Prestressed Concrete Design (SAB 4323)

Composite Beams

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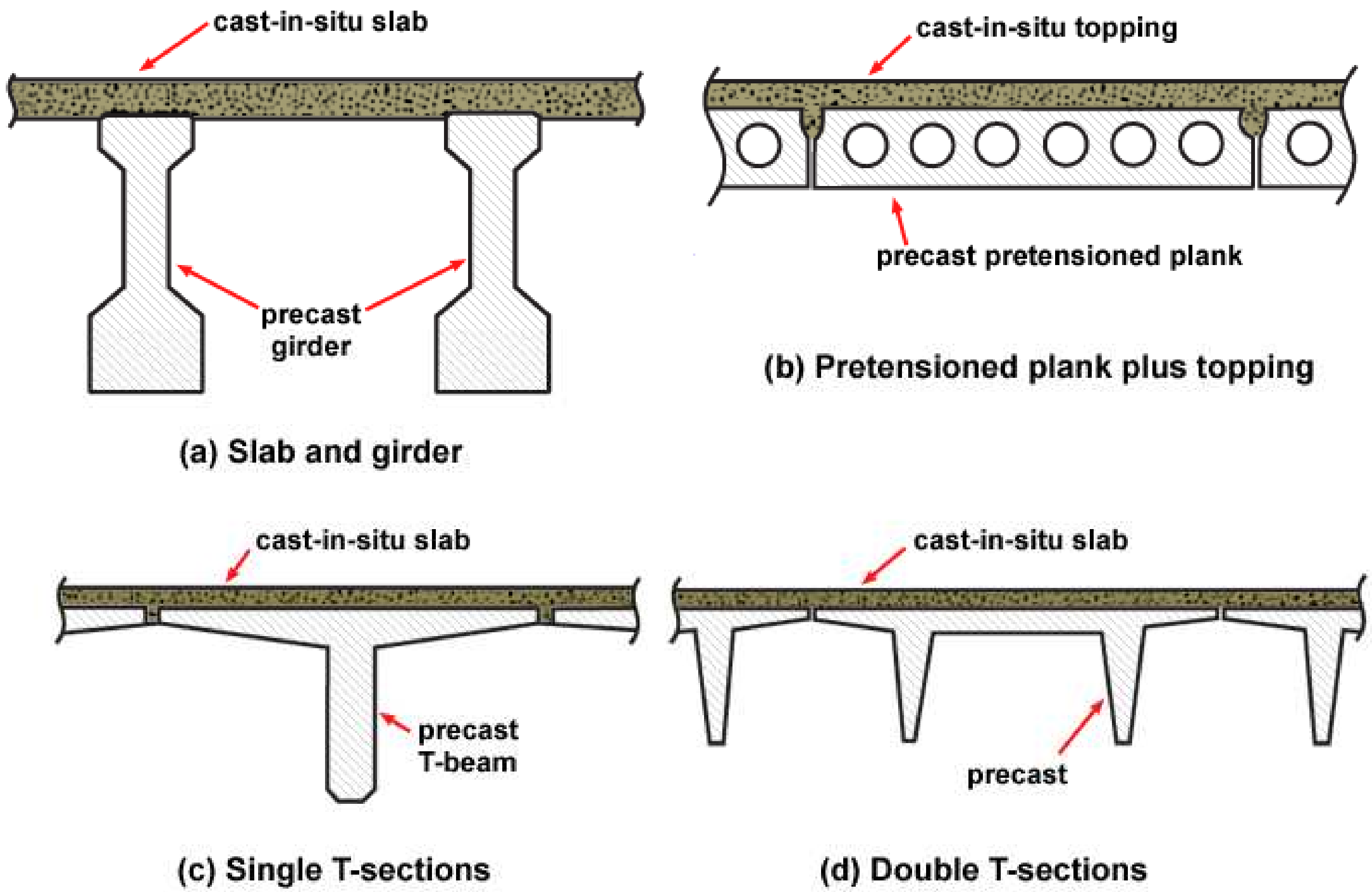
Introduction

- Composite construction implies the use, in a single structure acting as a unit, of different structural element made with similar or different structural materials.
- In a composite member where only concrete is used as a material, the concrete is placed in at least two separate stages generally leading to two different unit weights and/or properties.
- This is the case of composites made with precast reinforced or prestressed concrete element combined with a concrete element cast in situ at a different time.
- Typical composite cross-sections are as shown in the next slide

Typical Composite Cross-Sections



Advantages of Composite Construction

- Total construction time is substantially reduced when precast concrete elements are used.
- Pre-tensioning in plant is more cost-effective than post-tensioning on site. Because the precast prestressed concrete element is factory-produced and contains the bulk of reinforcement, rigorous quality control and higher mechanical properties can be achieved at relatively low cost. The cast in situ concrete slab does not need to have high mechanical properties and thus is suitable to field conditions.

Advantages of Composite Construction – cont'd

- The precast prestressed concrete units are erected first and can be used to support the formwork needed for the cast in situ slab without additional scaffolding (or shoring)
- In addition to its contribution to the strength and stiffness of the composite member, the cast in situ slab provides an effective means to distribute loads in the lateral direction.
- The cast in situ slab can be poured continuously over the supports of precast units placed in series, thus providing continuity to a simple span system.

UTM

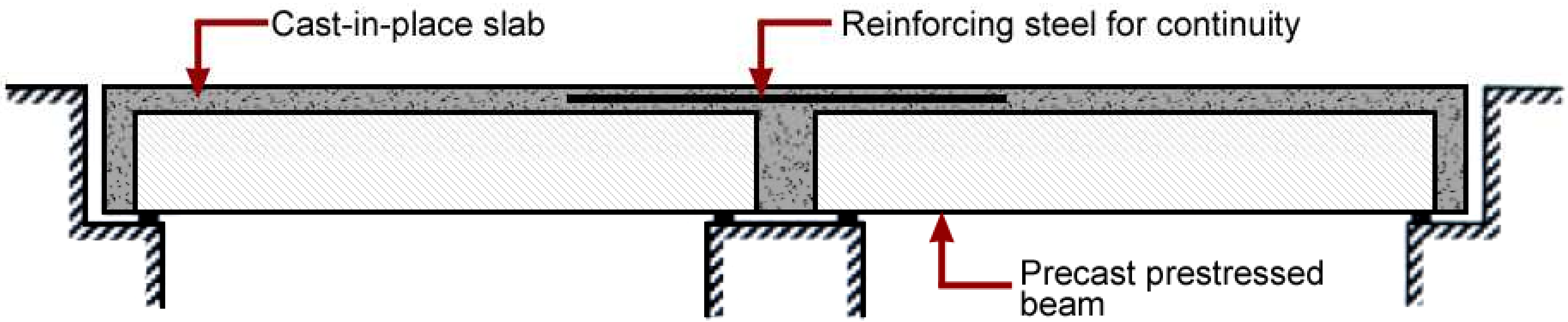
Temporary Slab Formwork

Permanent Slab Formwork

Shored Construction

Unshored Construction

Continuous Construction



Continuous Construction

Construction Sequence And Loading Stages

Loading Stage	Loads Considered in Shored Construction	Loads Considered in Unshored Construction	Section
1	αPi & Wsw	αPi & Wsw	Precast
2	βPi & Wsw	βPi, Wsw & Wslab	Precast
3	βPi,Wsw,Wslab, W_{DL} & W_{LL}	βPi,Wsw,Wslab, W_{DL} & W_{LL}	Composite

Transformed Section Properties

Need to consider

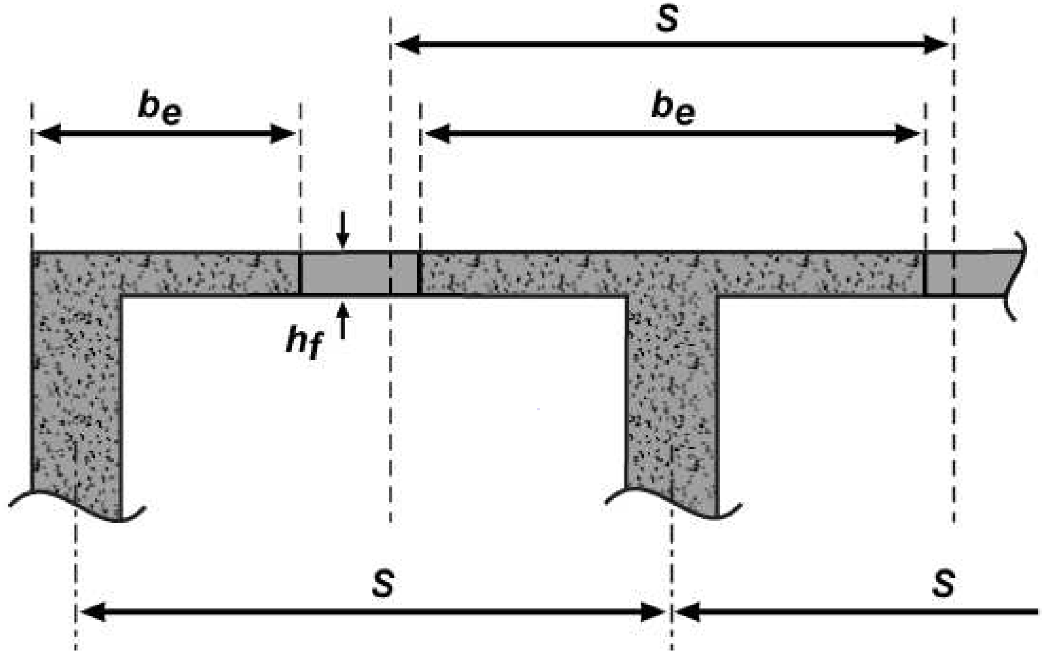
- Effective Flange Width
- Transformed Section

Effective Flange Width

- Only part of the slab participates in enhancing the strength and stiffness in the monolithic or composite beam-slab construction.

UTM

Effective Flange Width



Exterior Beam

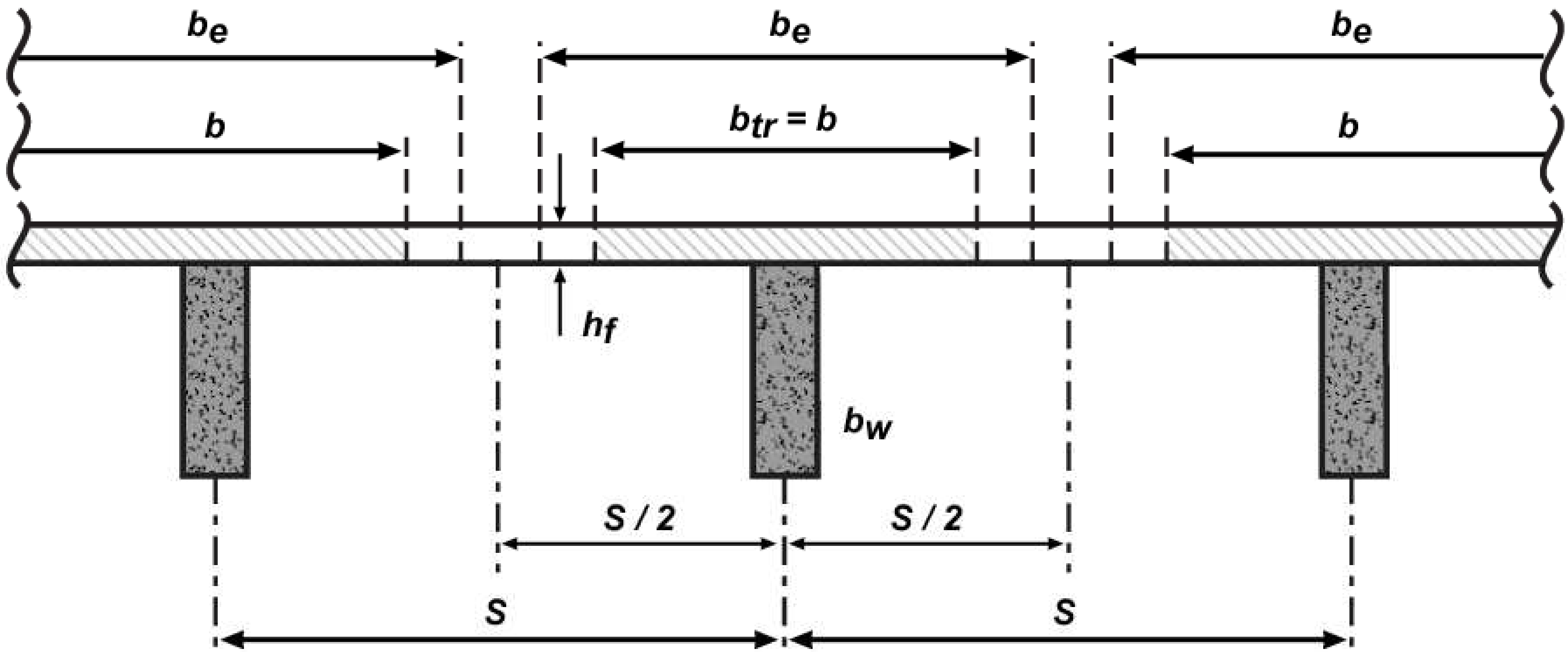
b_e = lesser of $(b_w + l_z/10)$
or $(b_w + S)/2$

Interior Beam

b_e = lesser of $(b_w + l_z/5)$
or S

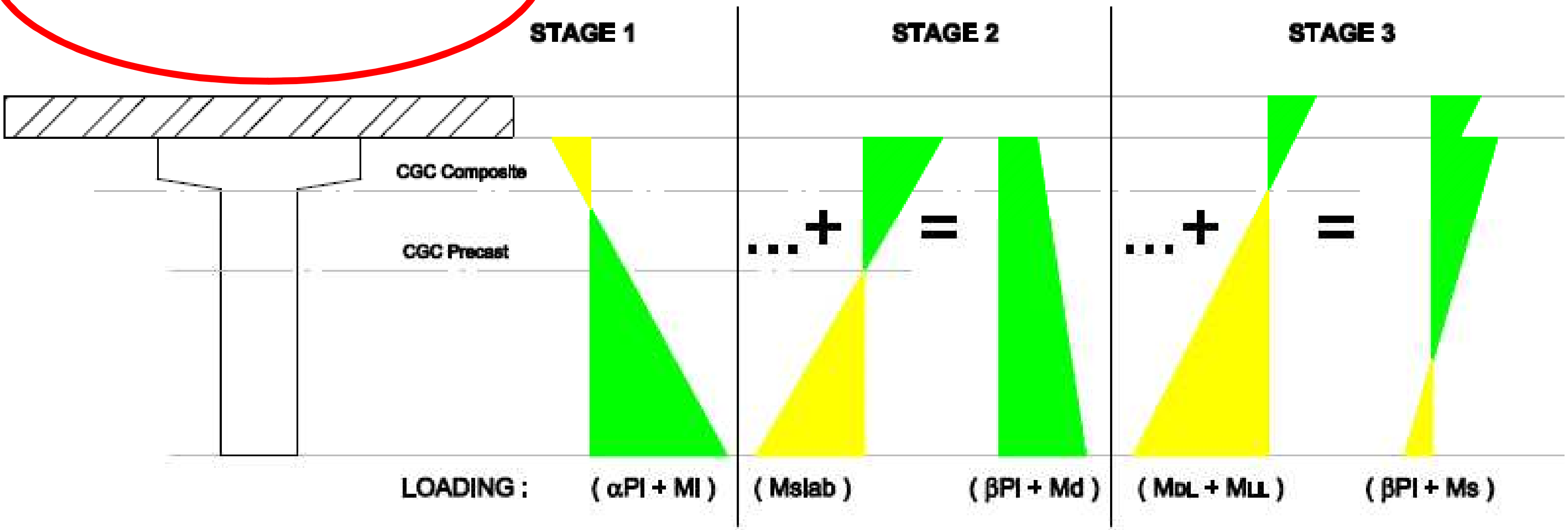
Where l_z = distance between points of zero moment

Transformed Vs Effective Flange Width



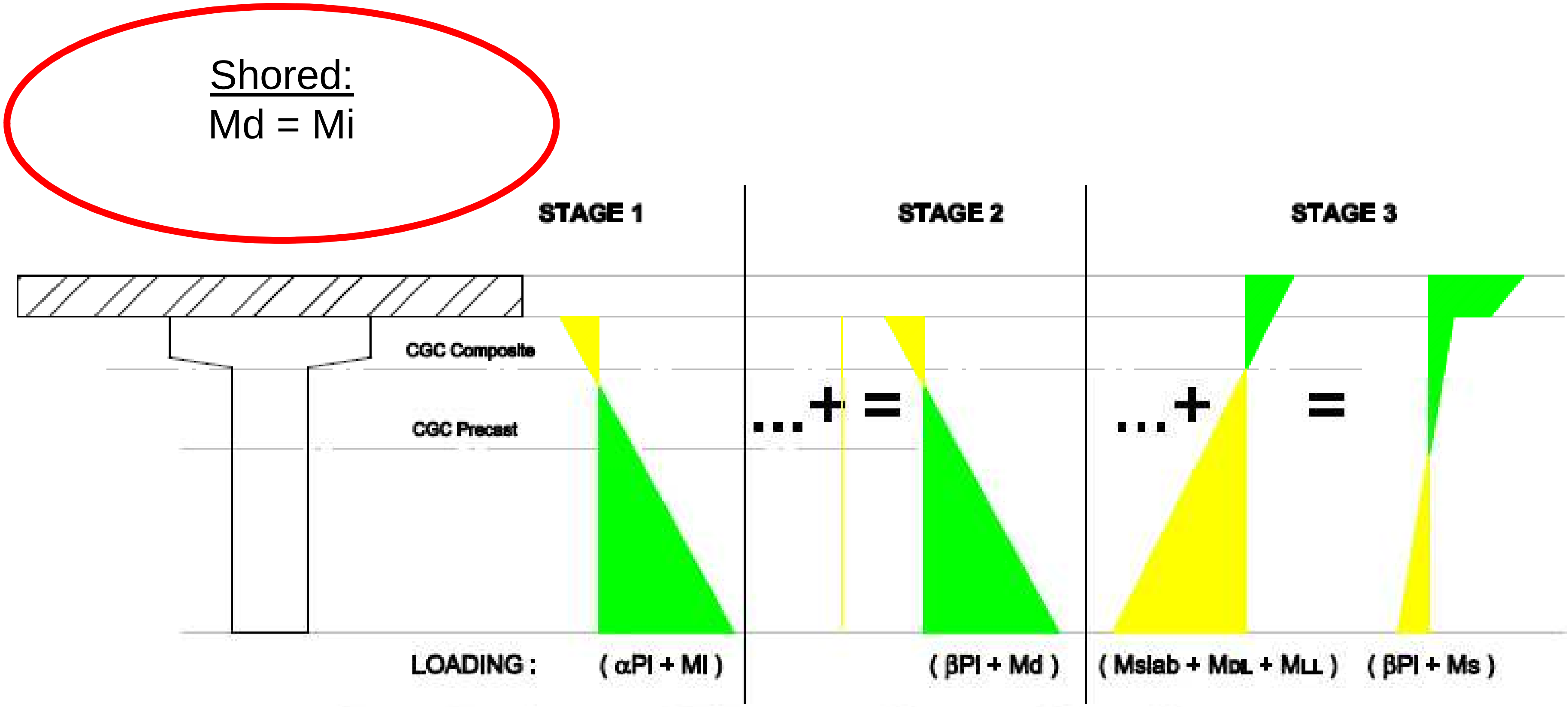
Working Stresses

Unshored:
Md = Mi + Mslab



Stress Distribution at Different Load Stages : Unshored Construction

Working Stresses



Stress Distribution at Different Load Stages : Shored Construction

Example 6-1

Determine the stress distributions at the various load stages of the floor slab shown below. Given the followings:

Span = 5m, Superimposed loads = 5 kN/m^2, Unshored Construction

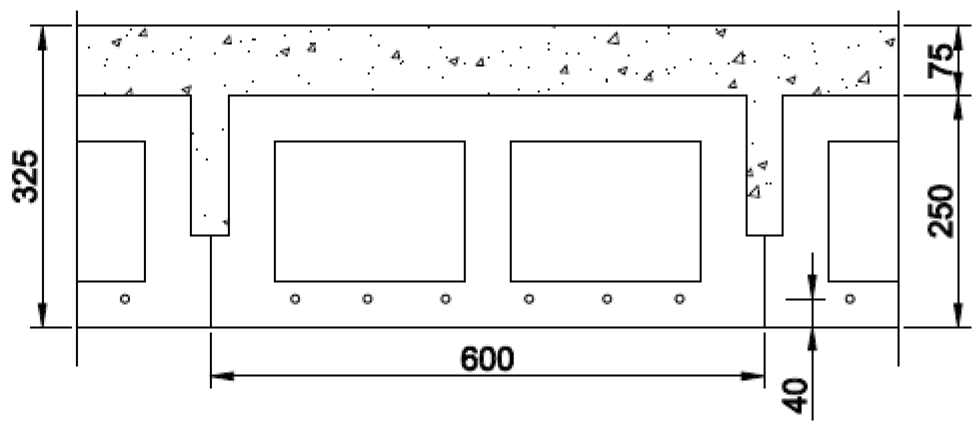
Prestress Force at Transfer = 24.3 kN/wire

Effective Prestress Force = 19.4 kN/wire

Section Properties:

A = 1.13 x 10^5 mm^2, I = 7.5 x 108 mm^4, $z_1 = z_2$ = 6.0 x 10^6 mm^3

fcu,precast = 40 N/mm^2, fcu,slab = 30 N/mm^2



Solution

Load Stage 1 – Precast Section

e = 125 – 40 = 85 mm, αPi = 6 x 24.3 = 145.8 kN

Beam self weight = 0.113 x 24 = 2.7 kN/m

Mi = 2.7 x 2.7²/8 = 8.4 kNm

$$f_1 = \frac{145.8 \times 10^3}{1.13 \times 10^5} - \frac{145.8 \times 85 \times 10^3}{6.0 \times 10^6} + \frac{8.4 \times 10^6}{6.0 \times 10^6}$$

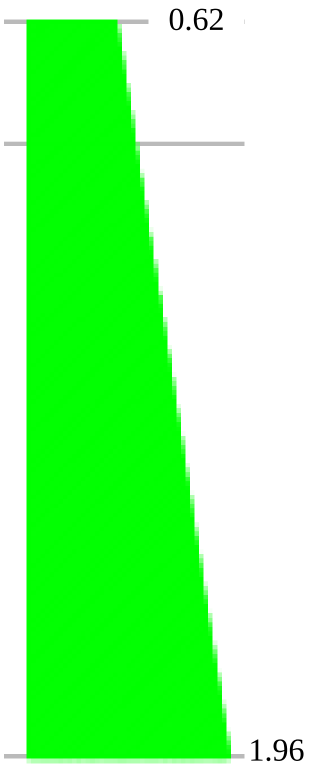
$$f_1 = 1.29 - 2.07 + 1.40$$

$$f_1 = 0.62 \text{ N/mm}^2$$

$$f_2 = \frac{145.8 \times 10^3}{1.13 \times 10^5} + \frac{145.8 \times 85 \times 10^3}{6.0 \times 10^6} - \frac{8.4 \times 10^6}{6.0 \times 10^6}$$

$$f_2 = 1.29 + 2.07 - 1.40$$

$$f_2 = 1.96 \text{ N/mm}^2$$



Solution

Load Stage 2 – Precast Section

βPi = 6 x 19.4 = 116.4 kN

Md = 8.4 + 0.075 x 0.6 x 24 x 5^2/8 = 11.8 kNm

$$f_1 = \frac{116.4 \times 10^3}{1.13 \times 10^5} - \frac{116.4 \times 85 \times 10^3}{6.0 \times 10^6} + \frac{11.8 \times 10^6}{6.0 \times 10^6}$$

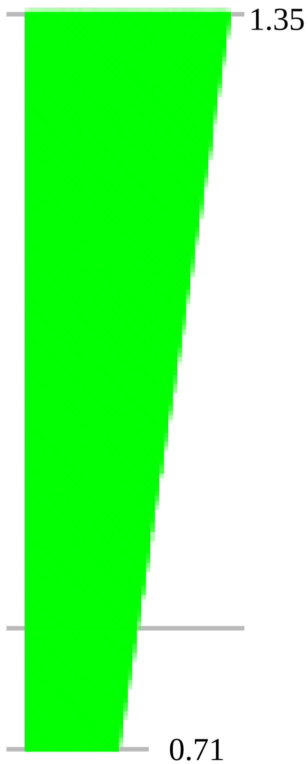
$$f_1 = 1.03 - 1.65 + 1.97$$

$$f_1 = 1.35 \text{ N/mm}^2$$

$$f_2 = \frac{145.8 \times 10^3}{1.13 \times 10^5} + \frac{145.8 \times 85 \times 10^3}{6.0 \times 10^6} - \frac{11.8 \times 10^6}{6.0 \times 10^6}$$

$$f_2 = 1.03 + 1.65 - 1.97$$

$$f_2 = 0.71 \text{ N/mm}^2$$



Solution

Load Stage 3 – Composite Section

Difference of fcu between precast and composite = 10N/mm^2

➔Assume a modular ratio of 1 (non-transformed section)

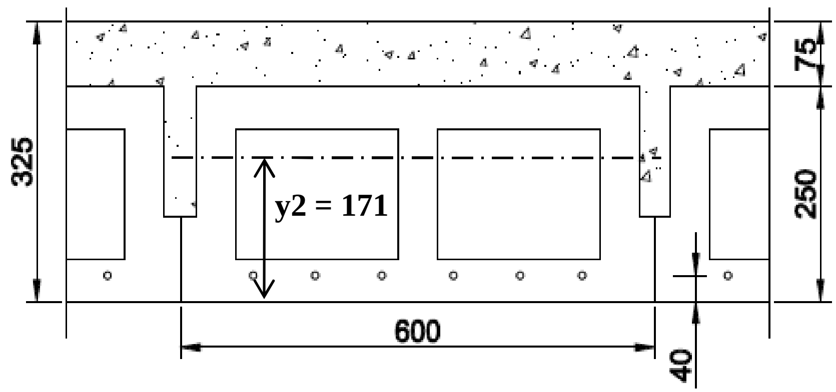
y_2 = 1.13 x10^5 + (600 x 75) / (1.13 x 10^5 x 125 + 600 x 75 x 288)

= 171 mm

I comp = 1.63 x 10^9 mm^4

Bending Moment for imposed load,

M_{IL} = 0.6 * 5 * 5^2/8 = 9.4 kNm



Solution to e.g. 16

Load Stage 3 – Composite Section

$$f_{1,slab} = \frac{9.4 \times 10^6}{1.63 \times 10^9} \times (325 - 171) = 0.89\ \text{N/mm}^2$$

$$f_{1,beam} = \frac{116.4 \times 10^3}{1.13 \times 10^5} - \frac{116.4 \times 85 \text{ x } 10^3}{6.0 \times 10^6} + \frac{11.8 \times 10^6}{6.0 \times 10^6} + \frac{9.4 \times 10^6}{1.63 \times 10^9} \times (250 - 171)$$

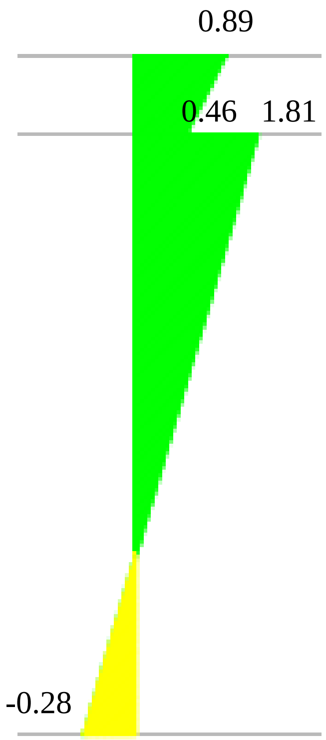
$$f_{1,beam} = 1.03 - 1.65 + 1.97 + 0.46$$

$$f_{1,beam} = 1.81\ \text{N/mm}^2$$

$$f_{2,beam} = \frac{116.4 \times 10^3}{1.13 \times 10^5} + \frac{116.4 \times 85 \text{ x } 10^3}{6.0 \times 10^6} - \frac{11.8 \times 10^6}{6.0 \times 10^6} - \frac{9.4 \times 10^6}{1.63 \times 10^9} \times 171$$

$$f_{2,beam} = 1.03 + 1.65 - 1.97 - 0.99$$

$$f_{2,beam} = -0.28\ \text{N/mm}^2$$



Allowable Stresses in Precast Section

Stage 1

- Flexural tensile stresses

  f_{tt} = 1.0 N/mm^2 (Class 1 members)

  f_{tt} = 0.45$(f_{ci})^{1/2}$ N/mm^2 (Class 2, pre-tensioned)

  f_{tt} = 0.36$(f_{ci})^{1/2}$ N/mm^2 (Class 2, post-tensioned)

- Flexural compressive stresses

  f_{ct} = 0.5f_{ci} N/mm^2 (flexural members)

  f_{ct} = 0.4f_{ci} N/mm^2 (near uniform distribution of prestress)

Where f_{ci} is the concrete strength at transfer of prestress

(> = 25 N/mm^2 - Cl 4.1.8.1)

Allowable Stresses in Composite Section

Stage 2 & Stage 3

- Flexural tensile stresses (Only for precast member)

  f_{ts} = 0 N/mm^2 (Class 1 members)

  f_{ts} = 0.45(f_{cu})$^{1/2}$ N/mm^2 (Class 2, pre-tensioned)

  f_{ts} = 0.36(f_{cu})$^{1/2}$ N/mm^2 (Class 2, post-tensioned)

- Flexural compressive stresses (Precast/Cast in situ member)

  f_{cs} = 0.33f_{cu} N/mm^2 (flexural members)

  f_{cs} = 0.4f_{cu} N/mm^2 (in statistically indeterminate structure)

Where f_{cu} is the design compressive strength of concrete

Inequalities for the 3 Load Stages

Combining the stresses at the 3 load stages and their stress limits, the following 7 inequalities were obtained:

Stage 1

$$\frac{\alpha P_i}{A} - \frac{\alpha P_i e}{z_1} + \frac{M_i}{z_1} \geq f_{tt} \; \ldots\ldots\ldots\ldots \; (31)$$

$$\frac{\alpha P_i}{A} + \frac{\alpha P_i e}{z_2} - \frac{M_i}{z_2} \leq f_{ct} \; \ldots\ldots\ldots\ldots \; (32)$$

Stage 2

$$\frac{\beta P_i}{A} - \frac{\beta P_i e}{z_1} + \frac{M_d}{z_1} \leq f_{cs} \; \ldots\ldots\ldots\ldots \; (33)$$

$$\frac{\beta P_i}{A} + \frac{\beta P_i e}{z_2} - \frac{M_d}{z_2} \geq f_{ts} \; \ldots\ldots\ldots\ldots \; (34)$$

Stage 3

$$\frac{\beta P_i}{A} - \frac{\beta P_i e}{z_1} + \frac{M_d}{z_1} + \frac{(M_s - M_d)}{z_{1,beam}} \leq f_{cs} \; \ldots\ldots \; (35)$$

$$\frac{\beta P_i}{A} + \frac{\beta P_i e}{z_2} - \frac{M_d}{z_2} - \frac{(M_s - M_d)}{z_{2,comp}} \geq f_{ts} \; \ldots\ldots \; (36)$$

$$\frac{(M_s - M_d)}{z_{1,comp}} \leq f_{cs,slab} \; \ldots\ldots\ldots\ldots \; (37)$$

Shored construction: Md = Mi

Unshored construction: Md = Mi + Mslab

Adequacy of a Composite Section

Combining (31) and (35),

$$z_{1,beam} \geq \frac{\alpha (M_s - M_d)}{(\alpha f_{cs} - \beta f_{tt}) + \frac{1}{z_1}(\beta M_i - \alpha M_d)} \quad \ldots \ldots \ldots (38)$$

Combining (32) and (36),

$$z_{2,comp} \geq \frac{\alpha (M_s - M_d)}{(\beta f_{ct} - \alpha f_{ts}) + \frac{1}{z_2}(\beta M_i - \alpha M_d)} \quad \ldots \ldots \ldots (39)$$

Also from (37),

$$z_{1,comp} \geq \frac{(M_s - M_d)}{f_{cs,slab}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (40)$$

Design of Prestress Force

The range of prestressing force for a given e can be found from the following inequalities:

$$P_i \geq \frac{(z_1 f_{tt} - M_i)}{\alpha\left(\frac{z_1}{A} - e\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

$$P_i \leq \frac{(z_2 f_{ct} + M_i)}{\alpha\left(\frac{z_2}{A} + e\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

$$P_i \leq \frac{(z_1 f_{cs} - M_d) - \frac{z_1}{z_{1,beam}}(M_s - M_d)}{\beta\left(\frac{z_1}{A} - e\right)} \quad \ldots\ldots\ldots\ldots\ldots (43)$$

$$P_i \geq \frac{(z_2 f_{ts} + M_d) + \frac{z_2}{z_{2,comp}}(M_s - M_d)}{\beta\left(\frac{z_2}{A} + e\right)} \quad \ldots\ldots\ldots\ldots\ldots (44)$$

The inequalities sign in (41) & (43) will be reversed if the denominator becomes -ve

Magnel Diagram

Magnel diagram can be plotted from the followings:

$$e \leq \frac{1}{\alpha P_i}(M_i - z_1 f_{tt}) + \frac{z_1}{A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

$$e \leq \frac{1}{\alpha P_i}(M_i + z_2 f_{ct}) - \frac{z_2}{A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

$$e \geq \frac{1}{\beta P_i}\left(M_d + \frac{z_1}{z_{1,beam}}(M_s - M_d) - z_1 f_{cs}\right) + \frac{z_1}{A} \ldots\ldots (47)$$

$$e \geq \frac{1}{\beta P_i}\left(M_d + \frac{z_2}{z_{2,comp}}(M_s - M_d) + z_2 f_{ts}\right) - \frac{z_2}{A} \ldots\ldots (48)$$

$$e_{mp} = y_2 - (h_c)_{min} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

Example 6-2

The following figure shows a cross section of a simply supported bridge made up of 6 nos JKR PRT2 class 1 precast post-tensioned prestressed beam and a 200mm cast in situ slab. Design the precast prestressed beam using the following information:

Span = 20.6m; Unshored Construction

Loading/beam: Wslab=8.11kN/m; SDL=3.73kN/m; LL=14.56kN/m

Beam: fci = 45N/mm^2; fcu = 50N/mm^2; E = 36 kN/mm^2; A = 488350 mm^2

y1 = 575.80 mm; y2 = 774.20 mm; I = 8.506 x 10^{10} mm^4

Slab : fcu = 40N/mm^2; E = 34 kN/mm^2; h = 200 mm

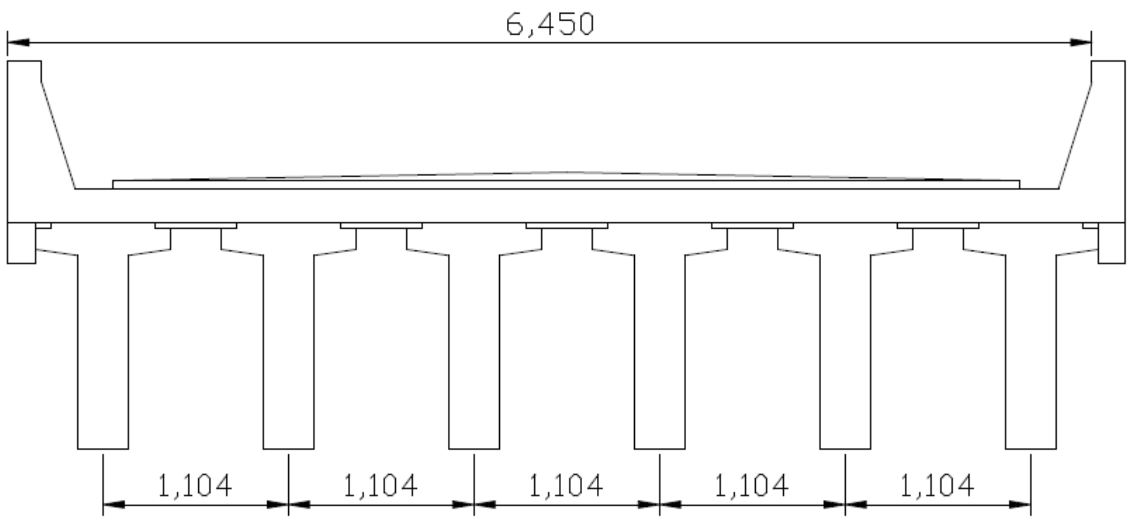
Prestress Steel (12.9mm dia 7-wire super strand):

Fpu = 186 kN

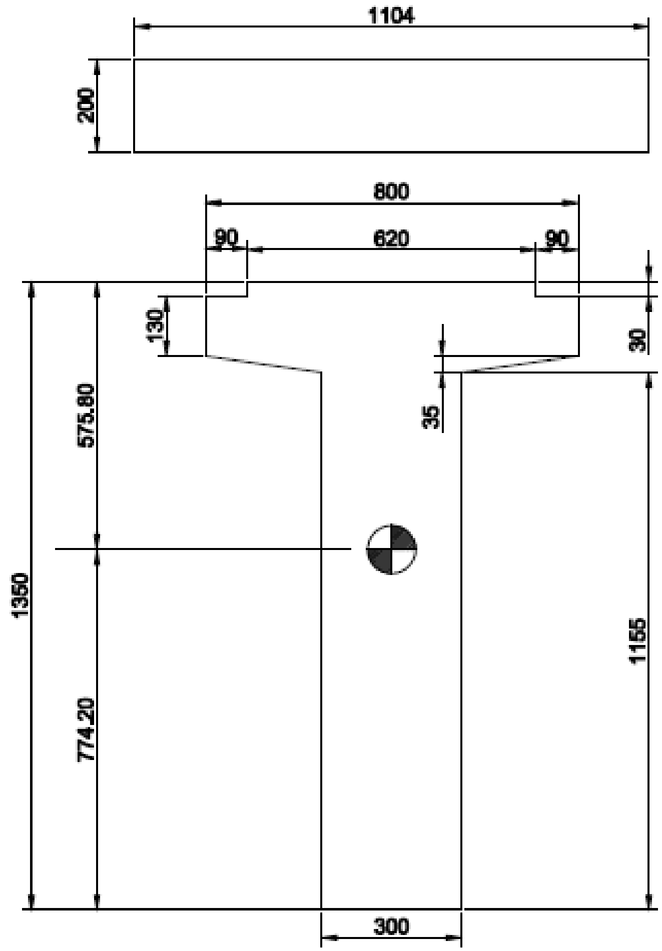
Aps = 100 mm^2

fpu = 1860 N/mm^2

%UTS = 70%



Solution



Modular ratio = 34/36 = 0.94

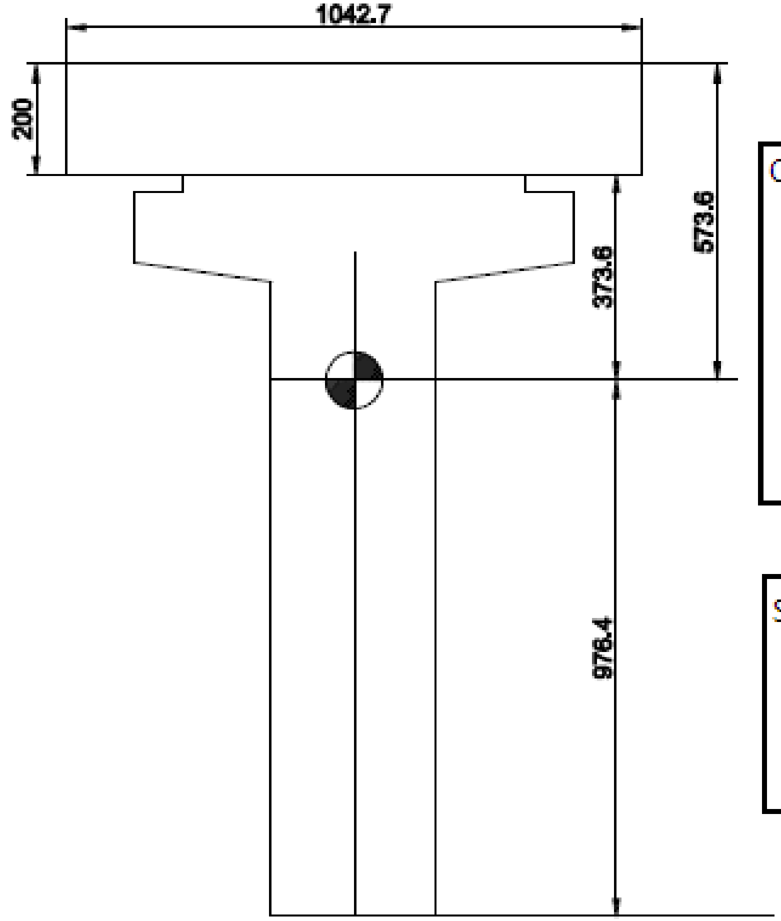
be the lesser of 20.6/5 = 4.12m and 1.104m

Take be = 1104 mm

btr = 0.94 * 1104 = 1042.7mm

Composite Section			btr =	1042.7 mm
m =	0.94		h_{comp} =	1550 mm
A_{comp} =	696883.3 mm^2		I_{comp} =	1.52E+11 mm^4
$y_{1,comp}$ =	573.6 mm		$z_{1,comp}$ =	2.66E+08 mm^3
$y_{2,comp}$ =	976.4 mm		$z_{2,comp}$ =	1.56E+08 mm^3
$y_{1,beam}$ =	373.6 mm		$z_{1,beam}$ =	4.08E+08 mm^3

Solution



Composite Section			btr =	1042.7 mm
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$y_{1,beam}$ =	373.6 mm		$z_{1,beam}$ =	4.08E+08 mm^3

Stress Limits			
ftt =	-1 N/mm^2	fts =	0 N/mm^2
fct =	22.5 N/mm^2	fcs =	16.5 N/mm^2
fcs,slab =	10 N/mm^2		

Solution

Moments

Wsw =	12.21 kN/m
Wslab =	8.11 kN/m
LL =	14.56 kN/m
DL =	3.73 kN/m
Span =	20.60 m
Mi =	647.61 kNm
Mslab =	430.19 kNm
Md =	1077.81 kNm (Unshored)
M_{IL} =	970.19 kNm
Ms =	2048.00 kNm

Check Section Adequacy

$\alpha(Ms-Md)$ =	8.732E+08	
$\alpha fcs - \beta ftt$ =	15.57	
$\beta fct - \alpha fts$ =	16.2	
$\beta Mi - \alpha Md$ =	-5.037E+08	
$z_{1,beam}$ =	7.181E+07	ok
$z_{2,comp}$ =	7.518E+07	ok
$z_{1,comp}$ =	9.702E+07	ok

Composite Section

		btr =	1042.7 mm
m =	0.94	h_{comp} =	1550 mm
A_{comp} =	696883.3 mm^2	I_{comp} =	1.52E+11 mm^4
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$y_{1,beam}$ =	373.6 mm	$z_{1,beam}$ =	4.08E+08 mm^3

UTM

Solution

Minimum Prestressing Force

$(z_1 f_{tt} - M_i)$=	-795338048.2	z_1/A - e =	-230.5
$(z_2 f_{ct} + M_i)$=	3119648793	z_2/A + e =	758.0
$(z_1 f_{cs} - M_d)$=	1359652830	Num43 =	1.01E+09
$(z_2 f_{ts} + M_d)$=	1077808094	Num44 =	1.76E+09

P_{41} =	<=	3833.8	kN
P_{42} =	<=	4573.1	kN
P_{43} =	>=	-6077.0	kN
P_{44} =	>=	3225.6	kN
Pmin	=	3225.6	kN

The range of prestressing force for a given e can be found from the following inequalities:

$$P_i \geq \frac{(z_1 f_{tt} - M_i)}{\alpha\left(\frac{z_1}{A} - e\right)} \quad \dots\dots (41)$$

$$P_i \leq \frac{(z_2 f_{ct} + M_i)}{\alpha\left(\frac{z_2}{A} + e\right)} \quad \dots\dots (42)$$

$$P_i \leq \frac{(z_1 f_{cs} - M_d) - \frac{z_1}{z_{1,beam}}(M_s - M_d)}{\beta\left(\frac{z_1}{A} - e\right)} \quad \dots\dots (43)$$

$$P_i \geq \frac{(z_2 f_{ts} + M_d) + \frac{z_2}{z_{2,comp}}(M_s - M_d)}{\beta\left(\frac{z_2}{A} + e\right)} \quad \dots\dots (44)$$

Solution

Magnel Diagram

e = m/Pi + c				
m	Nmm	kNmm	c	mm
m_{45} =	883708942.5	883.709	c_{45} =	302.50
m_{46} =	3466276437	3466.276	c_{46} =	-224.98
m_{47} =	-1400762182	-1400.762	c_{47} =	302.50
m_{48} =	2444898192	2444.898	c_{48} =	-224.98
1/Pi (kN^{-1})	0.31002459	0.284463	0.256016	
Pi (kN)	3225.6	3515.4	3906	
No of Strand	24.8	27	30	
Fpu=	186 kN			
%UTS =	70 %			

Solution

e
600
500
400
300
200
100
-100
-200
-0.02
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
0.28
0.3
0.32
0.34
0.36
0.38
1/Pi x 1e-3 (1/kN)
27 Nos of Strand
(Pi = 3515.4 kN, e = 533)
1/Pi = 0.31002459*1e-3
Pi = 3225.6 kN
f(x)=883.709x+302.5
f(x)=3466.276x-225
f(x)=-1400.762x+302.5
f(x)=2444.898x-225
f(x)=533
Suggested Pi & e
Practical min Pi

Solution

y bar = 774.2 mm

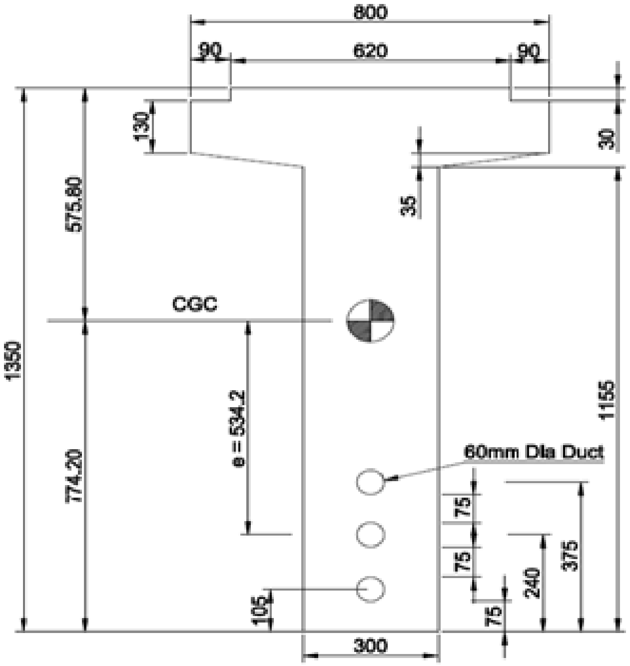
Arrangement of Tendons

No of strands =	27
No of strand / cable =	9
No of cable =	3
No of strand/row =	3
Diameter of strand =	12.9 mm
Aps =	100 mm^2
Dia of duct: The larger of	
(4*12.9 + 6) =	57.6 mm
and $(4*1.5*100/\pi)^{1/2}$ =	13.8
==>	57.6 mm
Use	60 mm
Duct Spacing: Sy =	60 mm
Sx =	60 mm
Use Sx = Sy =	75 mm

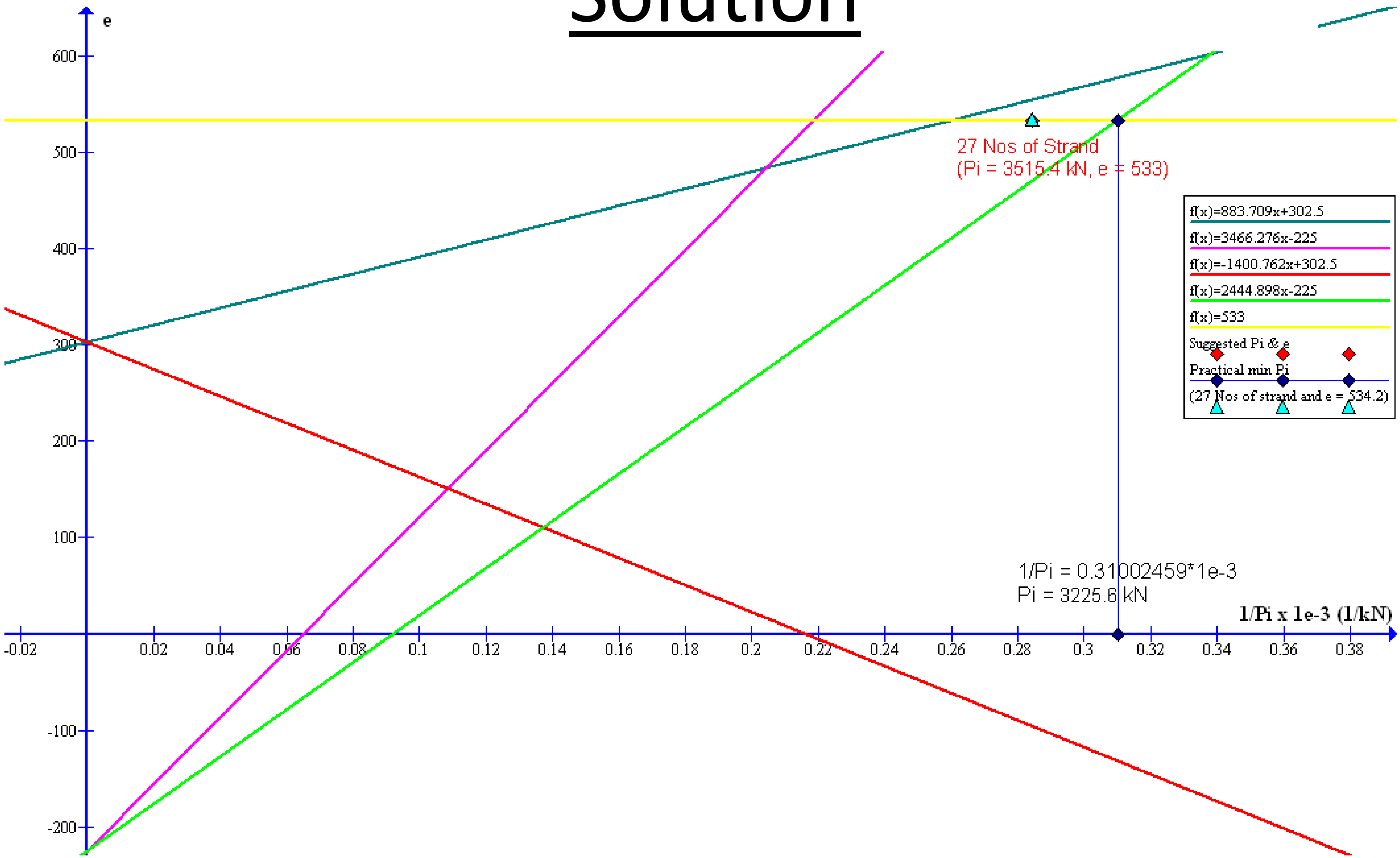
The arrangement of cables produced a new value of e = 534.2mm (old e = 533mm). From Magnel Diagram, this value is still within the safe zone.

row	Bil	Dist	Ay
1	1	105	105
2	1	240	240
3	1	375	375
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8			0
9			0
10			0
	3		720
	yms =	240	

e = 534.2 mm



Solution


e
600
500
400
300
200
100
-100
-200
-0.02
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
0.28
0.3
0.32
0.34
0.36
0.38
1/Pi x 1e-3 (1/kN)
27 Nos of Strand
(Pi = 3515.4 kN, e = 533)
1/Pi = 0.31002459*1e-3
Pi = 3225.6 kN
f(x)=883.709x+302.5
f(x)=3466.276x-225
f(x)=-1400.762x+302.5
f(x)=2444.898x-225
f(x)=533
Suggested Pi & e
Practical min Pi
(27 Nos of strand and e = 534.2)

Ultimate Strength

Remember to consider the followings:

- Ultimate Flexural Strength
- Ultimate Shear Strength
  - Horizontal Shear
  - Vertical Shear