Prestressed Concrete Design

(SAB 4323)

Design for Ultimate Strength in Flexure

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Introduction

- The most important single property of a structure is its strength
- Why? Because a member's strength relates directly to its safety!
- Adequate strength of a prestressed concrete member is not automatically insured by limiting stresses at service load

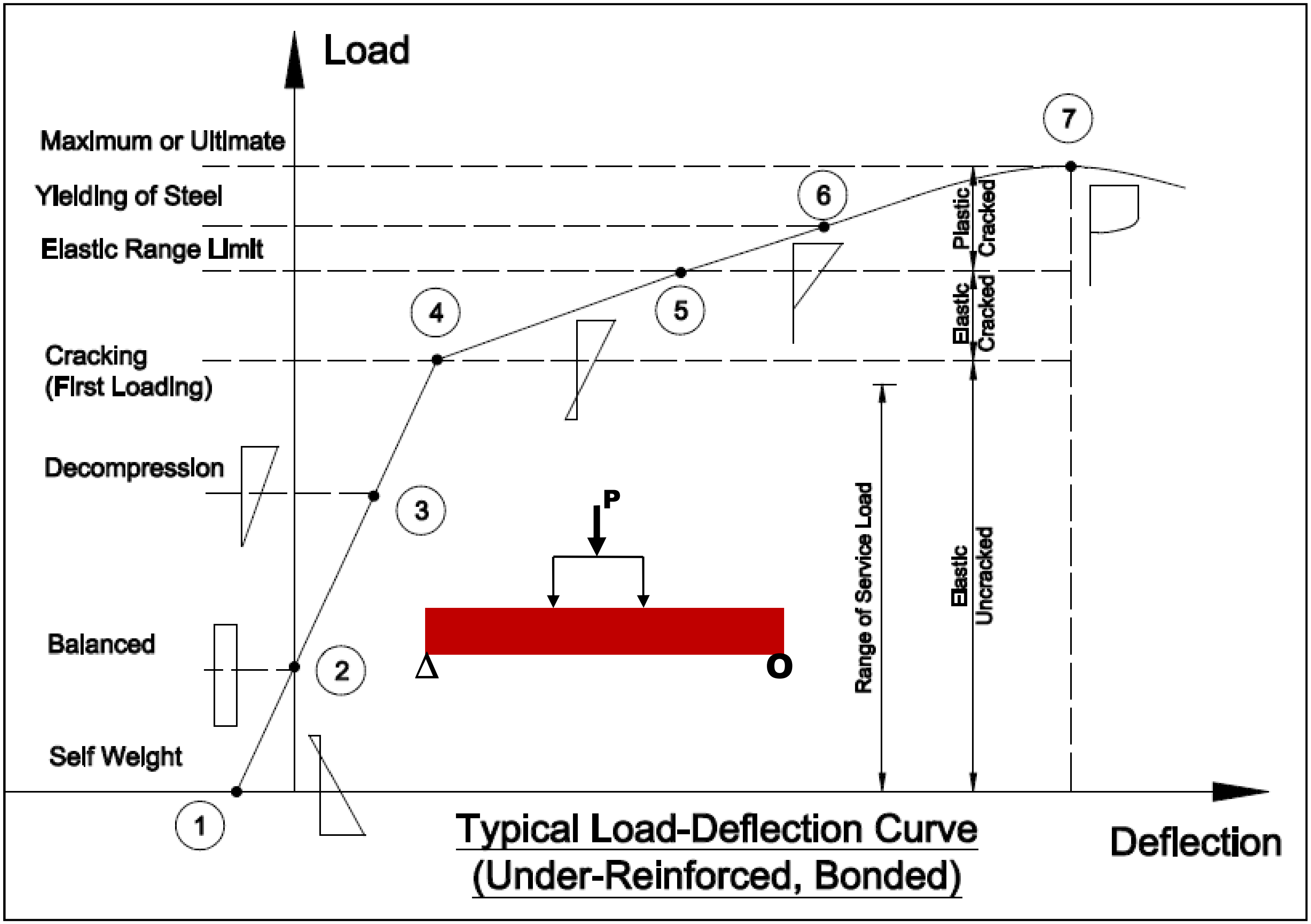
Introduction

- Should the member be overloaded, significant changes in behaviour result from cracking, and because one or both of the materials will be stressed into the inelastic range before failure
- The true factor of safety can be established only by calculating the strength of the member and comparing the load that would cause the member to fail with the load that is actually expected to act.

Ultimate Load Behaviour

- The overall behaviour of a simply supported prestressed beam subjected to a monotonically increasing load can be well described by its load-deflection curve as shown on the next slide
- Typical stress diagrams along the cross section of maximum moments corresponding to points 1 to 7 are also shown
- Point 1 – Upward deflection(camber) due to βPi and Wsw
- If additional load beyond self weight is applied, several points of interest can be identified until failure
- Point 2 – Zero deflection and corresponds to a uniform state of stress in the section
- Point 3 – Decompression or zero stress at the bottom fibre
- Point 4 – Beginning of cracking in the concrete ($f_{2s} = f_{tu}$)

Ultimate Load Behaviour



Typical Load-Deflection Curve
(Under-Reinforced, Bonded)

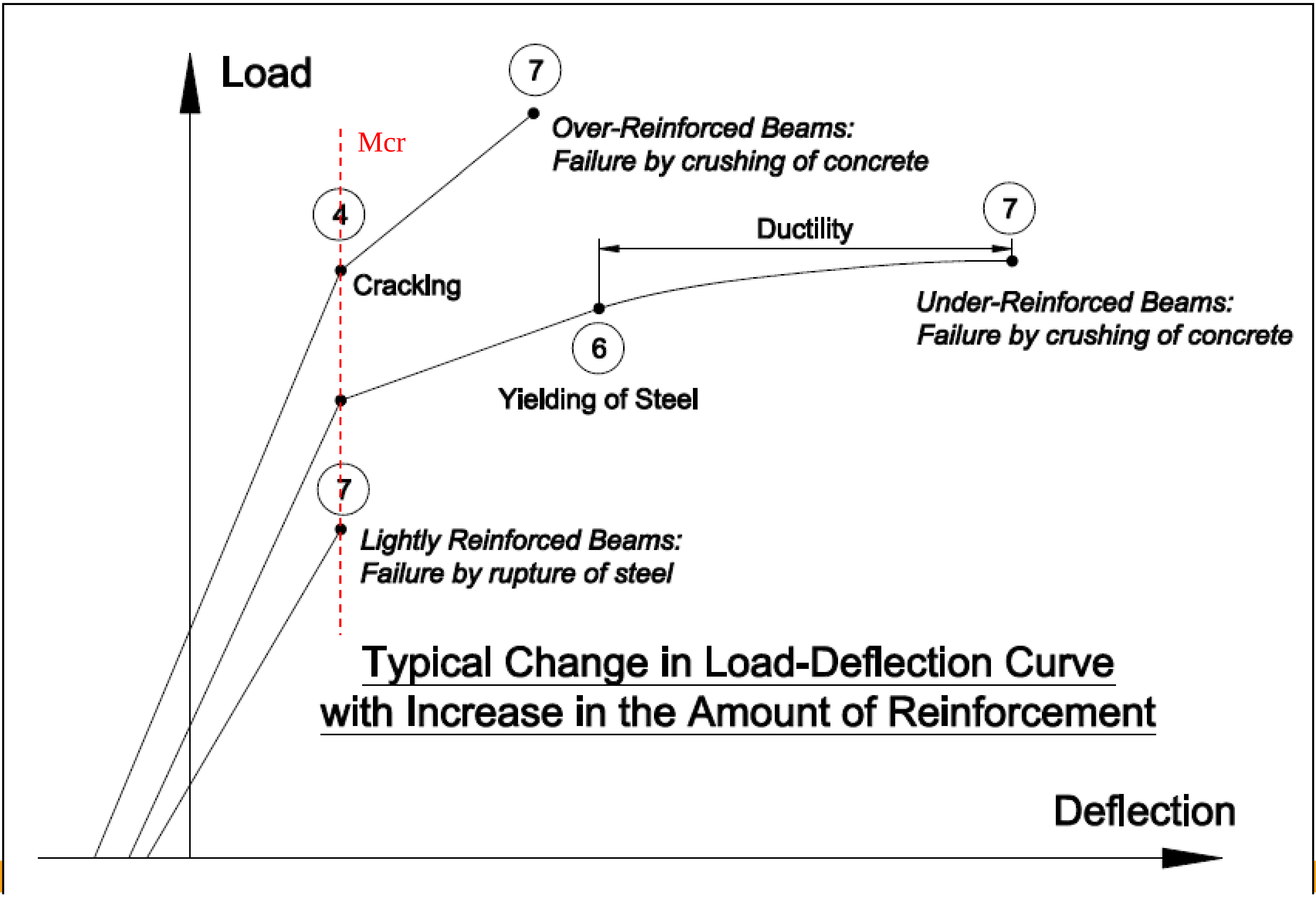
Ultimate Load Behaviour

- Beyond point 4, the prestressed concrete section behaves similar to reinforced concrete section subjected to combined bending and compression
- Point 5 – Either concrete or steel reaches its non elastic characteristics
- Point 6 – Steel has reach its yielding strength
- Point 7 – Maximum capacity of beam attained at ultimate load

Flexural Types of Failure

- Failure of prestressed concrete beam may occur either by rupture of steel or by crushing of concrete, depending on the amount of steel in the section
- Rupture of steel occurs when the beam contains reinforcement insufficient to carry the tensile stresses from the concrete at the instant of cracking. This type of failure is undesirable and is always avoided in design by providing a minimum amount of reinforcement (Clause 4.12.2 - when Mu > Mcr, taking f_{tu}=0.6$fcu^{0.5}$)
- When the beam contains reinforcement greater than the minimum amount, failure will always occur by crushing of the concrete

Flexural Types of Failure



Typical Change in Load-Deflection Curve with Increase in the Amount of Reinforcement

Over-Reinforced Beam

- At failure, the embedded steel may or may not yield depending on the relative amount of steel. If the amount of steel is such that yielding of steel (not rupture) and crushing of concrete occur simultaneously, the corresponding reinforcement ratio is said to be balanced reinforcement ratio, ρ_b.
- If $\rho > \rho_b$, the beam is said to be over-reinforced, i.e. steel will not yield at failure. The beam will fail suddenly by crushing of the concrete at small deflection before the cracks are fully developed.
- This type of failure is clearly undesirable in a practical situation, even if the beam has adequate margin of safety with respect to ultimate strength

Under-Reinforced Beam

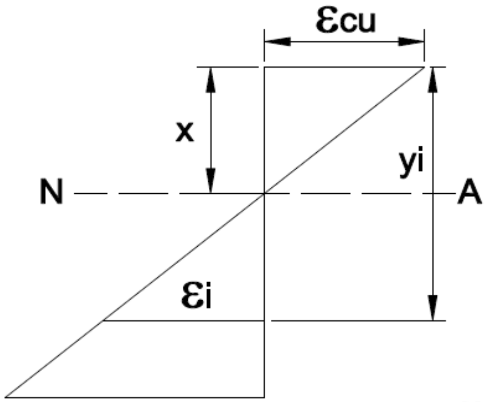
- Should a structure fail, it must exhibit visible signs of distress by displaying wide cracking and excessive deflection to serve as a warning to impending collapse so that occupants may take timely measures to save the structure, if possible and, protect lives and properties
- Hence, ductility or the ability of the structure to deform at or near the ultimate load is a vital consideration
- This is usually achieved by limiting the reinforcement ratio well below the balanced ratio ($\rho < \rho_b$) that results in an under-reinforced beam

Ultimate Strength Analysis

Assumptions

- Plane sections before bending remain plane after bending i.e. strain is proportional to the distance from neutral axis (Bernoulli’s Compatibility Condition)

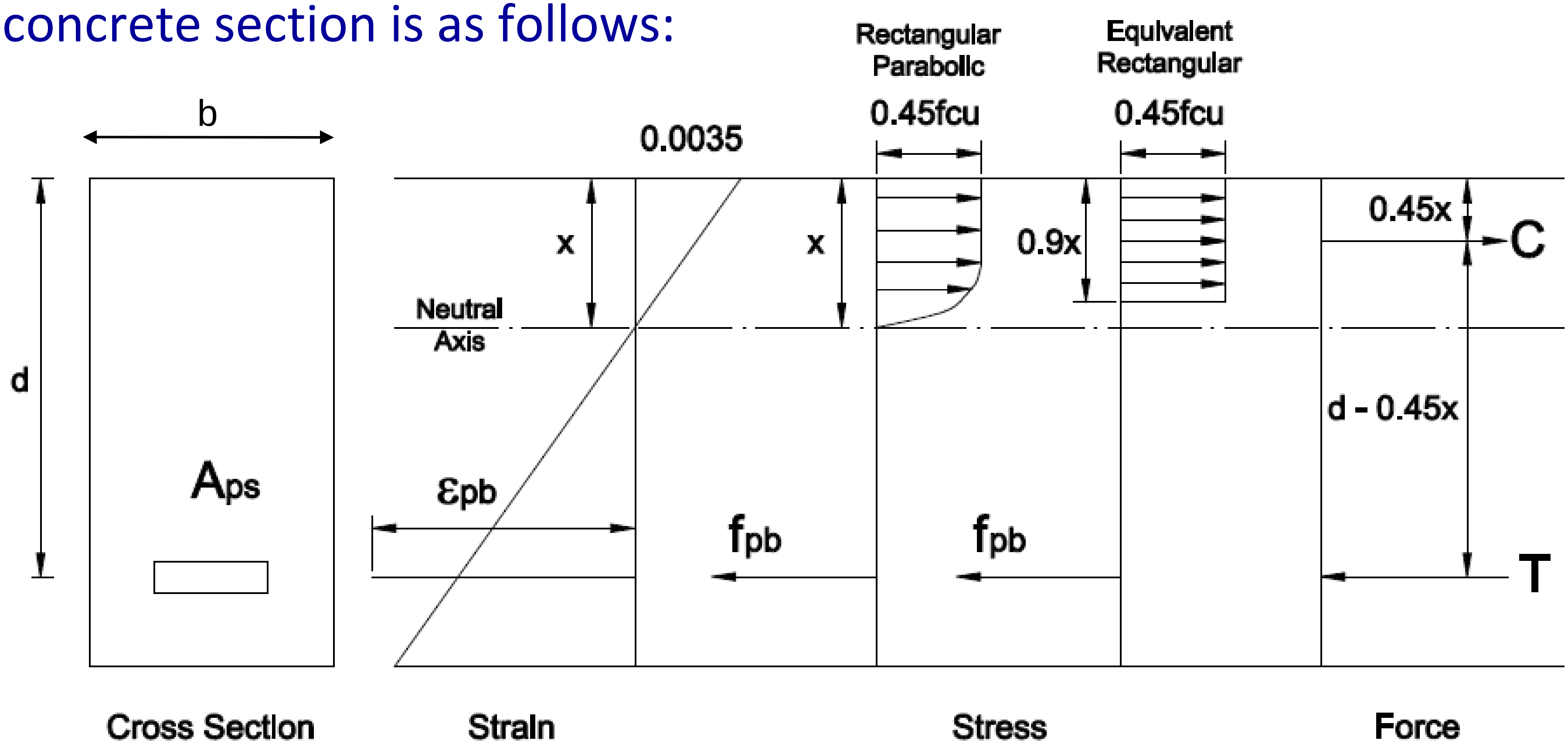
$$\varepsilon i = \varepsilon cu \frac{(yi - x)}{x}$$



- Perfect bond exists between concrete and prestressing steel or any additional reinforcements
- Tensile strength of concrete ignored

Conditions at Collapse

The strain, stress and force distributions across a prestressed concrete section is as follows:



Where :

f_{pb} – Tensile stress in tendons at failure

ε_{pb} – Ultimate strain in tendon

Components of Strains in Tendon

The ultimate strain in tendon, ε_{pb} is the sum of the followings:

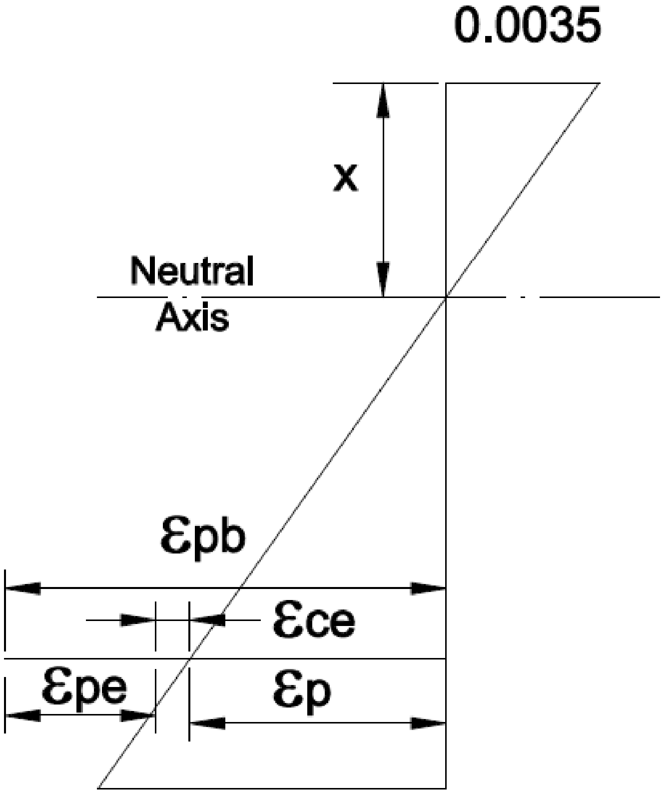
1. Effective prestrain in tendon, ε_{pe}
2. Effective prestrain in concrete, ε_{ce}
3. Strain in tendon due to flexure, ε_{p}

$$\varepsilon_{pe} = \frac{\beta Pi}{A_{ps}E_{ps}}$$

$$\varepsilon_{ce} = \frac{\beta Pi}{E_c}\left[\frac{1}{A_c} + \frac{e^2}{I_{xx}}\right]$$

$$\varepsilon_{p} = 0.0035\,\frac{(d-x)}{x}$$

$$\varepsilon_{pb} = \varepsilon_{pe} + \varepsilon_{ce} + \varepsilon_{p}$$



Ultimate Flexural Strength

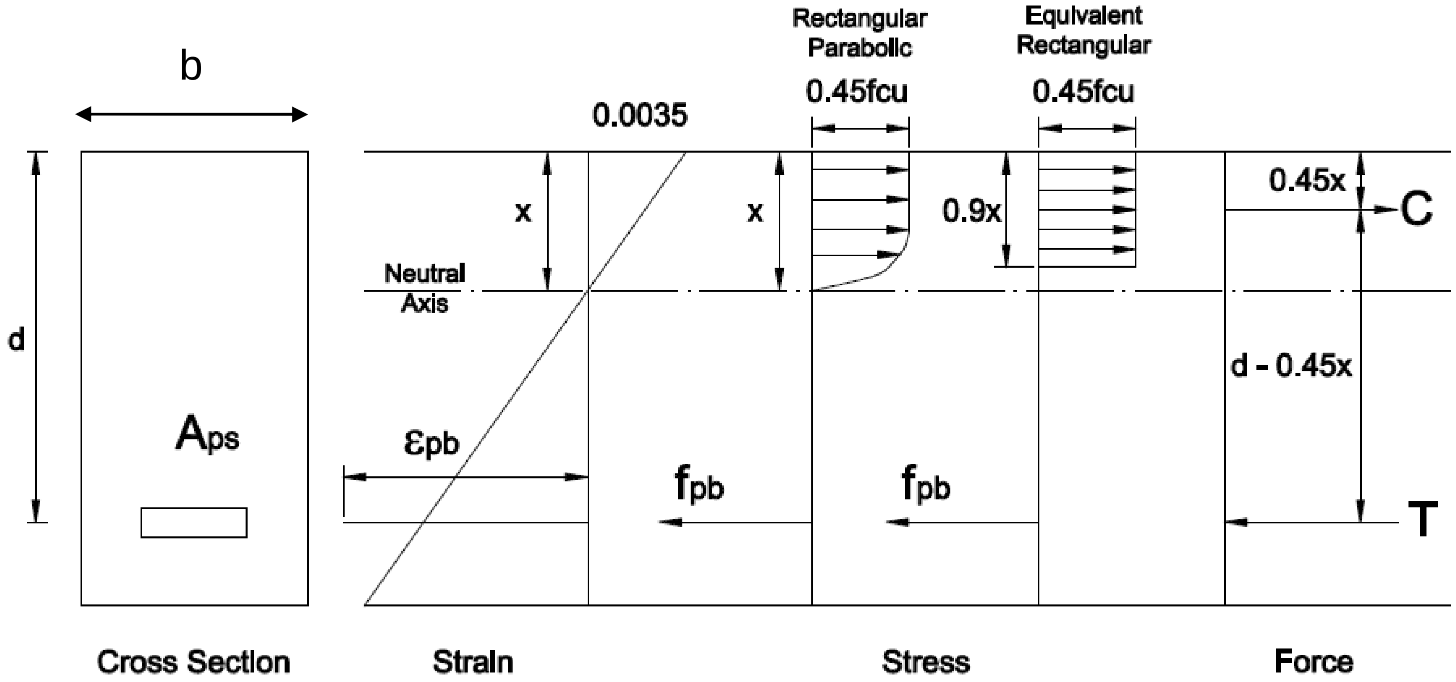
(Method of Strain Compatibility)

Equilibrium Equations

Rectangular Section

Using the Equivalent Rectangular Stress Block:

1. $T = f_{pb}A_{ps}$; $C = 0.45fcub(0.9x)$
2. $T = C \Rightarrow f_{pb}\ A_{ps} = 0.45f_{cu}b(0.9x)$
3. $Mu = f_{pb}A_{ps}(d - 0.45x)$ or $Mu = 0.405f_{cu}bx(d - 0.45x)$

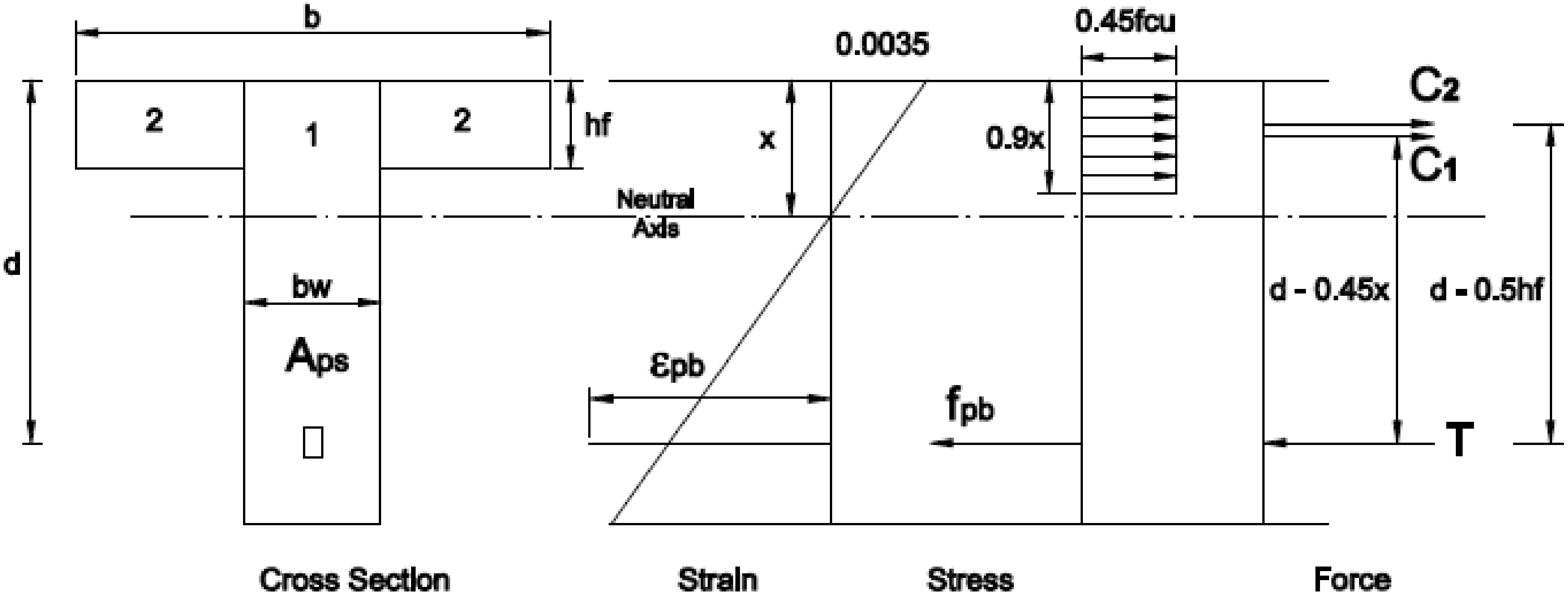


Equilibrium Equations

Tee Section

The following equations are valid for 0.9x >= hf:

1. $T = f_{pb}A_{ps}$; $C_1 = 0.405f_{cu}b_wx$; $C_2 = 0.45f_{cu}(b-b_w)hf$
2. $T = C \Rightarrow f_{pb}\ A_{ps} = 0.405f_{cu}b_wx + 0.45f_{cu}(b-b_w)hf$
3. $Mu = 0.405f_{cu}b_wx(d - 0.45x) + 0.45f_{cu}(b-b_w)hf\ (d-0.5hf)$



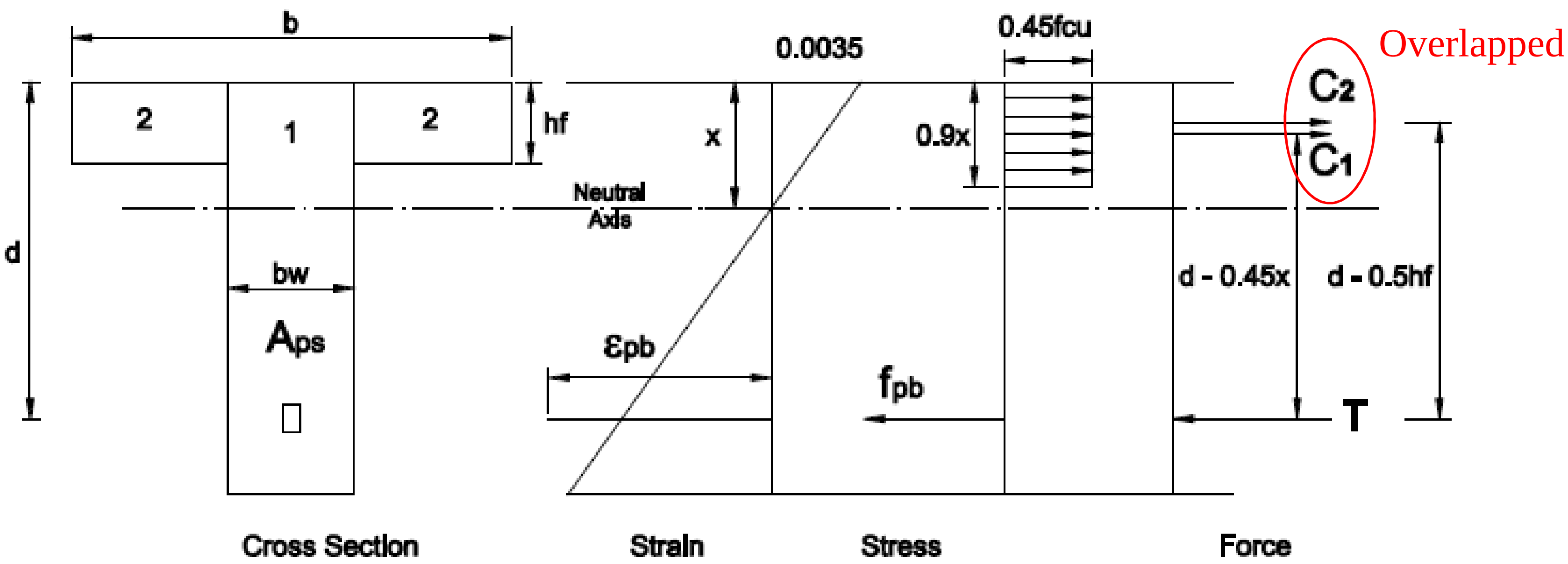
Equilibrium Equations

Tee Section

For $0.9x < hf$:

1. $T = f_{pb}A_{ps}$; $C_1 + C_2 = C = 0.405f_{cu}bx$
2. $T = C \Rightarrow f_{pb}\, A_{ps} = 0.405f_{cu}bx$
3. $Mu = 0.405f_{cu}bx(d - 0.45x)$ or $Mu = f_{pb}A_{ps}(d – 0.45x)$

Note: This case is similar to the Rectangular Section



Compatibility Equations

$$\varepsilon_{pe} = \frac{\beta Pi}{A_{ps}E_{ps}}$$

$$\varepsilon_{ce} = \frac{\beta Pi}{E_c}\left[\frac{1}{A_c} + \frac{e^2}{I_{xx}}\right]$$

$$\varepsilon_p = 0.0035\,\frac{(d - x)}{x}$$

$$\varepsilon_{pb} = \varepsilon_{pe} + \cancelto{0}{\varepsilon_{ce}} + \varepsilon_p$$

Note:

1. ε_{pe} & ε_{ce} depend on the level of effective prestress and is independent of the neutral axis position
2. ε_{ce} is relatively small and can be neglected

Stress-Strain Relationship

The tri-linear relationship for prestressing tendon may be expressed mathematically as ·

If $\varepsilon_{pb} \leq \dfrac{0.8 f_{pu}}{\gamma_m E_{ps}}$ then

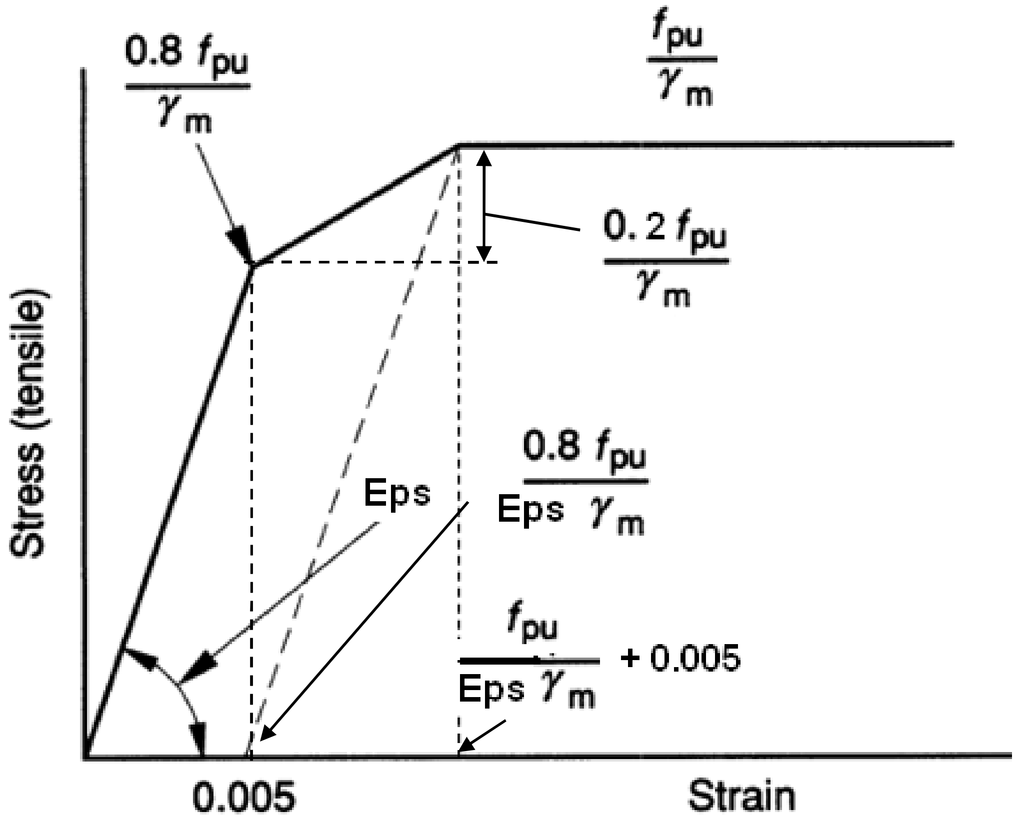
$$f_{pb} = E_{ps}\varepsilon_{pb}$$

If $\dfrac{0.8 f_{pu}}{\gamma_m E_{ps}} < \varepsilon_{pb} < 0.005 + \dfrac{0.8 f_{pu}}{\gamma_m E_{ps}}$ then

$$f_{pb} = \frac{0.8 f_{pu}}{\gamma_m} + \frac{0.2 f_{pu}}{\gamma_m}\left[\frac{\varepsilon_{pb} - \dfrac{0.8 f_{pu}}{\gamma_m E_{ps}}}{0.005 + \dfrac{0.2 f_{pu}}{\gamma_m E_{ps}}}\right]$$

If $\varepsilon_{pb} \geq 0.005 + \dfrac{0.8 f_{pu}}{\gamma_m E_{ps}}$ then

$$f_{pb} = \frac{f_{pu}}{\gamma_m}$$



Trial and Error Technique

1. Assume a trial value for the neutral axis depth, x
2. Calculate ε_{pb} from compatibility equation $\varepsilon_{pb} = \varepsilon_{pe} + \cancelto{0}{\varepsilon_{ce}} + \varepsilon_{p}$
3. Obtain f_{pb} from the stress-strain relationship
4. Repeat the above steps until T = C
5. Calculate M_u from the moment equilibrium equation,

   $M_u = f_{pb}A_{ps}(d – 0.45x)$ or $M_u = 0.405f_{cu}bx(d - 0.45x)$

Example 7-1

Determine the design ultimate moment of resistance of the following beam:

Prestressing tendons

f_{pu} = 1860 N/mm^2

γ_m = 1.15

E_{ps} = 195 kN/mm^2

A_{ps} = 845 mm^2

P_e = 880 kN

Section Properties

A_c = 213000 mm^2

I_{xx} = 13170000000 mm^4

e = 325 mm

d = 700 mm

b_f = 400 mm

h_f = 200 mm

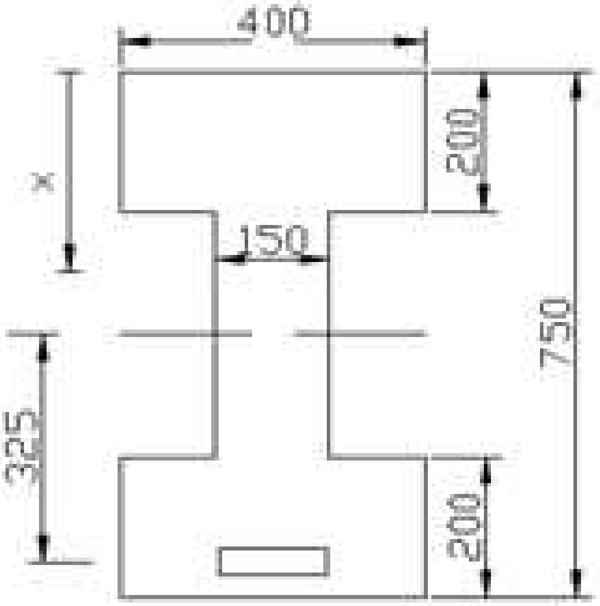
b_w = 150 mm

Concrete

f_{cu} = 40 N/mm^2

γ_m = 1.5

E_c = 28.0 kN/mm^2



Solution

1. Since it is a Tee Section, try x = hf/0.9 = 200/0.9=222.222mm
2. Calculate ε_{pb} from compatibility equation $\varepsilon_{pb} = \varepsilon_{pe} + \varepsilon_{ce} + \varepsilon_{p}$

$\varepsilon_{pe} = P_e/(A_{ps}\ E_{ps}) = 880\ /(845*195) = 0.00534$

$\varepsilon_{ce} = (P_e/E_c)(1/A_c + e^2/I_x) = (880/28)(1/2.13x10^5 + 325^2/1.317x10^{10})$

$= 0.00040$

$\varepsilon_{p} = 0.0035*(d\text{-}x)/x = 0.0035 * (700\text{-}222.222)/222.222 = 0.00753$

$\varepsilon_{pb} = 0.00534+0.00040+0.00753 = 0.01327$

3. Calculate f_{pb} from stress-strain relationship curve:

$0.8f_{pu}/\gamma_m E_{ps} = 0.8*1860/1.15*195x10^3 = 0.00664$

$0.005 + f_{pu}/\gamma_m E_{ps} = 0.005 + 1860/1.15*195x10^3 = 0.01329$

Solution

3. Calculate f_{pb} from stress-strain relationship curve:

$0.8f_{pu}/\gamma_m E_{ps} = 0.8*1860/1.15*195x10^3 = 0.00664$

$0.005 + f_{pu}/\gamma_m E_{ps} = 0.005 + 1860/1.15*195x10^3 = 0.01329$

From the curve, f_{pb} is between

$0.8f_{pu}/\gamma_m$ and f_{pu}/γ_m, (steel not yield)

$= 1615.978$ N/mm^2

4. $T = A_{ps}f_{pb}$=1615.978*845/1000

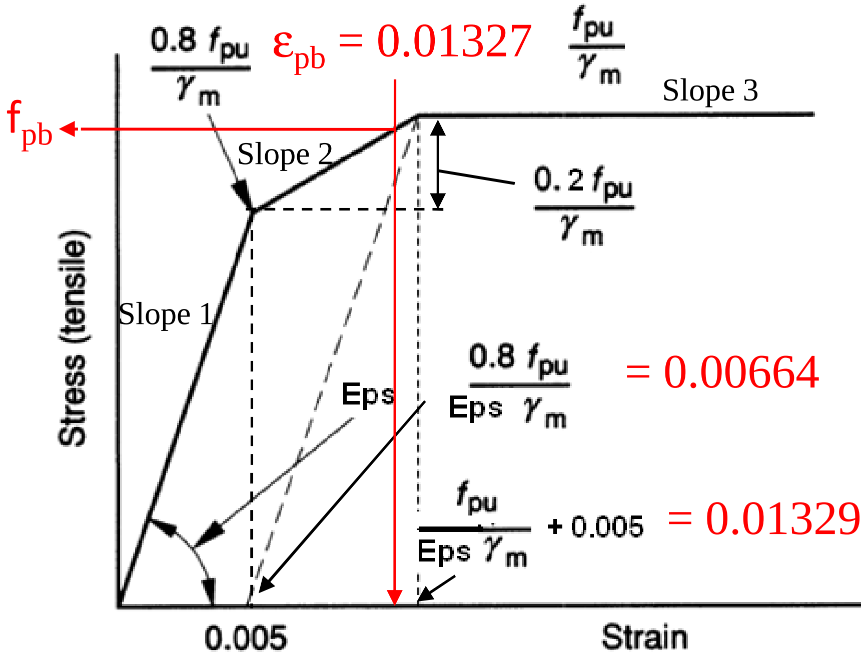
= 1365.5 kN

$$C = 0.405f_{cu}bx = \frac{0.405*40*400*222.222}{1000}$$

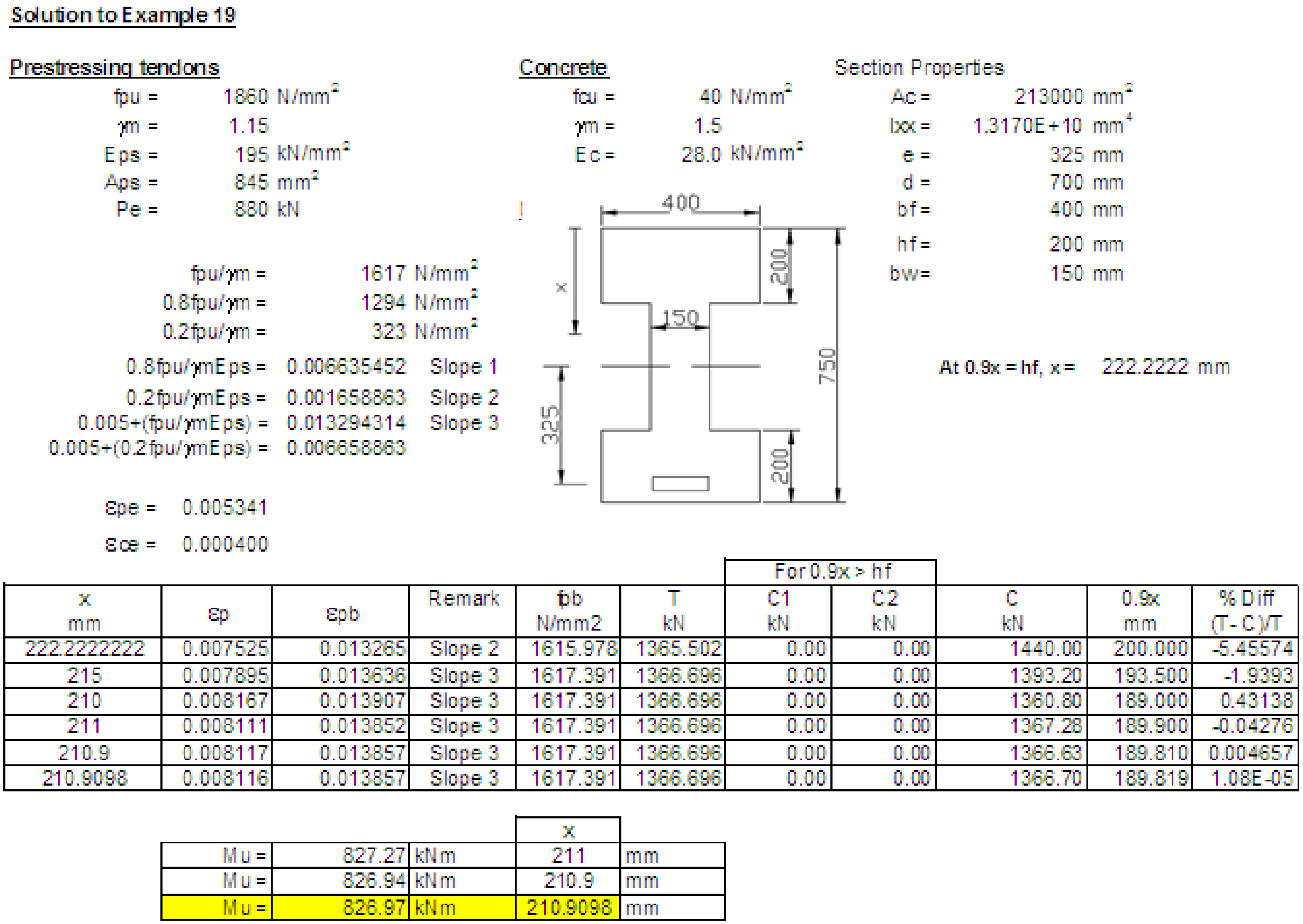
= 1440 kN

% Difference = 1365.5-1440/1440 = -5.4%

Decrease x! Try x = 215mm, REPEAT ABOVE STEPS!!!



Solution



Solution to Example 19

Prestressing tendons

fpu = 1860 N/mm^2
γm = 1.15
Eps = 195 kN/mm^2
Aps = 845 mm^2
Pe = 880 kN

fpu/γm = 1617 N/mm^2
0.8fpu/γm = 1294 N/mm^2
0.2fpu/γm = 323 N/mm^2

0.8fpu/γmEps = 0.006635452 Slope 1
0.2fpu/γmEps = 0.001658863 Slope 2
0.005+(fpu/γmEps) = 0.013294314 Slope 3
0.005+(0.2fpu/γmEps) = 0.006658863

εpe = 0.005341

εce = 0.000400

Concrete

fcu = 40 N/mm^2
γm = 1.5
Ec = 28.0 kN/mm^2

!

Section Properties

Ac = 213000 mm^2
Ixx = 1.3170E+10 mm^4
e = 325 mm
d = 700 mm
bf = 400 mm
hf = 200 mm
bw = 150 mm

At 0.9x = hf, x = 222.2222 mm

x mm	εp	εpb	Remark	fpb N/mm2	T kN	For 0.9x > hf C1 kN	C2 kN	C kN	0.9x mm	% Diff (T - C)/T
222.2222222	0.007525	0.013265	Slope 2	1615.978	1365.502	0.00	0.00	1440.00	200.000	-5.45574
215	0.007895	0.013636	Slope 3	1617.391	1366.696	0.00	0.00	1393.20	193.500	-1.9393
210	0.008167	0.013907	Slope 3	1617.391	1366.696	0.00	0.00	1360.80	189.000	0.43138
211	0.008111	0.013852	Slope 3	1617.391	1366.696	0.00	0.00	1367.28	189.900	-0.04276
210.9	0.008117	0.013857	Slope 3	1617.391	1366.696	0.00	0.00	1366.63	189.810	0.004657
210.9098	0.008116	0.013857	Slope 3	1617.391	1366.696	0.00	0.00	1366.70	189.819	1.08E-05

			x	
Mu =	827.27	kNm	211	mm
Mu =	826.94	kNm	210.9	mm
Mu =	826.97	kNm	210.9098	mm

Example 7-2

Determine the ultimate moment capacity of the composite beam in example 17 and compare with the design moment.

Given:

Span = 20.6m; Unshored Construction

Loading/beam:

Wslab=8.11kN/m; SDL=3.73kN/m; LL=14.56kN/m

Beam:

fcu = 50N/mm^2; E = 36 kN/mm^2

Slab :

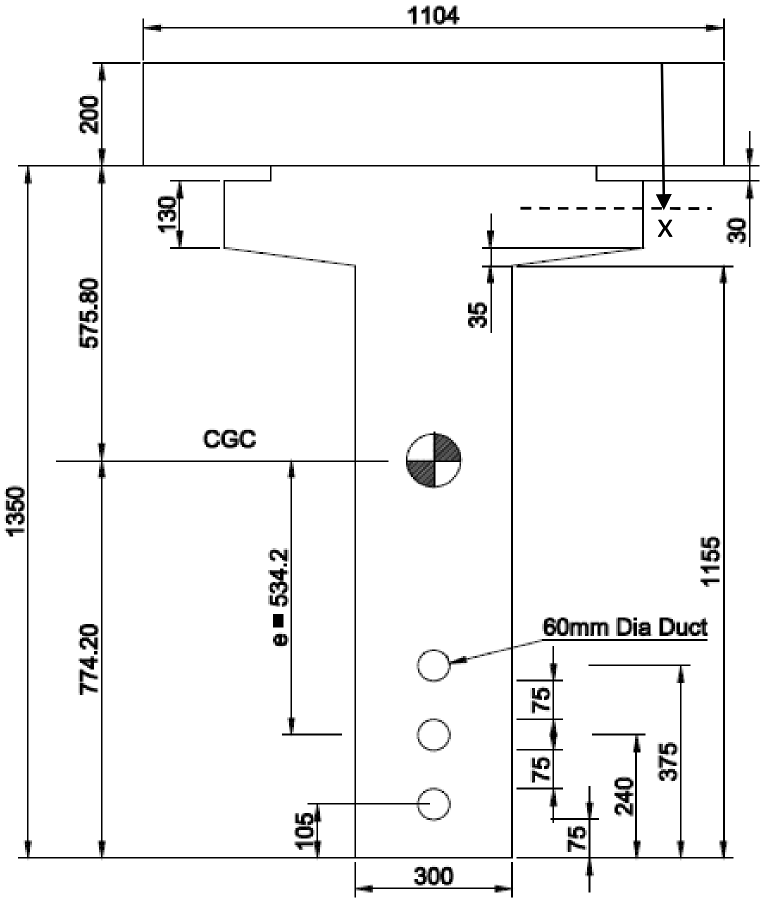
fcu = 40N/mm^2; E = 34 kN/mm^2; h = 200 mm

Prestress Steel (12.9mm dia 7-wire super strand):

Fpu = 186 kN; Aps = 100 mm^2

fpu = 1860 N/mm^2 ; %UTS = 70%

β=0.72



Solution

Compressive Force in Concrete

For $0.9x <= 200$

$C1 = 0.45*40 *1104*0.9x/1000$ kN

$y1 = 0.45x + 0.1x = 0.55x$

$C2=C3=0;y2=y3=0$

For $200<0.9x<=230$

$C1 = 0.45*40*1104*200/1000 = 3974.4$ kN

$y1 = x – 0.45*200 = x – 90$

$C2 = 0.45*50*620*(0.9x – 200)/1000$ kN; $C3=0$

$y2 = 0.5(0.9x-200)+0.1x ;y3=0$

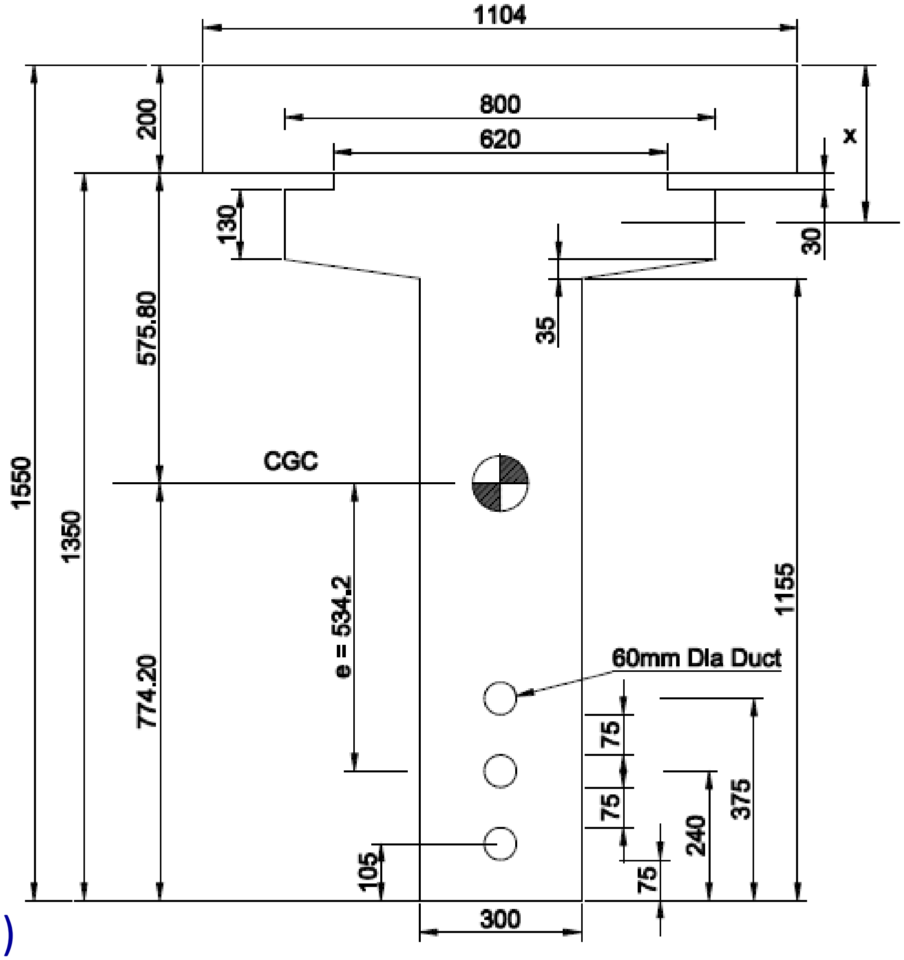
For $230<0.9x<=360$

$C1 = 3974.4$ kN; $y1 = x – 90$

$C2 = 0.45*50*620*30/1000 = 418.5$ kN;

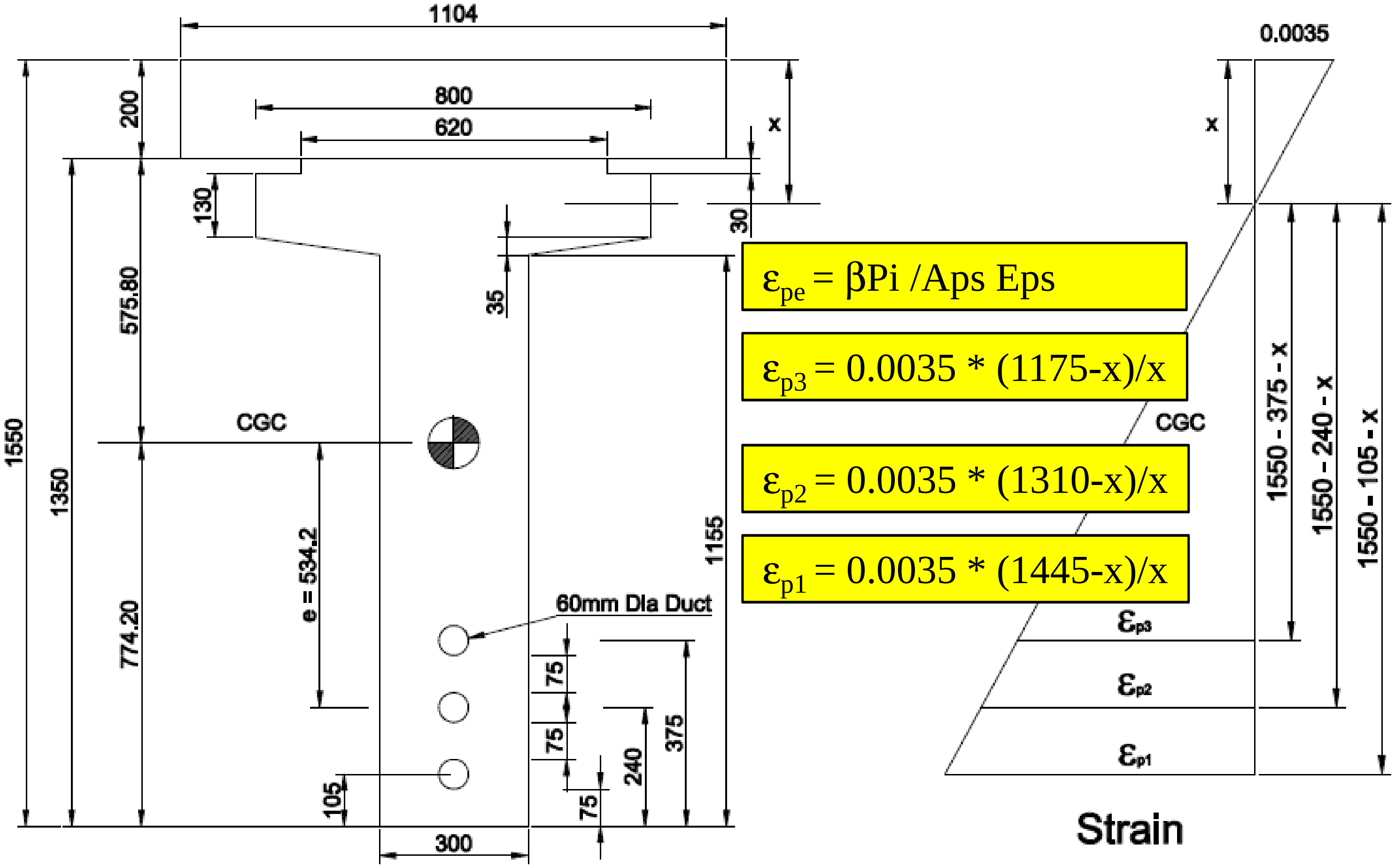
$C3 = 0.45*50*800*(0.9x-200-30)/1000$ kN

$y2=x-200-30/2=x – 215;y3=0.1x+0.5(0.9x-200-30)$



Solution

Strain in tendon due to flexure, ε_p and prestrain, ε_{pe}



$\varepsilon_{pe} = \beta Pi / Aps\ Eps$

$\varepsilon_{p3} = 0.0035 * (1175-x)/x$

$\varepsilon_{p2} = 0.0035 * (1310-x)/x$

$\varepsilon_{p1} = 0.0035 * (1445-x)/x$

Solution

Tensile Force in Tendon

With x = 253.5 mm

$$\left.\begin{aligned} \varepsilon_{pb1} &= 0.060338 \\ \varepsilon_{pb2} &= 0.059648 \\ \varepsilon_{pb3} &= 0.058958 \end{aligned}\right\} > 0.013295$$

∴ All the tendons have yield!

$f_{pb} = 1617.39 \text{ N/mm}^2$

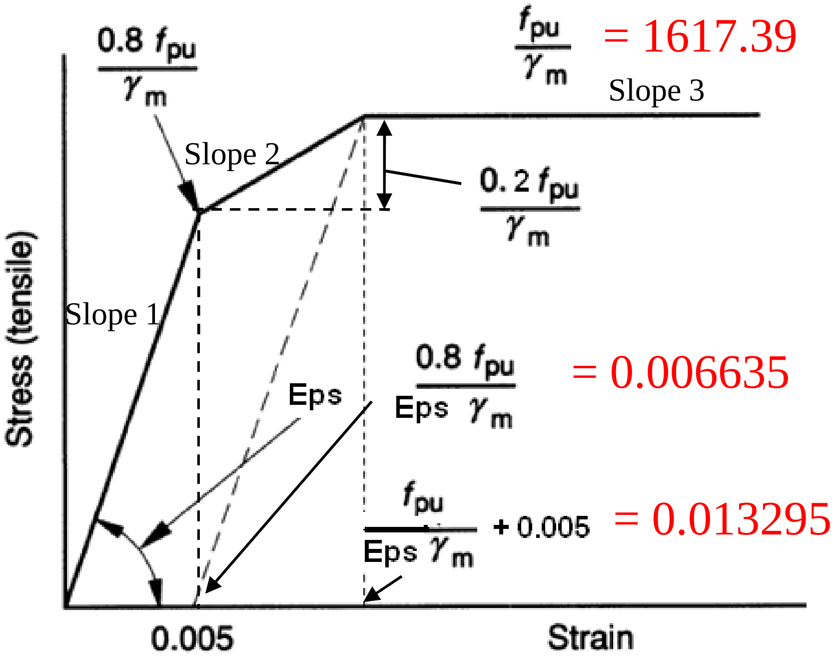
Tensile force, T per cable

= 1617.39*100*9/1000

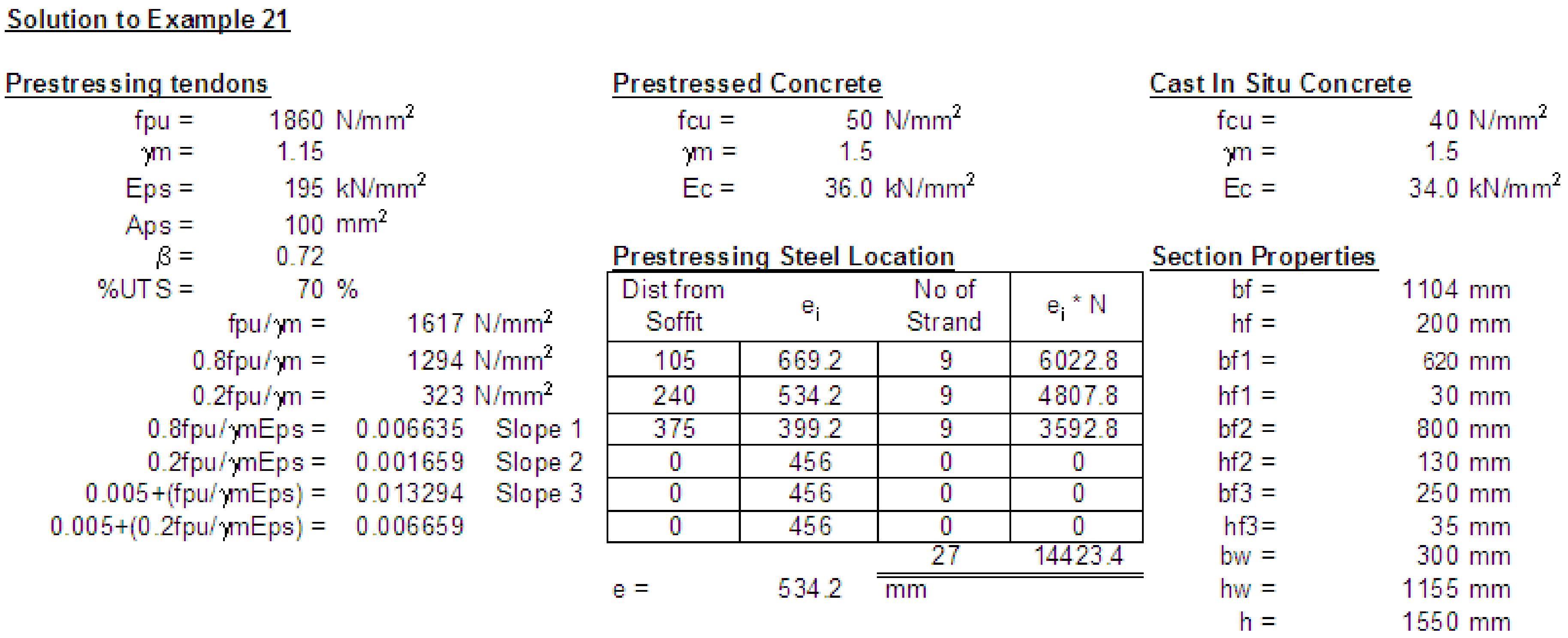
= 1455.65 kN

Taking moment about Neutral Axis:

$Mu = \Sigma C_i y_i + \Sigma T_i z_i$



Solution



Solution to Example 21

Prestressing tendons

fpu = 1860 N/mm^2

γm = 1.15

Eps = 195 kN/mm^2

Aps = 100 mm^2

β = 0.72

%UTS = 70 %

fpu/γm = 1617 N/mm^2

0.8fpu/γm = 1294 N/mm^2

0.2fpu/γm = 323 N/mm^2

0.8fpu/γmEps = 0.006635 Slope 1

0.2fpu/γmEps = 0.001659 Slope 2

0.005+(fpu/γmEps) = 0.013294 Slope 3

0.005+(0.2fpu/γmEps) = 0.006659

Prestressed Concrete

fcu = 50 N/mm^2

γm = 1.5

Ec = 36.0 kN/mm^2

Prestressing Steel Location

Dist from Soffit	e_i	No of Strand	e_i * N
105	669.2	9	6022.8
240	534.2	9	4807.8
375	399.2	9	3592.8
0	456	0	0
0	456	0	0
0	456	0	0
		27	14423.4

e = 534.2 mm

Cast In Situ Concrete

fcu = 40 N/mm^2

γm = 1.5

Ec = 34.0 kN/mm^2

Section Properties

bf = 1104 mm

hf = 200 mm

bf1 = 620 mm

hf1 = 30 mm

bf2 = 800 mm

hf2 = 130 mm

bf3 = 250 mm

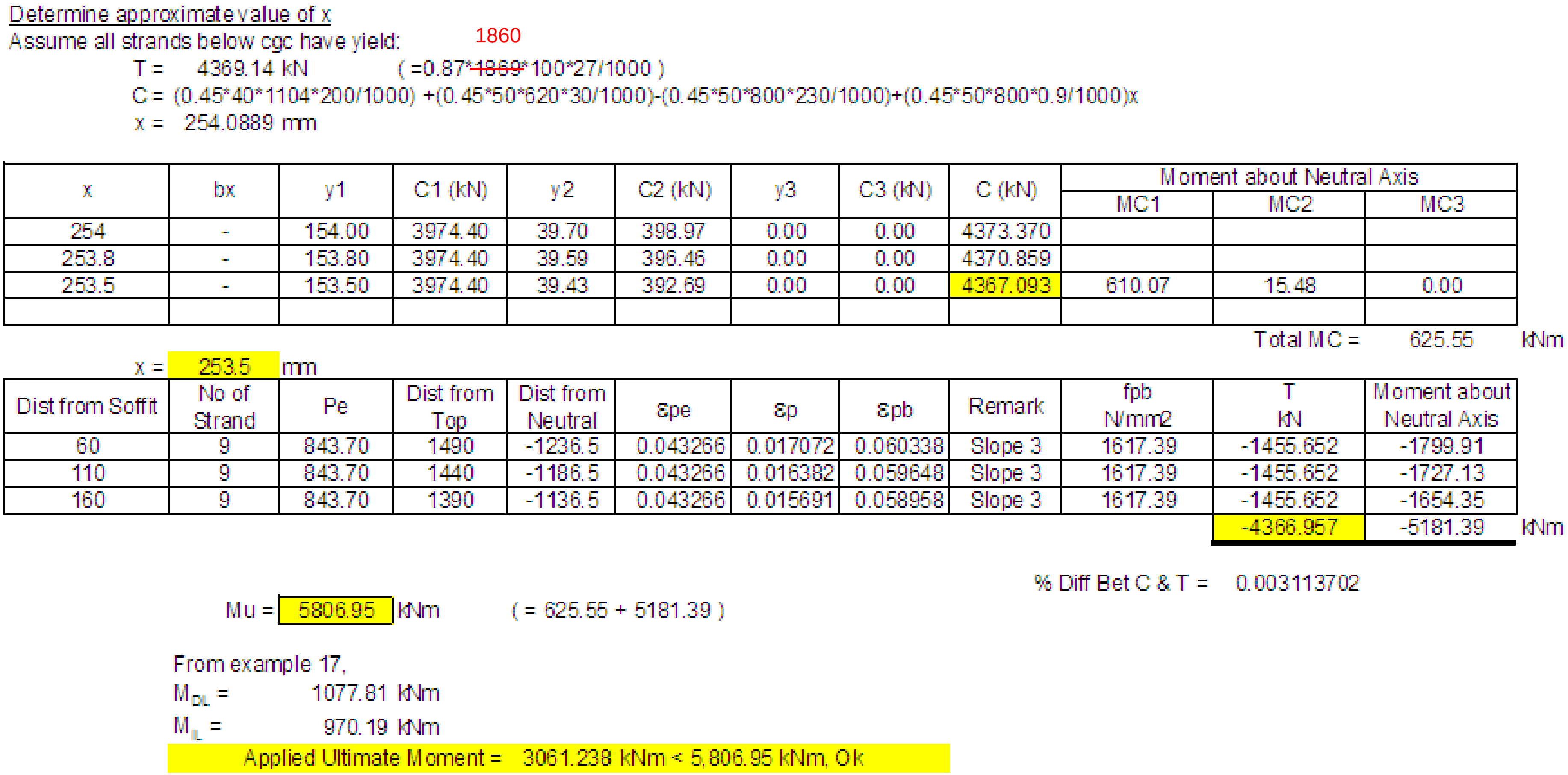
hf3 = 35 mm

bw = 300 mm

hw = 1155 mm

h = 1550 mm

Solution to e.g.21



Determine approximate value of x

Assume all strands below cgc have yield:

1860

T = 4369.14 kN (=0.87*~~1869~~*100*27/1000)

C = (0.45*40*1104*200/1000) +(0.45*50*620*30/1000)-(0.45*50*800*230/1000)+(0.45*50*800*0.9/1000)x

x = 254.0889 mm

x	bx	y1	C1 (kN)	y2	C2 (kN)	y3	C3 (kN)	C (kN)	Moment about Neutral Axis		
									MC1	MC2	MC3
254	-	154.00	3974.40	39.70	398.97	0.00	0.00	4373.370			
253.8	-	153.80	3974.40	39.59	396.46	0.00	0.00	4370.859			
253.5	-	153.50	3974.40	39.43	392.69	0.00	0.00	4367.093	610.07	15.48	0.00

Total MC = 625.55 kNm

x = 253.5 mm

Dist from Soffit	No of Strand	Pe	Dist from Top	Dist from Neutral	εpe	εp	εpb	Remark	fpb N/mm2	T kN	Moment about Neutral Axis
60	9	843.70	1490	-1236.5	0.043266	0.017072	0.060338	Slope 3	1617.39	-1455.652	-1799.91
110	9	843.70	1440	-1186.5	0.043266	0.016382	0.059648	Slope 3	1617.39	-1455.652	-1727.13
160	9	843.70	1390	-1136.5	0.043266	0.015691	0.058958	Slope 3	1617.39	-1455.652	-1654.35
										-4366.957	-5181.39 kNm

% Diff Bet C & T = 0.003113702

Mu = 5806.95 kNm (= 625.55 + 5181.39)

From example 17,

M_{DL} = 1077.81 kNm

M_{LL} = 970.19 kNm

Applied Ultimate Moment = 3061.238 kNm < 5,806.95 kNm, Ok