (Piggott's data)
IV For annuli in uncased hole (Piggott's data)

Fig. 7.1. Friction factor vs. Reynolds number for mud flow calculations. After Ormsby,[13] courtesy API.

Plastic Fluid Flow Calculations

- Drilling fluids is non-Newtonian fluid
- Newtonian fluid equations must be altered for application to typical drilling mud systems

Surface Equipment Losses (Δp_s)

- The surface equipment consist of standpipe, hose, swivel, kelly joint, and the piping between the pump and standpipe.
- In practice, there are only four types of surface equipment; each type is characterized by the dimensions of standpipe, kelly, rotary hose and swivel. Table 3.1 summarizes the four types of surface equipment.

Table 3.1: Types of surface equipment & value of constant E

Type	Standpipe		Hose		Swivel, etc.		Kelly		Eq. length, 3.826" ID	E
	ID	Length	ID	Length	ID	Length	ID	Length		
1	3"	40 ft.	2.5"	45 ft.	2"	20 ft.	2.25"	40 ft.	2,600 ft.	2.5×10^{-4}
2	3.5"	40 ft.	2.5"	55 ft.	2.5"	25 ft.	3.25"	40 ft.	946 ft.	9.6×10^{-5}
3	4"	45 ft.	3"	55 ft.	2.5"	25 ft.	3.25"	40 ft.	610 ft.	5.3×10^{-5}
4	4"	45 ft.	3"	55 ft.	3"	30 ft.	4"	40 ft.	424 ft.	4.2×10^{-5}

To determine surface equipment losses (Δp_s):

Use the following formula:

$$\Delta p_s = E \rho_m^{0.8} q^{1.8} \mu_p^{0.2} \quad \text{.......... (3.5)}$$

where:

Δp_s = surface pressure losses, psi
q = flow rate, gpm
ρ_m = mud density, ppg
E = a constant depending on type of surface equipment used
μ_p = mud plastic viscosity, cp

Fluid Flow Inside the Pipe

A. *Laminar Flow Region*

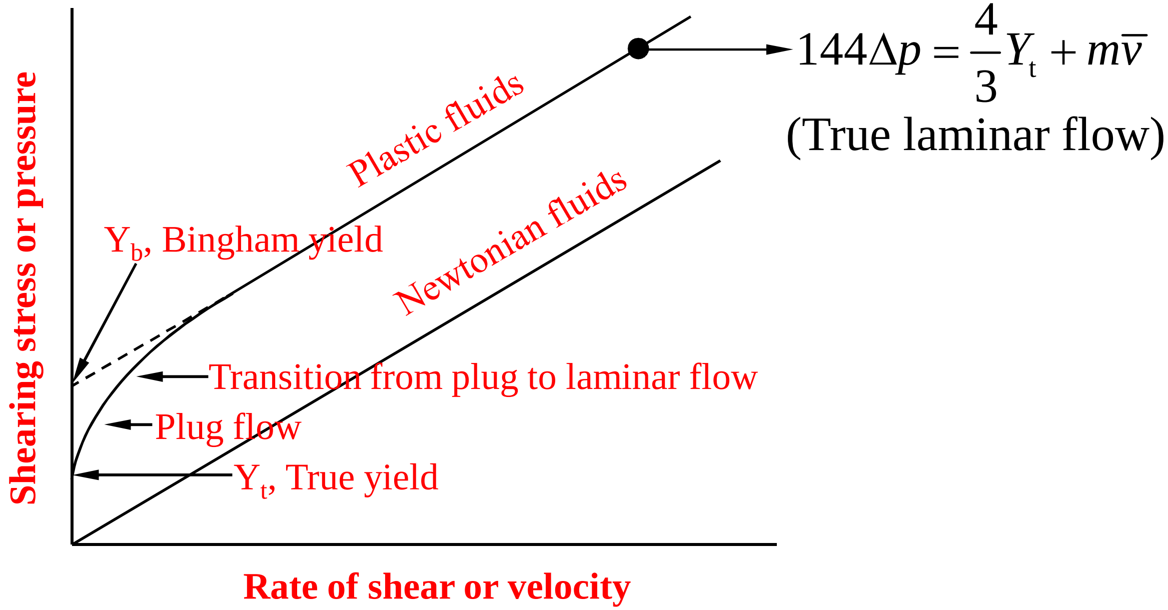


Fig. 3.1: Flow behavior of plastic and Newtonian fluids.

$$144\Delta p = \frac{4}{3} Y_t + m\bar{v}$$

where:

$144\Delta p$ = pressure drop, lb/ft^2

$\frac{4}{3} Y_t = Y_b$, lb/ft^2

$m = \mu L/(1500d^2)$, slope of linear portion (from Eq. (3.3))

- For practical values of $\bar{v}$, the behavior of plastic fluids may be expressed as:

$$\Delta p = \frac{LY_b}{300d} + \frac{\mu_p \bar{v} L}{1500d^2}$$

$$\therefore \boxed{\Delta p = \frac{L}{300d}\left(Y_b + \frac{\mu_p \bar{v}}{5d}\right)} \quad \text{.......... (3.6)}$$

laminar flow

where:

μ_p = plastic viscosity, cp.

Y_b = yield point, lb/100ft^2.

- Eq. (3.6) may be used in cases where laminar flow exists
- Determination of flow characteristic (laminar or turbulent) is made by comparing the actual velocity with a calculated critical velocity

Average Velocity Calculation

$$\bar{v} = \frac{q}{A}\,\frac{\text{ft}^3/\text{sec}}{\text{ft}^2}$$

$$= \frac{q\ \text{gal/min} \times \dfrac{1}{7.48}\dfrac{\text{ft}^3}{\text{gal}} \times \dfrac{1}{60}\dfrac{\text{min}}{\text{sec}}}{(\pi/4)(d/12)^2}$$

Avg. velocity inside the pipe

$$\bar{v} = \frac{q}{2.45\,d^2} \quad \text{....... (3.7a)}$$

Avg. velocity in the annulus

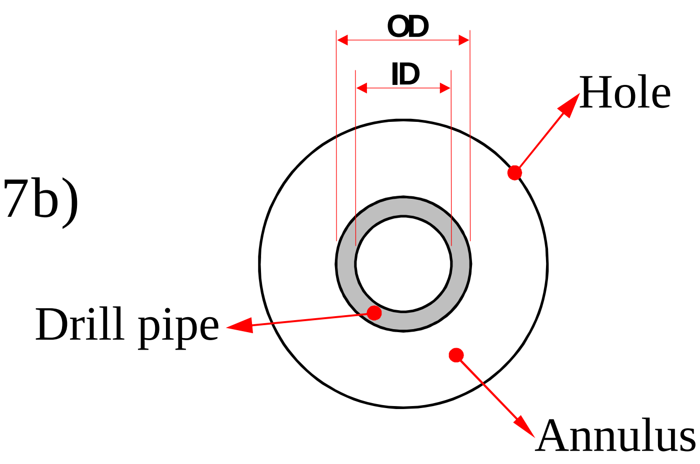
$$\bar{v} = \frac{q}{2.45\,(d_h^2 - d_p^2)} \quad \text{....... (3.7b)}$$

where:

$\bar{v}$ = average velocity, ft/sec.

q = flow rate, gpm

d = diameter, in.



$$\text{Annulus Area} = A_h - A_p = \frac{\pi}{4}(d_h^2 - d_{p(OD)}^2)$$

Critical Velocity Calculation

- If Eqs. (3.3) and (3.6) are equated, an equivalent Newtonian viscosity in terms of $d, \bar{v}, \mu_p$ and Y_b is obtained:

$$\mu = \frac{5dY_b}{\bar{v}} + \mu_p$$

- Substituting the above Eq. for μ in the Reynolds's number of Eq. (3.2), equating the resulting equation to 2000, and solving for $\bar{v}$ gives:

$$v_c = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 9.3\rho d^2 Y_b}}{\rho d} \quad \text{.......... (3.8)}$$

where:

v_c = critical velocity, ft/sec, above which turbulent flow exists and below which the flow is laminar.

- Eq. (3.8) assumes that turbulence occurs at N_{Re} = 2000. Therefore, if:

$$\bar{v} < v_c, \text{ flow is laminar}$$
$$\bar{v} > v_c, \text{ flow is turbulent}$$

B. Turbulent Flow Region

- Before Fanning Eq. can be used, alteration to N_{Re} expression have to be done (after *Beck, Nuss & Dunn*)

$$\mu_t = \frac{\mu_p}{3.2} \quad \text{.......... (3.9)}$$

where:

μ_t = turbulent viscosity of plastic fluids, cp

- Substitution of μ_t, for μ in the general N_{Re} expression (Eq. (3.2)) gives:

$$N_{Re} = \frac{928 \rho \bar{v} d}{\mu_t}$$

$$\boxed{N_{Re} = \frac{2{,}970 \rho \bar{v} d}{\mu_p}} \quad \text{.......... (3.10)}$$

- By using Fig. 7.1, determine *f*
- This *f* may then be used in Eq. (3.4) for calculation of pressure

In summary, Δp calculation for plastic fluid flow systems can be done as follows:

(1) Calculate the average velocity, $\bar{v}$, from Eq. (3.7a) or (3.7b)

(2) Calculate $\boldsymbol{v_c}$ from Eq. (3.8)

(3) If $\bar{v} < v_c \rightarrow$ flow is laminar, Eq. (3.6) applies

(4) If $\bar{v} > v_c \rightarrow$ flow is turbulent, requiring:

a. Calculation of $\boldsymbol{N_{Re}}$ from Eq. (3.10)

b. Determination of $\boldsymbol{f}$ from Fig. 7.1 at the calculated for the conduit in question

c. Calculation of pressure drop from Eq. (3.4)

Example 3.1

Mud is flowing through 4 1/2 inch OD, internal flush drill pipe. Calculate the frictional pressure drop per 1000 ft of pipe.

Mud properties

Mud density, ρ_m	=	10 lb/gal
Pipe ID	=	3.640 in.
Bingham yield, Y_b	=	10 lb/100 ft^2
Circulating rate, q	=	400 gal/min
Plastic viscosity, μ_p	=	30 cp

Solution 3.1

$$Eq.\ (3.7\text{a}):\ \bar{v} = \frac{q}{2.45d^2}$$

$$Eq.\ (3.8):\ v_\text{c} = \frac{1.08\mu_\text{p} + 1.08\sqrt{\mu_\text{p}^2 + 9.3\rho d^2 Y_\text{b}}}{\rho d}$$

(1) $\bar{v} = \dfrac{400}{2.45(3.64)^2} = 12.3\ \text{ft/sec}$

(2) $v_\text{c} = \dfrac{(1.08)(30) + (1.08)\sqrt{(30)^2 + (9.3)(10)(3.64)^2(10)}}{(10)(3.64)} = 4.3\ \text{ft/sec}$

(3) Since $\bar{v} > v_c$, flow is turbulent.

(a) $N_\text{Re} = \dfrac{(2{,}970)(10)(12.3)(3.64)}{30} = 44{,}300$

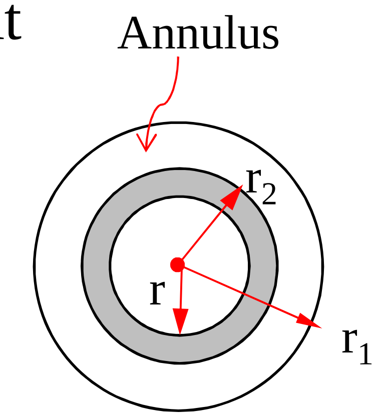
(b) f = 0.0062 from Curve II, Fig. 3.1

(c) $\Delta p_p = \dfrac{(0.0062)(10)(1000)(12.3)^2}{(25.8)(3.64)} = 100\ \text{psi/1000 ft}$

Hydraulically Equivalent Annulus Diameter

- For annular flow, it is necessary to use a hypothetical circular diameter, d_a, which is the hydraulic equivalent of the actual annular system
- The hydraulic radius is defined as:

$$\text{hydraulic radius, } r_h = \frac{\text{cross-sectional area of flow system}}{\text{wetted perimeter of conduit}}$$



for an annulus $\rightarrow$ $$r_h = \frac{\pi(r_1^2 - r_2^2)}{2\pi(r_1 + r_2)} = \frac{r_1 - r_2}{2}$$

for a circular pipe $\rightarrow$ $$r_h = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$$

- The frictional loss in an annulus is equal to the loss in a circular pipe having the same hydraulic radius; hence, in general terms:

$$r_e = r_1 - r_2 \quad or \quad d_e = d_1 - d_2 \qquad \text{.......... (3.11)}$$

where r_e and d_e are the hydraulically equivalent radius and diameter

Pressure Drop Across Bit Nozzles

- Consider the diagram below for incompressible fluid:

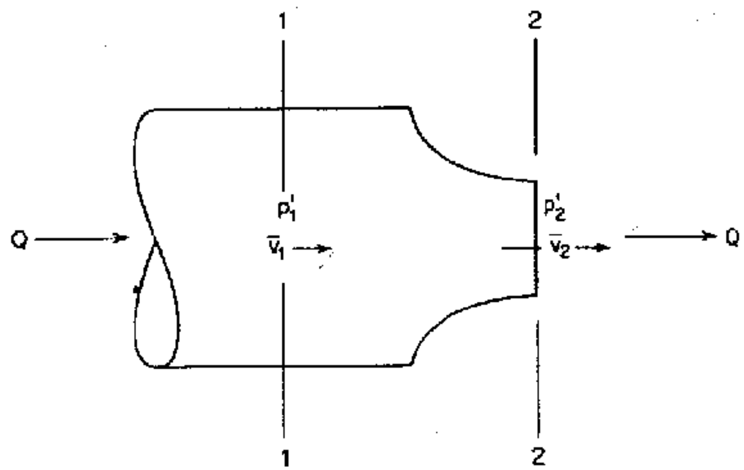


Fig. 3.2: Schematic sketch of incompressible fluid flowing through a converging tube or nozzle.

- Assuming *steady state, adiabatic, and frictionless*:

$$\frac{p_1}{\rho} + \frac{\bar{v}_1^2}{2g} = \frac{p_2}{\rho} + \frac{\bar{v}_2^2}{2g} \quad \text{..........} \; (a)$$

where:

p_1, p_2 = turbulent flow pressure drop, lb/ft^2

ρ = density, lb/ft^3

$\bar{v}_1, \bar{v}_2$ = velocities at points 1 and 2, ft/sec

$$\frac{p_1}{w} + \frac{\overline{v}_1^2}{2g} = \frac{p_2}{w} + \frac{\overline{v}_2^2}{2g} \qquad \text{..........} (a)$$

or

$$\frac{\Delta p}{\rho} = \frac{\overline{v}_2^2 - \overline{v}_1^2}{2g}$$

- Practically, $\overline{v}_2^2 - \overline{v}_1^2 \cong \overline{v}_2^2$, therefore:

$$\overline{v}_2^2 = 2g\frac{\Delta p}{\rho} \qquad \text{..........} (b)$$

- The ideal rate of flow, $q_i = A_2\overline{v}_2$. The actual flow rate q is:

$$q = Cq_i \qquad \text{..........} (c)$$

where C is the flow or nozzle coefficient for particular design.

- By substituting Eq. (c) into Eq. (b), and rearranging it, the equation becomes:

$$\Delta p = \frac{\rho q^2}{2gC^2 A_2^2} \quad \text{.......... (3.12)}$$

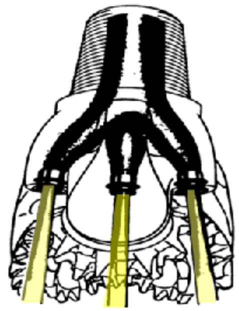
- Altering Eq. (3.12) to practical units for mud flow, we:

$$\Delta p_b = \frac{q^2 \rho}{7{,}430\, C^2 d_e^{\,4}} \quad \text{.......... (3.13)}$$

where d_e = hydraulically equivalent nozzle diameter, in.

- The value of C is around 0.8 – 0.98.

Multiple Nozzles



- The calculation of Δp across a multiple nozzle bit may be simplified by substituting the sum of the nozzle areas for ***A*** in Equation (3.12).
- For single nozzle:

$$\Delta p = \frac{\rho q^2}{2gC^2A^2}$$

- For several nozzles, each of area A_1:

$$\Delta p_m = \frac{\rho q_1^2}{2gC^2A_1^2}$$

- For parallel flow, $q_1 = \dfrac{q}{n}$, where n = number of nozzles.

therefore:

$$\frac{\Delta p_m}{\Delta p} = \frac{q_1^2}{q^2}\frac{A^2}{A_1^2} = \frac{q_1^2 A^2}{n^2 q_1^2 A_1^2}$$

- Cross sectional area of flow, A, is defined as

$$\frac{A^2}{n^2 A_1^2} = 1$$

$$\therefore\ A^2 = n^2 A_1^2$$

or

$$A = nA_1 \qquad \text{..........} \ (3.14)$$

- Similarly, for use in Eq. (3.13)

$$d_e = \sqrt{nd^2} \quad \text{.......... (3.15a)}$$

- If the multiple nozzles vary in size,

$$d_e = \sqrt{ad_1^2 + bd_2^2 + \text{ etc.}} \quad \text{.......... (3.15b)}$$

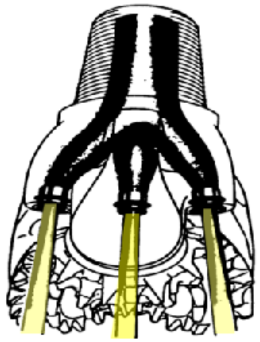
where:

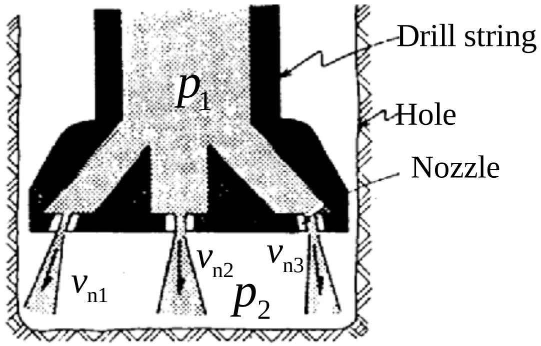
a = number of nozzles having diameter d_1.
b = number of nozzles having diameter d_2.
d_e = hydraulically equivalent single nozzle diameter, in.

Example 3.2

A 10 lb/gal mud is being circulated at the rate of 500 gal/min. through a tri-cone bit having three 3/8 in. diameter jets. What is the pressure drop across the bit?

Solution 3.2





$$d_e \text{ or } d = \sqrt{3(\tfrac{3}{8})^2} = 0.65 \text{ in. (equivalent single nozzle diameter)}$$

Using Eq. (3.13):

$$(p_1 - p_2) \text{ or } \Delta p = \frac{(500)^2(10)}{(7430)(0.95)^2(0.65)^4} = 2{,}100 \text{ psi}$$

Pressure Drop Calculations for a Typical Systems

Example 3.3

Operating Data

Depth = 6,000 ft (5,500 ft drill pipe, 500 ft drill collars)
Drill pipe = 4 ½-in. internal flush, 16.6 lb/ft (ID = 3.826 in.)
Drill collars = 6 ¾ in. (ID = 2.813 in.)
Mud density, ρ_m = 10 lb/gal
Plastic viscosity, μ_p = 30 cp
Bingham yield, Y_b = 10 lb/100ft^2
Bit = 7 7/8-in., 3 cone, jet rock bit
Nozzle velocity required = at least 250 ft/sec through each nozzle (this value is obtained by a commonly applied rule of thumb). Assume C = 0.95
Surface equipment type = 2

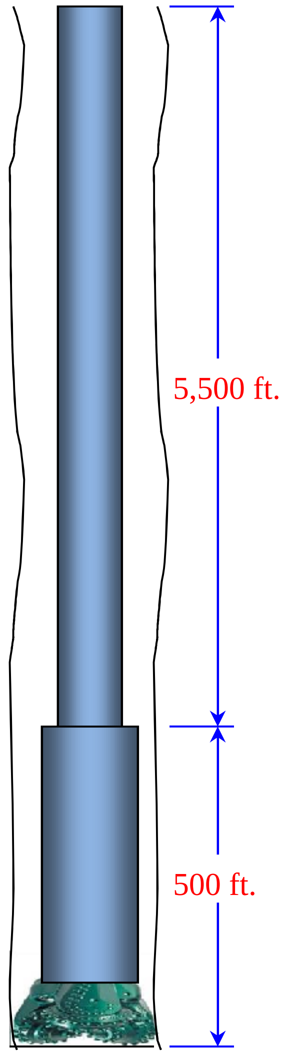
What hydraulic (pump output) horsepower will be required for these conditions?

Gbr ni tak perlu ubah

Solution 3.3

Circulation rate: This is obtained from the desired annular velocity necessary for proper hole cleaning (cutting removal).

Assume that this is a fast drilling, soft rock area and that 180 ft/min (3 ft/sec) upward velocity based on a gauge hole is required (i.e. annular velocity around the drill pipe).



The flow rate , q is:

$$
\begin{aligned}
q &= (\text{annulus area}) \times \text{velocity} \\
&= 2.45(d_h^2 - d_p^2)\bar{v} \\
&= 2.45\left[\left(7\tfrac{7}{8}\right)^2 - \left(4\tfrac{1}{2}\right)^2\right](3) \\
&= 307 \text{ gpm}
\end{aligned}
$$

(a) Surface equipment losses (Δp_s)

Eq. (3.16) $\Delta p_s = E\rho_m^{0.8} q^{1.8} \mu_p^{0.2}$

Surface equipment type 2 —Table 3.1→ $E = 9.6 \times 10^{-5}$

$$\therefore \quad \Delta p_s = (9.6 \times 10^{-5})(10)^{0.8}(307)^{1.8}(30)^{0.2} = 36 \text{ psi}$$

(b) Pressure losses inside drill pipe (Δp_p)

The average velocity inside the drill pipe:

$$\bar{v} = \frac{q}{2.45 d^2} = \frac{307}{2.45(3.826)^2} = 8.56 \text{ ft/sec}$$

The critical velocity:

$$v_c = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 9.3\rho_m d^2 Y_b}}{\rho_m d}$$

$$= \frac{1.08(30) + 1.08\sqrt{(30)^2 + (9.3)(10)(3.826)^2(10)}}{(10)(3.826)}$$

$$= 4.25 \text{ ft/sec}$$

$\bar{v} > v_c \quad \Rightarrow \quad \therefore$ turbulent flow (use Eq. 3.4)

$$N_{\mathrm{Re}} = \frac{2{,}970\rho\bar{v}d}{\mu_{\mathrm{p}}} = \frac{(2{,}970)(10)(8.58)(3.826)}{30} = 32{,}423 \cong 32{,}400$$

$$\left.\begin{array}{l} N_{\mathrm{Re}} = 32{,}400 \\ \text{Curve II} \end{array}\right\} \xrightarrow{\text{Fig. 7.1}} f = 0.0066$$

Applying Eq. (3.4):

$$\Delta p_{\mathrm{p}} = \frac{f\rho L\bar{v}^2}{25.8\,d} = \frac{(0.0066)(10)(5{,}500)(8.56)^2}{(25.8)(3.826)} = 269\ \text{psi}$$

(c) Pressure losses inside drill collar (Δp_c)

The average velocity inside the drill collar:

$$\bar{v} = \frac{q}{2.45d^2} = \frac{307}{2.45(2.813)^2} = 15.84 \text{ ft/sec}$$

The critical velocity:

$$v_c = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 9.3\rho_m d^2 Y_b}}{\rho_m d}$$

$$= \frac{1.08(30) + 1.08\sqrt{(30)^2 + (9.3)(10)(2.813)^2(10)}}{(10)(2.813)}$$

$$= 4.64 \text{ ft/sec}$$

$\bar{v} > v_c \quad \Rightarrow \quad \therefore$ turbulent flow (use Eq. 3.4)

$$N_{\text{Re}} = \frac{2{,}970\rho\bar{v}d}{\mu_{\text{p}}} = \frac{(2{,}970)(10)(15.84)(2.813)}{30} = 44{,}112 \cong 44{,}100$$

$$\left.\begin{array}{l} N_{\text{Re}} = 44{,}100 \\ \text{Curve II} \end{array}\right\} \xrightarrow{\text{Fig. 7.1}} f = 0.0062$$

Applying Eqn. (3.4):

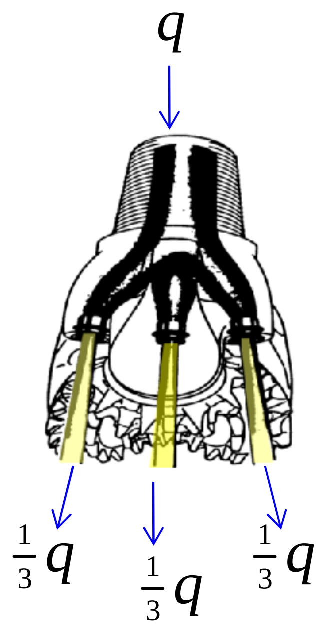
$$\Delta p_c = \frac{f\rho L\bar{v}^2}{25.8\,d} = \frac{(0.0062)(10)(500)(15.84)^2}{(25.8)(2.813)} = 107 \text{ psi}$$

(d) ***Pressure losses through bit*** *(Δp_b)*

Three nozzles (one for each cone) will be used, hence *1/3 q* will flow through each. For $\bar{v}$ = at least 250 ft/sec through each nozzle,

$$d = \sqrt{\frac{\frac{1}{3}q}{2.45\bar{v}}} = \sqrt{\frac{307/3}{(2.45)(250)}} = 0.41 \text{ in.}$$

compare



Nozzle sizes are sell in multiples of 1/32 in. Therefore, the nearest stock nozzle available is 13/32 in. (i.e. 0.40625 in.):

$\therefore$ nozzle diameter of $\frac{13}{32}$ in. is chosen

This nozzle allows an actual velocity of:

$$\bar{v} = \frac{102}{2.45(\frac{13}{32})^2} = 252 \text{ ft/sec}$$

Eq. (3.15a) $d_e = \sqrt{nd^2}$

Eq. (3.15b) $d_e = \sqrt{ad_1^2 + bd_2^2 + \text{ etc.}}$

Using Eq. (3.15) or (3.15a), the actual nozzle diameter:

$$d = \sqrt{3\left(\tfrac{13}{32}\right)^2} = 0.704 \text{ in.}$$

Eq. (3.13) $\Delta p_b = \dfrac{q^2 \rho_m}{7,430 C^2 d^4}$

$\therefore$ Pressure drop across the bit, Δp_b :

$$\Delta p_b = \frac{(307)^2 (10)}{7,430 (0.95)^2 (0.704)^4} = 573 \text{ psi}$$

(e) Pressure losses around drill collar (Δp_{ac})

The average velocity around the drill collar:

$$\bar{v} = \frac{307}{(2.45)\left[(7\tfrac{7}{8})^2 - (6\tfrac{3}{4})^2\right]} = 7.62 \text{ ft/sec}$$

The hydraulically equivalent diameter of the annulus:

$$d_a = d_1 - d_2$$
$$d = 7\tfrac{7}{8} - 6\tfrac{3}{4} = 1\tfrac{1}{8} \text{ in.}$$

The critical velocity:

$$v_c = \frac{1.08(30) + 1.08\sqrt{(30)^2 + (9.3)(10)(1\tfrac{1}{8})^2(10)}}{(10)(1\tfrac{1}{8})} = 7.26 \text{ ft/sec}$$

$\bar{v} > v_c \quad \Rightarrow \quad \therefore$ turbulent flow (use Eq. 3.4)

$$N_{\text{Re}} = \frac{2,970 \rho \bar{v} d}{\mu_p} = \frac{(2,970)(7.62)(1\frac{1}{8})}{30} = 8,487 \cong 8,500$$

$N_{\text{Re}} = 8,400$

Curve IV (for annuli in uncased hole)

$\xrightarrow{\text{Fig. 7.1}}$ $f = 0.0098$

Applying Eqn. (3.4):

$$\Delta p_{ac} = \frac{f \rho L \bar{v}^2}{25.8\, d} = \frac{(0.0098)(10)(500)(7.62)^2}{(25.8)(1\frac{1}{8})} = 98 \text{ psi}$$

(f) Pressure losses around drill pipe (Δp_{ap})

The average velocity around the drill collar (as assume/given earlier):

$$\bar{v} = 3 \text{ ft/sec}$$

The hydraulically equivalent diameter of the annulus:

$$d_a = d_1 - d_2$$
$$d = 7\tfrac{7}{8} - 4\tfrac{1}{2} = 3\tfrac{3}{8} \text{ in.}$$

The critical velocity:

$$v_c = \frac{1.08(30) + 1.08\sqrt{(30)^2 + (9.3)(10)(3\tfrac{3}{8})^2(10)}}{(10)(3\tfrac{3}{8})} = 4.39 \text{ ft/sec}$$

$\bar{v} < v_c \;\Rightarrow\; \therefore$ laminar flow (use Eq. 3.6)

$$\Delta p = \frac{L}{300d}\left(Y_b + \frac{\mu_p \bar{v}}{5d}\right)$$

$$\therefore\ \Delta p_{ap} = \frac{5,500}{300(3\tfrac{3}{8})}\left[10 + \frac{30(3)}{5(3\tfrac{3}{8})}\right] = 83 \text{ psi}$$

(g) The total pressure drop in the system (Δp_t)

$$\Delta p_t = 36 + 269 + 107 + 573 + 98 + 83 \cong 1{,}166 \text{ psi}$$

(h) Horsepower output at the pump

$$HP = \frac{q \times \Delta p}{1{,}714 \times \eta_v \times \eta_m} \quad \text{.......... (3.17)}$$

where:

q = flow rate, gpm

η_v = volumetric efficiency

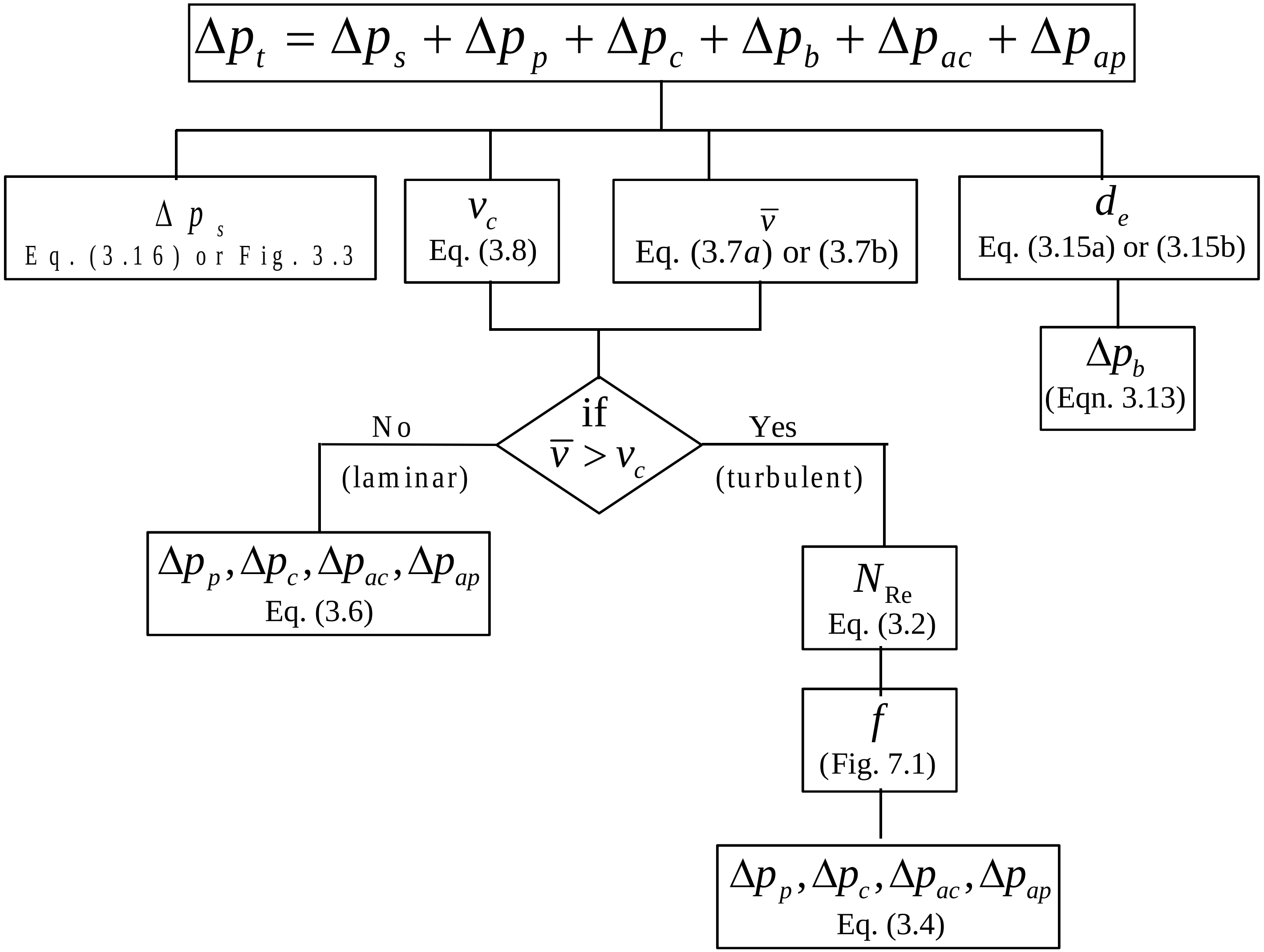
η_m = mechanical efficiency

Assuming volumetric and mechanical efficiencies of the pump are 90% and 85% respectively:

$$\therefore\ HP = \frac{307\,(1{,}166)}{1{,}714(0.90)(0.85)} = 273 \textit{ horsepower}$$

Summary

Bingham Plastic Model: Calculation Steps



$$\Delta p = \Delta p_u \times \frac{\rho_m}{9.5} \times \left[\frac{\mu_p}{3.2(3)}\right]^{0.14}$$

Example 3.4

Using a data as in Example 3.3, calculate the circulating pressure required.

Solution 3.4

From Example 3.3: q = 307 gpm, bit = 3 13/32 in. nozzles

(a) Surface equipment losses (Δp_s)

q = 307 gpm
Curve type 2

Fig. 7.3 → $\Delta p_u = 27$ psi

$$\therefore \ \Delta p_s = 27 \times \frac{10}{9.5} \times \left[\frac{30}{3.2(3)}\right]^{0.14} = 33 \text{ psi}$$

(b) Pressure losses inside drill pipe (Δp_p)

q = 307 gpm
Curve 7

Fig. 7.5 (for 4.5" d/p) (assume ID = 3 ¾") → $\Delta p_u = \frac{32}{1{,}000} \times 5{,}500 = 176$ psi

$$\therefore \ \Delta p_p = 176 \times \frac{10}{9.5} \times \left[\frac{30}{3.2(3)}\right]^{0.14} = 217 \text{ psi}$$

$$\Delta p = \Delta p_u \times \frac{\rho_m}{9.5} \times \left[\frac{\mu_p}{3.2(3)}\right]^{0.14}$$

(c) Pressure losses inside drill collar (Δp_c)

q = 307 gpm
Curve 2 ¾ bore

Fig. 7.7 → (assume ID = 2 ¾")

$$\Delta p_u = \frac{15}{100} \times 500 = 75 \text{ psi}$$

$$\therefore \; \Delta p_c = 75 \times \frac{10}{9.5} \times \left[\frac{30}{3.2(3)}\right]^{0.14} = 93 \text{ psi}$$

$$\Delta p = \Delta p_u \times \frac{\rho_m}{9.5}$$

(d) Pressure losses through bit (Δp_b)

q = 307 gpm
$3 - \frac{13}{32}''$ nozzle

Fig. 7.9 → (no viscosity effect)

$$\Delta p_u = 550 \text{ psi}$$

$$\therefore \; \Delta p_b = 550 \times \frac{10}{9.5} = 579 \text{ psi}$$

$$\Delta p = \Delta p_u \times \frac{\rho_m}{9.5} \times \left[\frac{\mu_p}{3.2(3)}\right]^{0.14}$$

(e) Pressure losses around drill collar (Δp_{ac})

q = 307 gpm
6 ¾ drill collar

Fig. 7.10 → (bit size = 7 7/8")

$$\Delta p_u = \frac{25}{100} \times 500 = 125 \text{ psi}$$

$$\therefore \ \Delta p_{ac} = 125 \times \frac{10}{9.5} \times \left[\frac{30}{3.2(3)}\right]^{0.14} = 154 \text{ psi}$$

(f) Pressure losses around drill pipe (Δp_{ap})

q = 307 gpm
4 ½ drill pipe

Fig. 7.10 → (bit size = 7 7/8")

$$\Delta p_u = \frac{1.4}{100} \times 5{,}500 = 77 \text{ psi}$$

$$\therefore \ \Delta p_{ap} = 77 \times \frac{10}{9.5} \times \left[\frac{30}{3.2(3)}\right]^{0.14} = 95 \text{ psi}$$

(g) The total pressure drop in the system (Δp_t)

$$\Delta p_t = 33 + 217 + 107 + 579 + 154 + 95 \cong 1{,}185 \text{ psi}$$

Comparison of Δp Calculation Methods

System component	Plastic flow calculation (psi)	Hughes Tools Co. charts (psi)
Surface connections, Δp_s	**36**	**33**
Inside drill pipe, Δp_p	**269**	**217**
Inside drill collar, Δp_c	**107**	**107**
Bit nozzles, Δp_b	**573**	**579**
Outside drill collar, Δp_{ac}	**98**	**154**
Outside drill pipe, Δp_{ap}	**83**	**95**
Total circulating pressure, Δp_t	**1,166**	**1,185**

Additional Information

- Besides Newtonian and Bingham Plastic Models, there are several other model used to predict pressure losses in mud circulating systems.
- Generally, each model is based on a set of assumptions which cannot be completely fulfilled in any drilling situation.
- Power law, Herschel-Bulkley (Yield Power Law @ API Power Law) models are the most widely used in the oil industry.
- Table 3.3 shows a summary of pressure loss equations