Prestressed Concrete Design

(SAB 4323)

Design for Ultimate Strength in Shear

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Introduction

- The behaviour of prestressed concrete beams at failure in shear is distinctly different from their behaviour in flexure
- The beams fail abruptly without sufficient warning, and the diagonal cracks that develop are considerably wider than the flexural cracks
- Shear forces result in shear stress. Such a stress can result in principal tensile stresses at the critical section which can exceed the tensile strength of the concrete.
- When the tensile strength of the concrete is exceeded, cracks will form

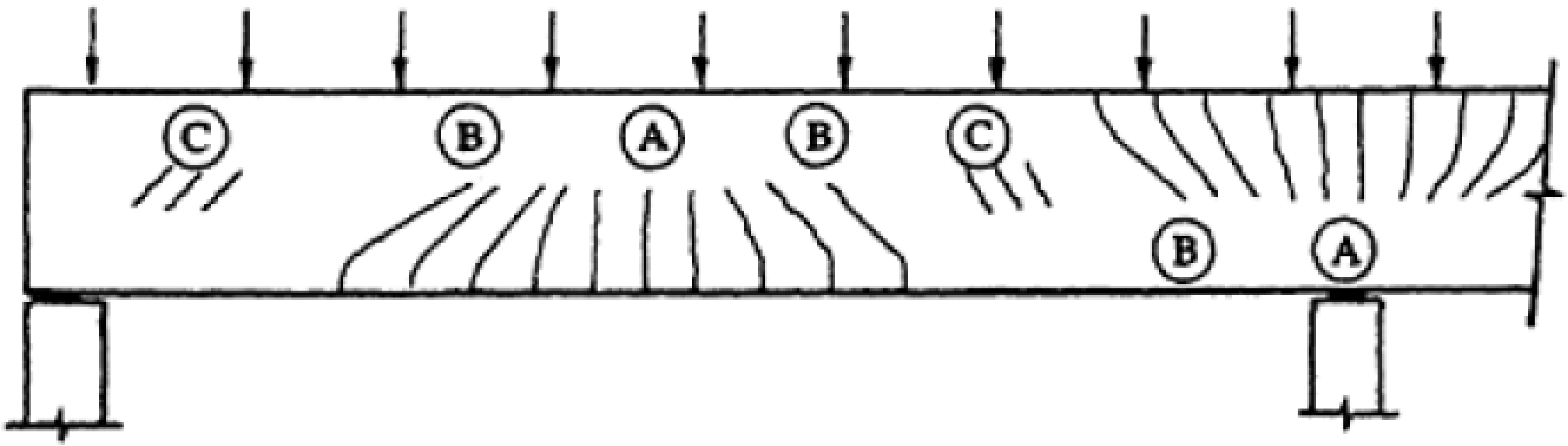
Cracking Patterns & Failure Modes

- Cracking in prestressed concrete beams at ultimate load depends on the local magnitudes of moment and shear as shown on the next two slides
- In regions where the moment is large and the shear is small, vertical flexural cracks appear after the normal tensile stress in the extreme concrete fibres exceeds the tensile strength of concrete. These are the cracks shown as type A
- Where both the moment and shear force are relatively large, flexural cracks which are vertical at the extreme fibres become inclined as they extend deeper into the beam owing to the presence of shear stresses in the beam web. These inclined cracks, which are often quite flat in a prestressed beam, are called flexure-shear cracks and are designated crack type B

Cracking Patterns & Failure Modes

- If adequate shear reinforcement is not provided, a flexure-shear crack may lead to a so-called shear-compression failure, in which the area of concrete in compression above the advancing inclined crack is so reduced as to be no longer adequate to carry the compression force resulting from flexure.
- A second type of inclined crack sometimes occurs in the web of a prestressed beam in the regions where moment is small and shear is large, such as the cracks designated type C adjacent to the discontinuous support and near the point of contraflexure in the Figure.
- In such locations, high principal tensile stress may cause inclined cracking in the mid-depth region of the beam before flexural cracking occurs in the extreme fibres. These cracks are known as web-shear cracks and occur most often in beams with relatively thin webs.

Cracking Patterns & Failure Modes



Region A - Flexural cracks (*M* /*V* is high) — ULS in Flexure!

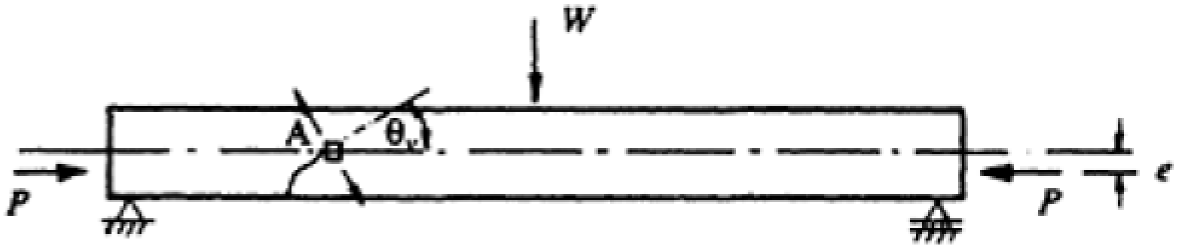
Cracked Section → Region B - Flexure-shear cracks (*M* /*V* is moderate) — ULS in Shear!

Unracked Section → Region C - Web-shear cracks (*M* /*V* is low)

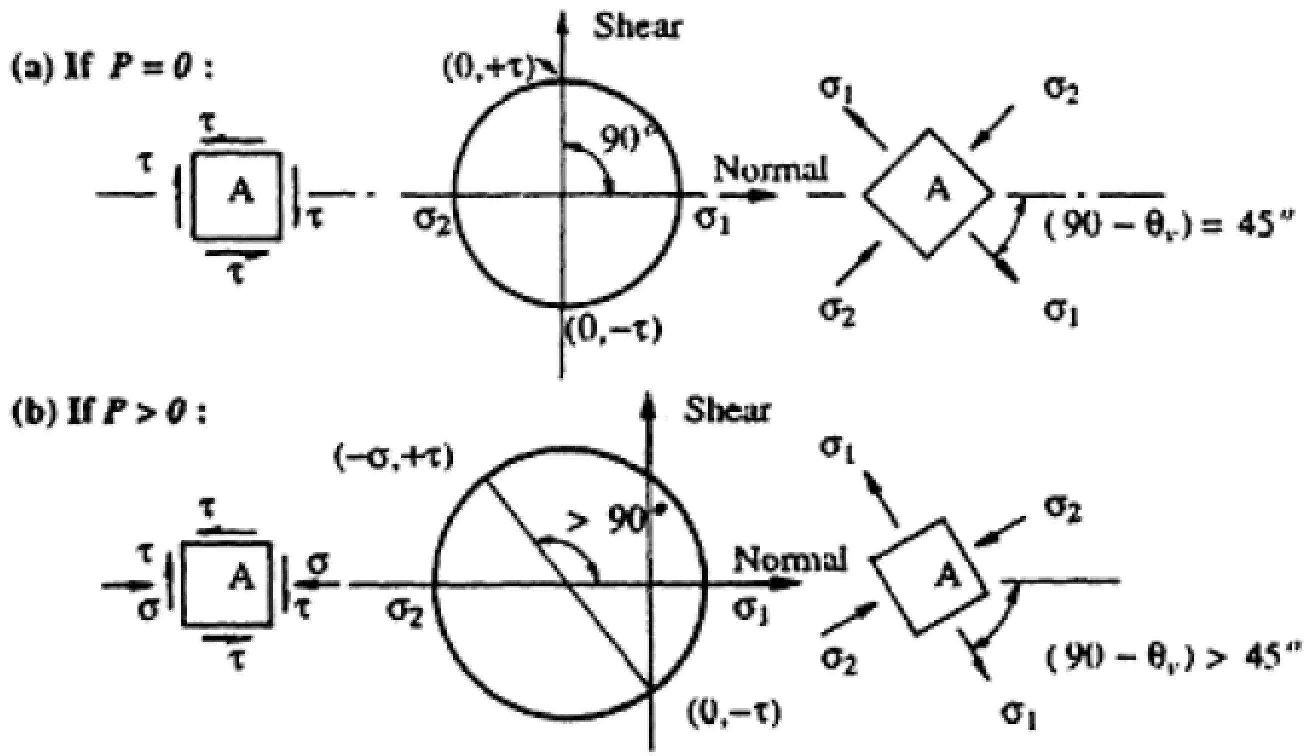
Effect of Prestressing in Shear

- The longitudinal compression introduced by prestress delays the formation of each of the crack types shown previously. The effect of prestress on the formation and direction of inclined cracks can be seen by examining the stresses acting on a small element located at the centroidal axis of the uncracked beam as shown on the next slide
- Using a simple Mohr's circle construction, the principal stresses and their directions are readily found.
- When the principal tensile stress $\sigma 1$ reaches the tensile strength of concrete, cracking occurs and the cracks form in the direction perpendicular to the direction of $\sigma 1$.

Effect of Prestressing in Shear



At A: $\sigma = -\frac{P}{A}$ and $\tau = \frac{VQ}{Ib}$ = V A$\bar{y}$/Ib



Effect of Prestressing in Shear

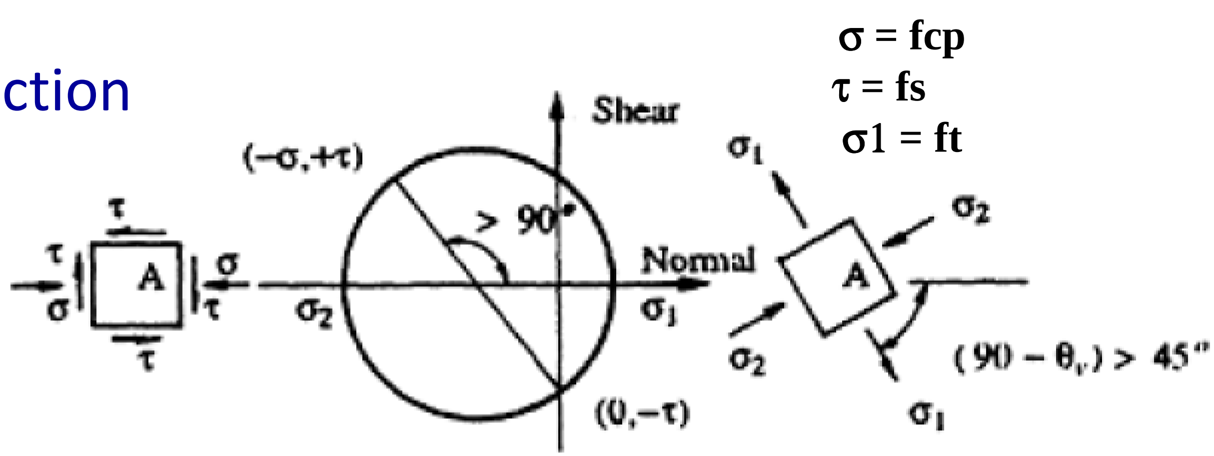
- When the prestress is zero, σ1 is equal to the shear stress and acts at 45° to the beam axis. If diagonal cracking occurs, it will be perpendicular to the principal tensile stress, i.e. at 45° to the beam axis
- When the prestress is not zero, the normal compressive stress σ(=P/A) reduces the principal tension σ1. The angle between the principal stress direction and the beam axis increases, and consequently if cracking occurs, the inclined crack is flatter. Prestress therefore improves the effectiveness of any transverse reinforcement (stirrups) that may be used to increase the shear strength of a beam.
- With prestress causing the inclined crack to be flatter, a larger number of the vertical stirrup legs are crossed by the crack and consequently a larger tensile force can be carried across the crack.

Uncracked Sections (Web Shear)

- Figure below shows the small element A at the centroid of a simply supported prestressed concrete member subjected to a compresive stress, fcp and a shear stress, fs.
- fs = V_{co}.Ay/Ib and from Mohr Circle, ft = -fcp/2 + $(fcp^2/4 + fs^2)^{0.5}$
- V_{co} = (Ib/Ay)$(ft^2 + ftfcp)^{0.5}$
- The allowable principal tensile stress is given in BS 8110 by

  ft = $0.24fcu^{0.5}$
- For a rectangular section

  Ib/Ay = 0.67bh



Uncracked Sections (Web Shear)

- $V_{co} = (Ib/Ay)(ft^2 + ftfcp)^{0.5}$
- $V_{co} = 0.67b_v h(ft^2 + 0.8fcpft)^{0.5}$

where

V_{co} – design ultimate shear resistance of a section uncracked in flexure

b_v – breadth of the member or for T-, I- and L-beams, width of the web

if grouted duct is present in the web,

$b_v = b_w - 0.67d_d$ (d_d – diameter of duct)

$ft = 0.24fcu^{0.5}$

Cracked Sections (Flexural Shear)

- Clause 4.3.8.5 of BS 8110 gives the following empirical equation for the ultimate shear resistance of a section cracked in flexure:

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_{pu}}\right) v_c b_v d + M_o \frac{V}{M}$$

M_o – moment which produces zero stress at extreme tension fibre;
M_o = 0.8fptI/y
where fpt is the level of prestress in concrete at the tensile face

The value of V_{cr} should be taken as not less than $0.1 b_v d \sqrt{f_{cu}}$.

$$v_c = 0.79\{100A_s/(b_v d)\}^{1/3} (400/d)^{1/4}/\gamma_m \quad (\gamma_m = 1.25.)$$

where

$\frac{100A_s}{b_v d}$ should not be taken as greater than 3;

As – sum of area of prestressing steel and non-prestressing steel
d –depth from compression face to centroid of total steel

$\left(\frac{400}{d}\right)^{\frac{1}{4}}$ should not be taken as less than 0.67 for members without shear reinforcement;

$\left(\frac{400}{d}\right)^{\frac{1}{4}}$ should not be taken as less than 1 for members with shear reinforcement providing a design shear resistance of ≥ 0.4 N/mm^2.

For characteristic concrete strengths greater than 25 N/mm^2, the values in this table may be multiplied by $(f_{cu}/25)^{1/3}$.
The value of f_{cu} should not be taken as greater than 40.

Design Ultimate Shear Resistance, V_c

- According to Clause 4.3.8.3 BS 8110,

  $V_c = V_{co}$ at uncracked section ($M < M_o$)

  V_c = is the smaller of V_{co} and V_{cr} at cracked section ($M >= M_o$)
- For deflected tendon, the vertical component of the prestressing force will help to resist the shear force.
- The total shear resistance then becomes:

  $V_c + P_e \sin \alpha$ where α is the angle of inclination of the prestressing tendon
- For Parabolic Profile, $e(x) = (- 4\delta/L^2)x^2 + (4\delta/L)x$

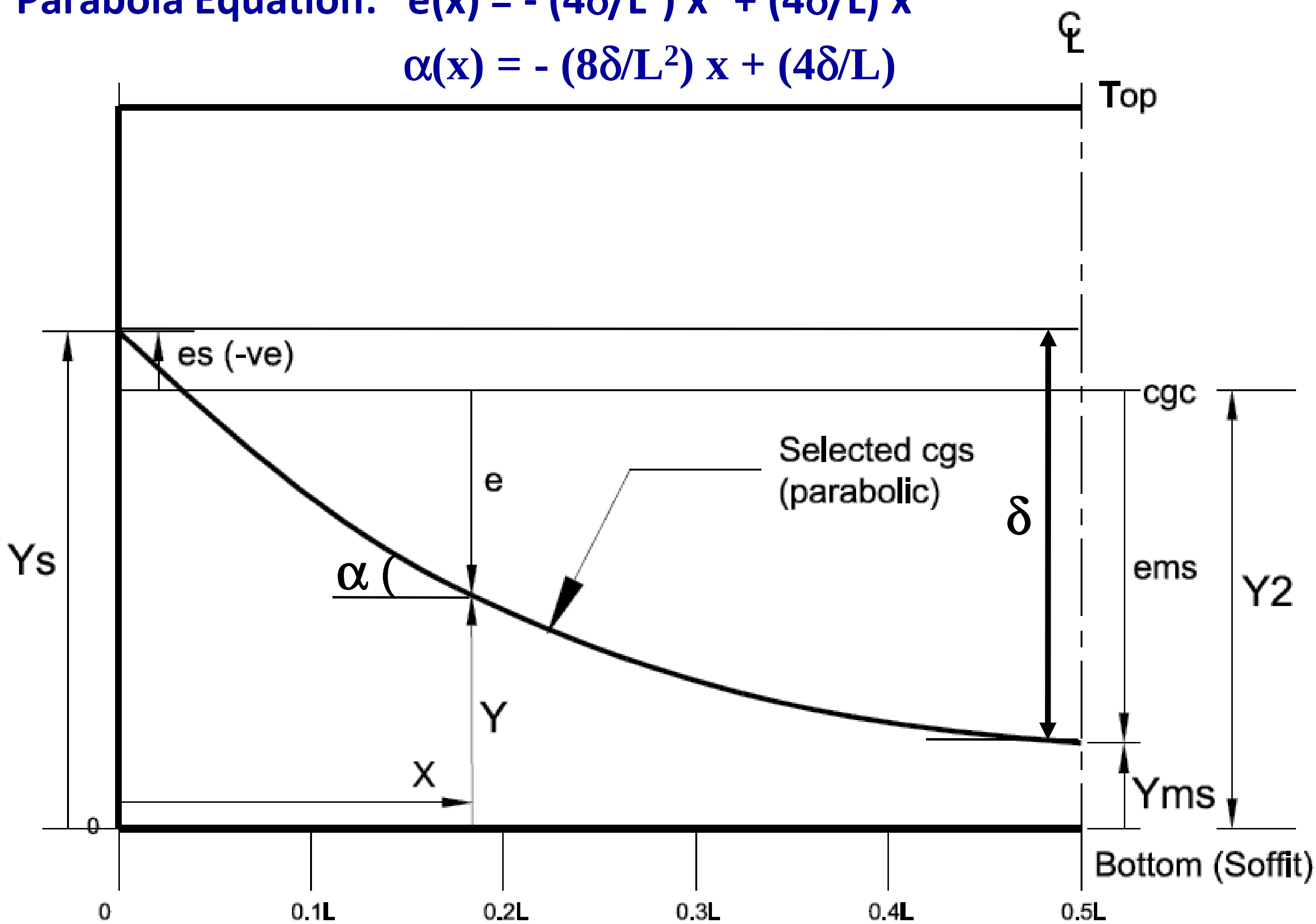
  $\alpha(x) = (-8d/L^2)x + (4\delta/L)$ in radian

  where $\delta = abs(es – ems)$

Parabolic Profile

Parabola Equation: $e(x) = -(4\delta/L^2)\,x^2 + (4\delta/L)\,x$

$\alpha(x) = -(8\delta/L^2)\,x + (4\delta/L)$



Shear Design Steps

1. Draw the bending moment and shear force diagram
2. Check maximum allowable shear stress (Clause 4.3.8.2)

   v (= $V/b_v d$) <= the lessser of $0.8fcu^{0.5}$ or 5 N/mm^2
3. Plot M_o on the BMD. M_o will varies along the span if the tendon profile is not straight. Determine the cracked and uncracked region.

   ($M >= M_o$) ($M < M_o$)
4. Calculate V_{co} and V_{cr} .
5. Determine the ultimate shear resistance of the prestressed beam as follows:

   Cracked Region: $V_c = V_{cr}$ or $V_{co} + P_e \sin\alpha$

   Uncracked Region: $V_c = V_{co} + P_e \sin\alpha$
6. If V <= 0.5 V_c , shear reinforcement is not required (Clause 4.3.8.6)
7. If $0.5 V_c < V <= V_c + 0.4 b_v d$, shear reinforcement is required

Shear Design Steps

6. If $V <= 0.5 V_c$, shear reinforcement is not required (Clause 4.3.8.6)
7. If $0.5 V_c < V <= V_c + 0.4b_v d$, shear reinforcement is required (Clause 4.3.8.7). Use Asv/Sv = 0.4bv/0.87fyv
8. If $V > V_c + 0.4b_v d$, shear reinforcement is required (Clause 4.3.8.8). Use $A_{sv}/S_v = (V – V_c)/0.87f_{yv}d_t$

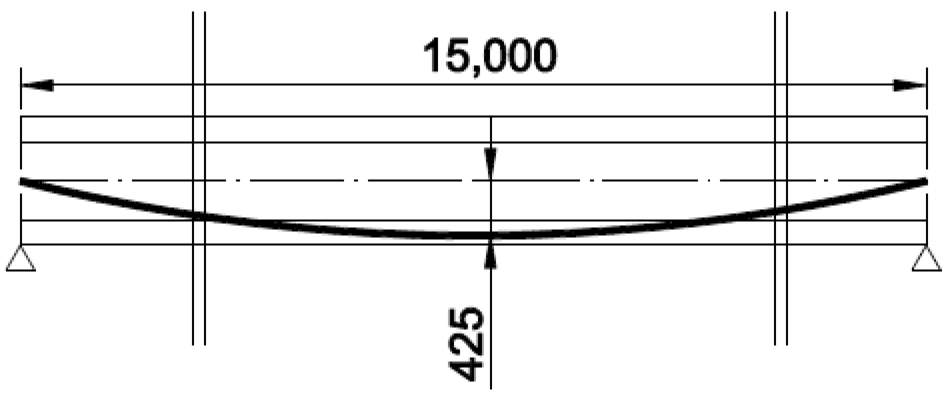
   Where d_t is the depth from the extreme compression fibre either to the longitudinal bars or to the centroid of the tendons, whichever is the greater
9. Sv ≯ the lesser of $0.75d_t$ or 4*web thickness
10. When V > 1.8Vc, the maximum spacing should be reduced to 0.5dt.
11. The lateral spacing of the individual legs of the links provided at a cross-section should not exceed d_t

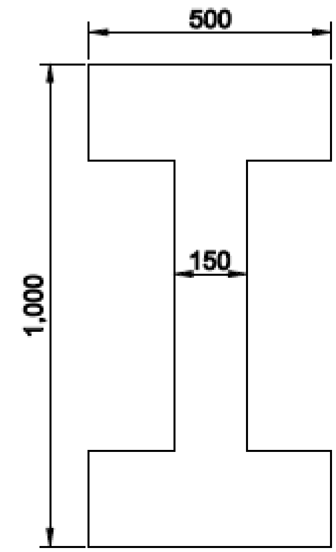
Example 8-1

The beam shown below supports an ultimate load, including self weight of 85kN/m over a span of 15m and has a final prestress force of 2000kN. Determine the shear reinforcement required. Use the following data:

fcu = 40N/mm^2; A = 2.9 x 10^5 mm^2 ; I = 3.54 x 10^{10} mm^4

Aps = 2010 mm^2 ; fpe/fpu = 0.6





Solution

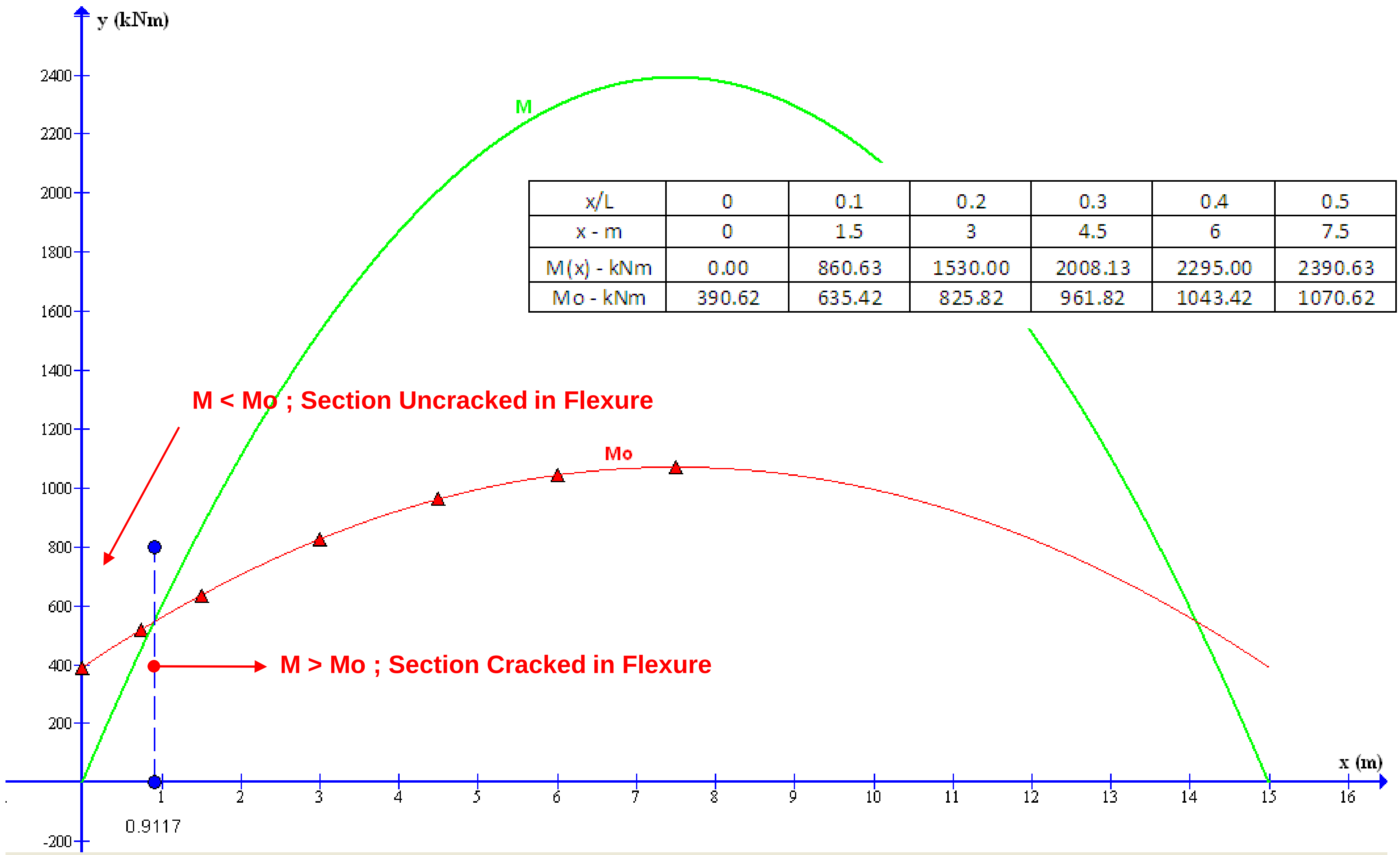
1. Draw BMD and SFD
   - $M(x)=0.5w(Lx-x^2)$ & $V(x)=w(0.5L – x)$ where w = 85 kN/m

x/L	0	0.1	0.2	0.3	0.4	0.5
x - m	0	1.5	3	4.5	6	7.5
M(x) - kNm	0.00	860.63	1530.00	2008.13	2295.00	2390.63
V(x) - kN	637.5	510	382.5	255	127.5	0

2. Check maximum allowable shear stress (Clause 4.3.8.2)
   - V = 637.5 kN, $v = 637.5*10^3/(500*500) = 2.55$ N/mm²
   - vmax = lesser of ($0.8*40^{0.5} = 5.06$N/mm² and 5N/mm²) ➔ ok
3. Plot Mo on BMD.
   - $Mo = 0.8fpt\ I/y$ & $fpt = P_e/A + P_e\ e/Z_2$
   - $e(x) = (-4\delta/L^2)x^2 + (4\delta/L)x$ where $\delta = 425$mm

Solution

x/L	0	0.1	0.2	0.3	0.4	0.5
x - m	0	1.5	3	4.5	6	7.5
M(x) - kNm	0.00	860.63	1530.00	2008.13	2295.00	2390.63
Mo - kNm	390.62	635.42	825.82	961.82	1043.42	1070.62



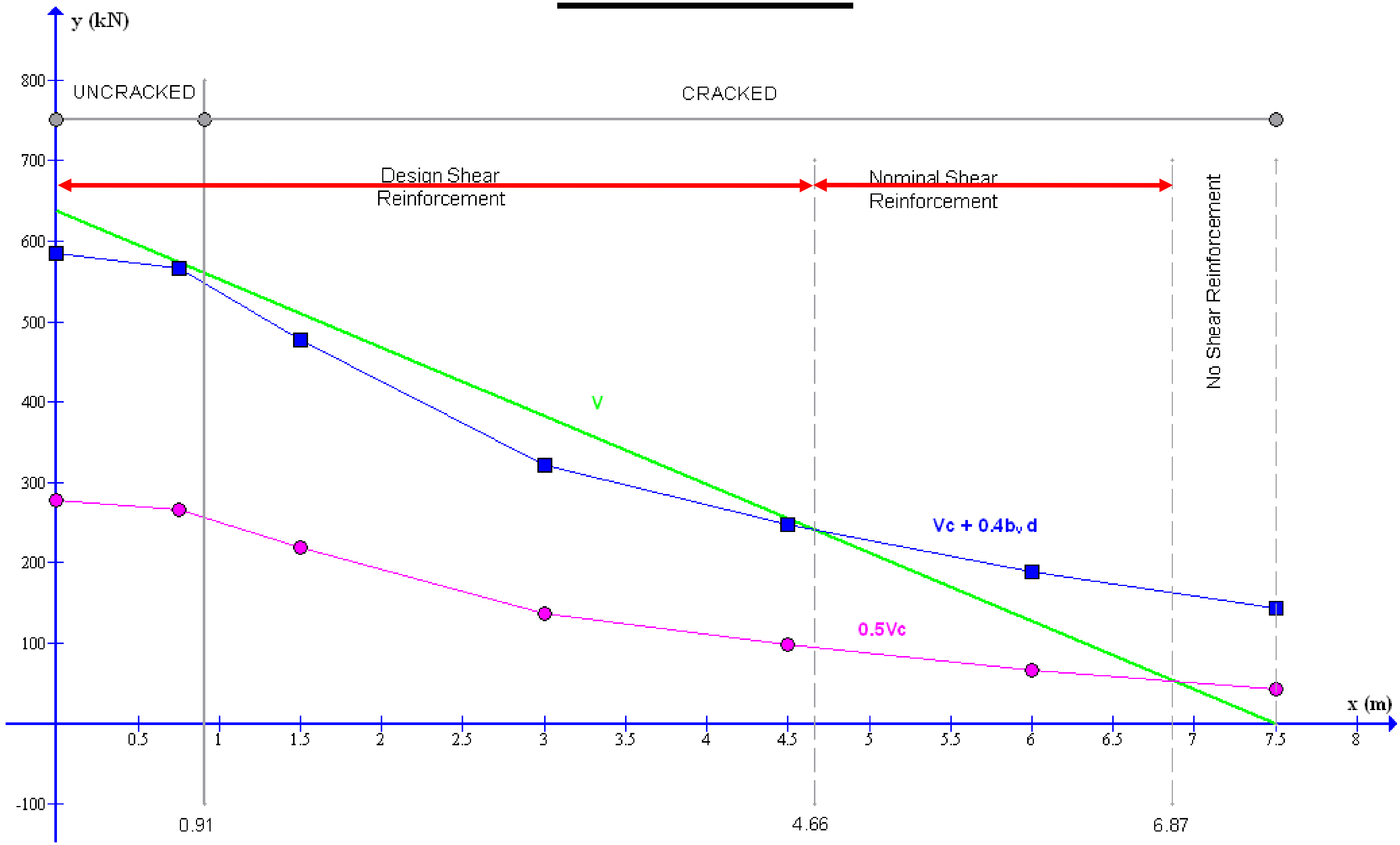
Solution

Steps 4 to 8

Vp = P_eSin α

x/L	0	0.05	C/UC	0.1	0.2	0.3	0.4	0.5
x - m	0	0.75	0.9117261	1.5	3	4.5	6	7.5
M(x) - kNm	0.00	454.22	545.90	860.63	1530.00	2008.13	2295.00	2390.63
V(x) - kN	637.5	573.75	560.00	510	382.5	255	127.5	0
e(x) - mm	0	80.75	97.05	153	272	357	408	425
α(x) - rad	0.1133	0.1020	0.0996	0.0907	0.0680	0.0453	0.0227	0.0000
α(x) - deg	6.4935	5.8442	5.7041	5.1948	3.8961	2.5974	1.2987	0.0000
dt - mm	500	580.75	597.05	653	772	857	908	925
(100As/bd)^1/3	1.39	1.32	1.31	1.27	1.20	1.16	1.14	1.13
(400/d)^1/4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(fcu/25)^1/3	1.17	1.17	1.17	1.17	1.17	1.17	1.17	1.17
vc - N/mm^2	1.03	0.98	0.97	0.94	0.89	0.86	0.84	0.84
fpt - N/mm^2	6.90	9.18	9.64	11.22	14.58	16.98	18.42	18.90
Mo - kNm	390.62	519.82	545.90	635.42	825.82	961.82	1043.42	1070.62
Vco - kN	328.41	328.41	328.41	328.41	328.41	328.41	328.41	328.41
Vcr - kN	#########	713.63	618.08	438.19	275.38	196.03	134.77	87.75
Vco + Vp	554.60	532.06	527.20	509.50	464.31	419.05	373.74	328.41
Vc	554.60	532.06	527.20	438.19	275.38	196.03	134.77	87.75
Vc+0.4bvd	584.60	566.91	563.02	477.37	321.70	247.45	189.25	143.25
0.5Vc	277.30	266.03	263.60	219.10	137.69	98.02	67.38	43.88

Solution



y (kN)
x (m)
UNCRACKED
CRACKED
Design Shear Reinforcement
Nominal Shear Reinforcement
No Shear Reinforcement
V
Vc + 0.4b$_v$ d
0.5Vc
800
700
600
500
400
300
200
100
-100
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
0.91
4.66
6.87

Solution

Nominal Shear Reinforcement

Use R8 ; Asv = $2*0.25*\pi*8^2 = 101\ mm^2$

Sv = 101*0.87*250/ 0.4*150 = 366 mm

Sv mak = 0.75d = 0.75*908 = 681 mm

∴ Use R8 – 350 mm

Design Shear Reinforcement

Use R8 ; Asv = $2*0.25*\pi*8^2 = 101\ mm^2$

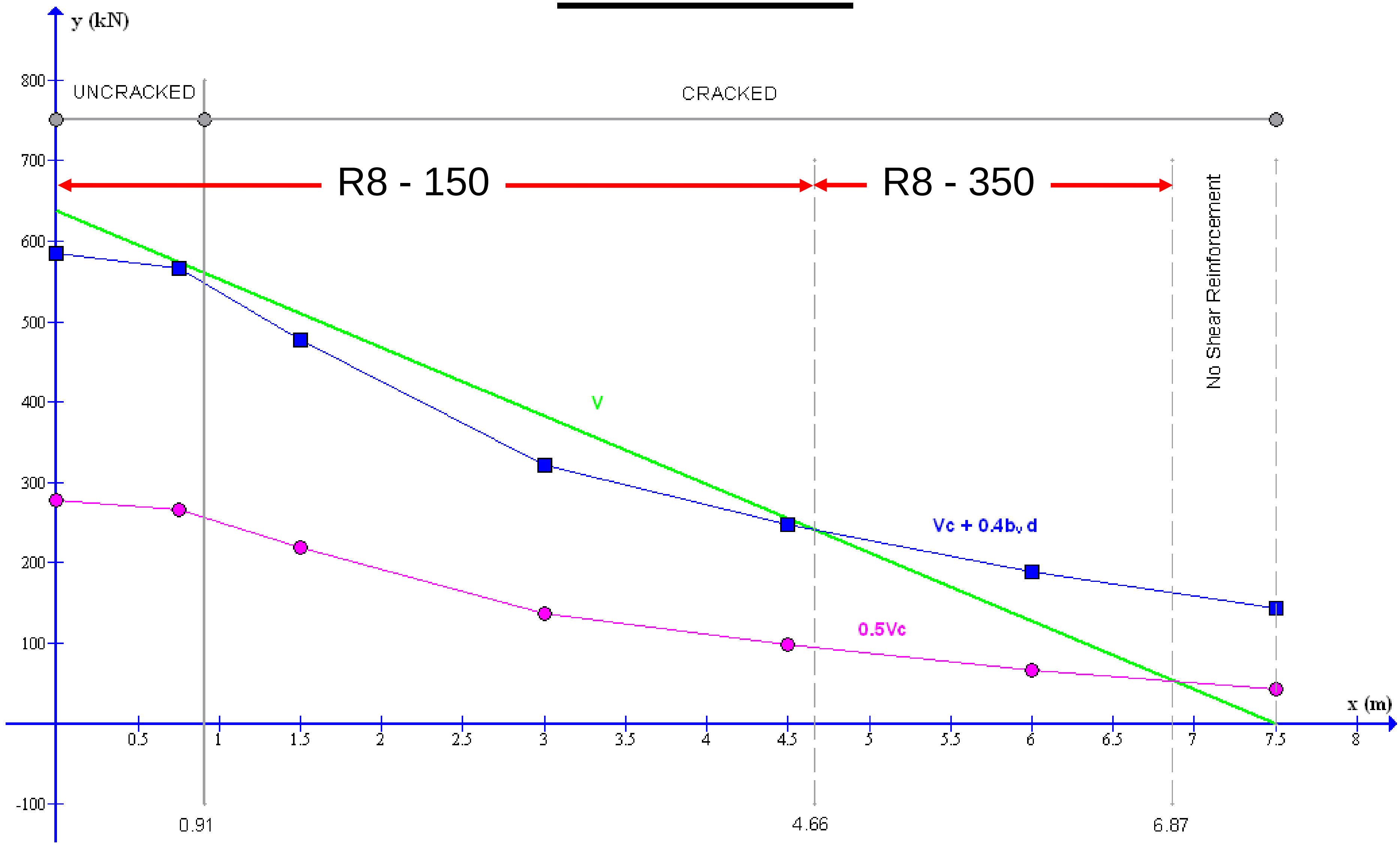
V - Vc = 107.12 kN

Sv = 101*0.87*250*772/ (107.12*1000) = 158 mm

Sv mak = 0.75d = 0.75*772 = 579 mm

∴ Use R8 – 150 mm

Solution



y (kN)
UNCRACKED
CRACKED
R8 - 150
R8 - 350
No Shear Reinforcement
V
Vc + 0.4bv d
0.5Vc
x (m)
800
700
600
500
400
300
200
100
-100
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
0.91
4.66
6.87

Example 8-2

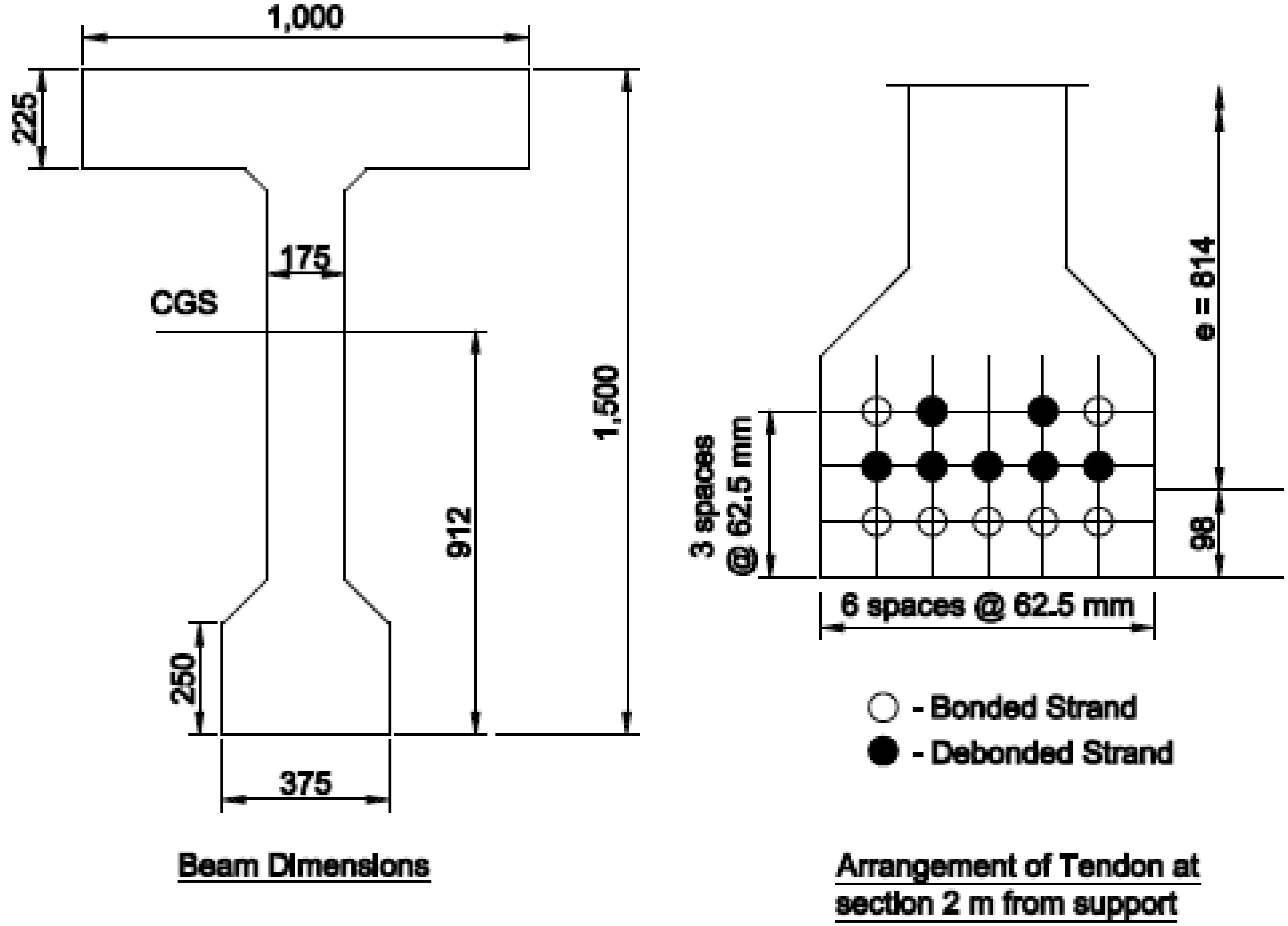
A prestressed concrete T-beam shown on next slide is simply supported over a span of 28m, has been designed to carry, in addition to its own weight, a characteristic dead load of 4 kN/m and a characteristic imposed load of 10 kN/m. The beam is pre-tensioned with 14 Nos 15.7 mm diameter 7-wire super strands (Aps = 150 mm^2) but due to debonding only 7 of the strands are active at a section 2 m from the support. The effective prestressing force for these 7 strands is 1044 kN. Use the following data:

fcu = 50 N/mm^2; A = 5.08 x 10^5 mm^2 ; I = 134 x 10^9 mm^4;

y_2 = 912 mm; fpu = 1770 N/mm^2 ; fyv = 250 N/mm^2

Design the section for shear.

Example 8-2



1,000
225
175
CGS
912
1,500
250
375
Beam Dimensions
e = 814
3 spaces @ 62.5 mm
98
6 spaces @ 62.5 mm
- Bonded Strand
- Debonded Strand
Arrangement of Tendon at section 2 m from support

Solution

1. Loads
   - Self weight, Wsw = 0.508 x 24 = 12.19 kN/m
   - Ultimate Load, Wu = 1.4(12.19 + 4) + 1.6 x 10 = 38.67 kN/m
2. Shear and Moment at 2m from support
   - V = 38.67(0.5 x 28 – 2) = 464 kN
   - M = 0.5 x 38.67(28 x 2 - 2^2) = 1005 kNm
3. Calculate M_o
   - e = 814 mm; d = 1500 – 98 = 1402 mm
   - fpt = (1044×10^3/508000)+(1044×10^3 x 814 x 912 / 134 x 10^9)
   - fpt = 7.84 N/mm^2
   - M_o = 0.8 x 7.84 x 134 x 10^9 / 912 = 922 kNm < M = 1005 kNm
   - ∴ Section is cracked in flexure

Solution

4. Calculation of Vco
   - $V_{co} = 0.67b_v h(ft^2 + 0.8fcpft)^{0.5}$
   - $fcp = 1044 \times 10^3/50800 = 2.06$ N/mm^2
   - $ft = 0.24 \times 50^{0.5} = 1.70$ N/mm^2 ; h = 1500 mm; b_v = 175 mm
   - $V_{co} = 0.67x175x1500(1.70^2 + 0.8x2.06x1.70)^{0.5} /10^3 = 420$ kN
5. Calculation of Vcr
   - $fpe = 1044 \times 10^3 / 7 \times 150 = 994$ N/mm^2
   - 100As/bvd= 100x7x150/175x1402 = 0.43 <= 3 ok
   - 400/d = 400/1402 = 0.29 < 1 use 400/d = 1
   - fcu = 50 N/mm^2 > 40 N/mm^2; Use fcu = 40 N/mm^2
   - $vc = 0.79 \times 0.43^{1/3} \times 1 \times (40/25)^{1/3} /1.25 = 0.556$ N/mm^2

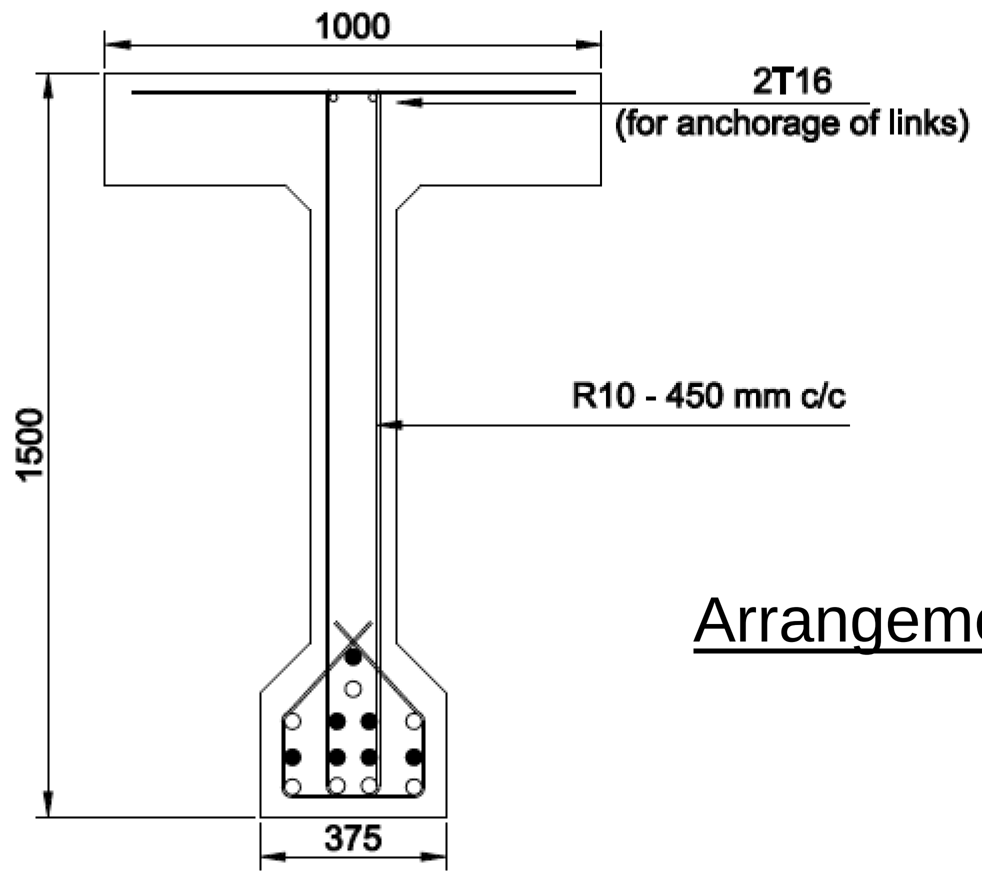
Solution

5. Calculation of Vcr – cont'd
   - $V_{cr} = (1 – 0.55x994/1770)x0.556x175x1402x10^{-3} + 992x464/1005 = 520$ kN
   - $V_{cr} \nless 0.1x175x1402x50^{1/2}x10^{-3} = 174$ kN ok
6. Shear resistance provided by the concrete, Vc
   - Vc = smaller between Vco (420 kN) and Vcr (520 kN)
   - Vc = 420 kN
7. Design of shear reinforcement
   - V = 464 kN > 0.5Vc = 210 kN but < Vc + 0.4bvd = 518 kN
   - ∴ Only nominal links required
   - Asv/Sv = 0.4 x 175 / 0.87 x 250 = 0.322 mm^2/mm

Solution

7. Design of shear reinforcement
   - V = 464 kN > 0.5Vc = 210 kN but < Vc + 0.4bvd = 518 kN
   - ∴ Only nominal links required
   - Asv/Sv = 0.4 x 175 / 0.87 x 250 = 0.322 mm^2/mm
   - Using 10 mm diameter stirrup/links, Asv = 157 mm2
   - Sv = 157/0.322 = 487mm<0.75dt = 0.75(1500–62.5) = 1078mm
   - ∴ Use R10 – 450 mm c/c

Solution



Arrangement of Shear Links

Shear in Composite Beams

Introduction

Need to design for two types of shear

- Horizontal Shear At Interface between Precast and Cast In Situ Concrete
- Vertical Shear

→ Both design at ULS

Horizontal Shear at Interface

- The composite behaviour of precast beam and in situ slab is only effective if the horizontal shear stresses at the interface between the two regions can be resisted
- For shallower members, there is usually no mechanical key between the two types of concrete, and reliance is made on the friction developed between the contact surfaces
- For deeper section, mechanical shear connectors in the form of links projecting from the beam are used
- The determination of the horizontal shear resistance is based on the ultimate limit state, and if this condition is satisfied, it may be assumed that satisfactory horizontal shear resistance is provided at the serviceability limit state

Horizontal Shear Reinforcements

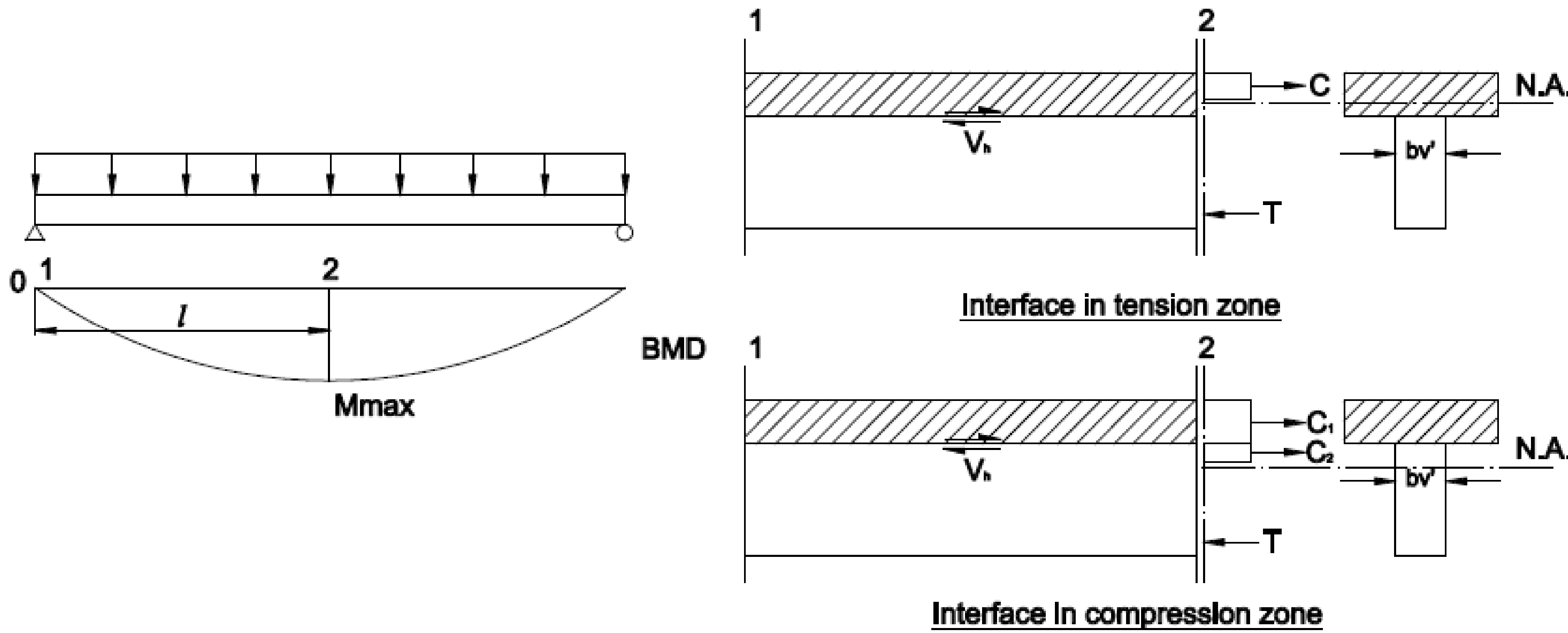
Horizontal Shear Reinforcements

Horizontal Shear Reinforcements

Design Horizontal Shear Force

- To determine the horizontal shear force acting at the interface, the segments of the beam between points of maximum moment and zero moment are considered
- For a particular segment, the neutral axis position and the distribution of stresses at the maximum moment section due to ultimate loads acting on the beam are established by the ultimate strength analysis procedure
- Two situation may arise depending on the location of the neutral axis. As shown on the next slide, the interface may be either in the tension zone or in the compression zone.
- The horizontal shear force, V_h, acting at the interface can then be calculated from horizontal equilibrium of the cast in situ component

Horizontal Shear at Interface



HORIZONTAL SHEAR FORCE

Design Horizontal Shear Force

Interface in the tension zone

- V_h = C or T

Interface in the compression zone

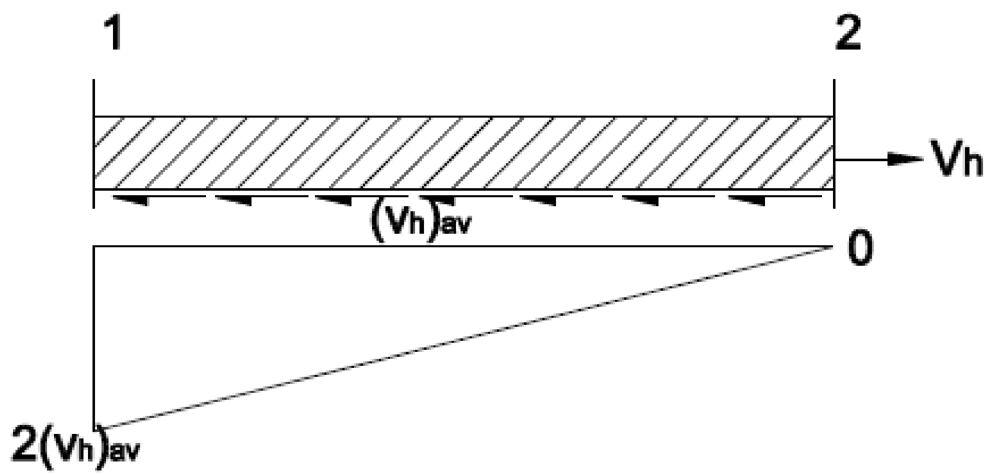
- $V_h = C_1$

The average horizontal shear stress is given as:

- $(v_h)av = V_h / b_v' \, l$
- Where b_v' is the width of contact surface and *l* is the distance between point of maximum moment and the point of zero moment

Horizontal Design Shear Stress at a Section

- The horizontal design shear stress, v_h at a section along the length of the member is obtained by distributing $(v_h)av$ in proportion to the vertical shear force diagram
- For a uniformly distributed load, this stress distribution is linear as shown below



Distribution of design horizontal shear stress for segment 1 - 2

Design for Horizontal Shear

- The allowable ultimate horizontal shear resistance is given below.
- The extent of shear resistance depend on the strength class of in situ concrete and the texture of precast beam at the interface

Table 5.5 — Design ultimate horizontal shear stresses at interface

Precast unit	Surface type	Strength class of in-situ concrete		
		C20/25	C25/30	C32/40 and over
Without links	As-cast or as-extruded	0.4	0.55	0.65
	Brushed, screeded or rough-tamped	0.6	0.65	0.75
	Washed to remove laitance or treated with retarder and cleaned	0.7	0.75	0.80
With nominal links projecting into in-situ concrete	As-cast or as-extruded	1.2	1.8	2.0
	Brushed, screeded or rough-tamped	1.8	2.0	2.2
	Washed to remove laitance or treated with retarder and cleaned	2.1	2.2	2.5

Design for Horizontal Shear

- Where higher resistance to horizontal shear is needed, the beam may be provided with nominal links that are projected through the interface and are anchored in the cast in situ concrete
- Nominal links are defined as
  - $A_h = 0.15 \times b_v' \times l / 100$
  - Sv – the smaller between $4h_f$ and 600 mm
- If the maximum horizontal design shear stress exceeds the allowable values, provide the following area of steel:
  - $A_h = 1000\ b_v'\ v_h / 0.87fyv$