Logic Gates and Boolean Algebra

- Logic Gates
  - Inverter, OR, AND, Buffer, NOR, NAND, XOR, XNOR
- Boolean Theorem
  - Commutative, Associative, Distributive Laws
  - Basic Rules
- DeMorgan's Theorem
- Universal Gates
  - NAND and NOR
- Canonical/Standard Forms of Logic
  - Sum of Product (SOP)
  - Product of Sum (POS)
  - Minterm and Maxterm

SOP and POS

- All boolean expressions can be converted to two standard forms:
  - SOP: Sum of Product
  - POS: Product of Sum
- Standardization of boolean expression makes evaluation, simplification, and implementation of boolean expressions more systematic and easier

Sum of Product (SOP)

- Boolean expressions are expressed as the sum of product, example:

minterm

$$ABC + CDE + \overline{B}C\overline{D}$$

literal

- Each variable or their complements is called *literals*
- Each product term is called *minterm*

SOP (cont.)

- In SOP, a single overbar cannot extend over more than one variable, example:

$$AB + A\overline{BC}$$ ← Not SOP because $\overline{\text{BC}}$

- *Standard SOP* forms must contain all of the variables in the domain of the expression for each product term, example:

$$\bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$$

SOP (cont.)

- In the following SOP form,

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

  - How many minterms are there? => 3
  - How many literals in the second product term? => 2
  - Is it in a standard SOP form? => No
  - How do we convert the boolean expression to standard SOP form?

SOP (cont.)

- To convert SOP to its standard form, we use the boolean rules
  - $A + \overline{A} = 1$
  - $A(B + C) = AB + AC$
- We have

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

- The first product term is missing the variable D, and the second product term is missing C and D

SOP (cont.)

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Apply $D + \overline{D} = 1$ and $C + \overline{C} = 1$

$$= A\overline{B}C(D+\overline{D}) + \overline{A}\,\overline{B}(C+\overline{C})(D+\overline{D}) + AB\overline{C}D$$

Apply the distributive law

$$= A\overline{B}CD + A\overline{B}C\overline{D} + (\overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C})(D+\overline{D}) + AB\overline{C}D$$

$$= A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\overline{D} +$$

$$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$ ← Standard SOP form

Product of Sum (POS)

- Boolean expressions are expressed as the product of sum, example:

$$(\overline{A}+B)(A+\overline{B}+C)$$

literal

maxterm

POS (cont.)

- In POS, a single overbar cannot extend over more than one variable, example:

$$(\overline{A}+B)(A+\overline{B+C})$$

Not SOP because $\overline{B+C}$

- *Standard POS* forms must contain all of the variables in the domain of the expression for each sum term, example:

$$(A+B+C)(A+B+\overline{C})(A+\overline{B}+C)$$

POS (cont.)

- In the following POS form,

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

  - Is it in a standard POS form? => No
  - How do we convert the boolean expression to standard POS form?

POS (cont.)

- To convert POS to its standard form, we use the boolean rules
  - $A \cdot \overline{A} = 0$
  - $A + BC = (A + B)(A + C)$
- We have

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

- The first sum term is missing the variable D, and the second sum term is missing A

POS (cont.)

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

Apply D.$\overline{\text{D}}$ = 0 and A.$\overline{\text{A}}$ = 0 to first and second terms

$$(A+\overline{B}+C+D.\overline{D})(A.\overline{A}+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

Expand first and second terms

$$(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})$$
$$(A+\overline{B}+\overline{C}+D)$$

Standard POS form

Minterm and Maxterm

- Minterm: Product terms in SOP
- Maxterm: Sum terms in POS
- Standard forms of SOP and POS can be derived from truth tables

A	B	C	Z	
0	0	0	0	$A+B+C$
0	0	1	1	$\overline{A}\overline{B}C$
0	1	0	0	$A+\overline{B}+C$
0	1	1	0	$A+\overline{B}+\overline{C}$
1	0	0	0	$\overline{A}+B+C$
1	0	1	1	$A\overline{B}C$
1	1	0	1	$AB\overline{C}$
1	1	1	1	ABC

For SOP form,

$$Z = \overline{A}\overline{B}C + A\overline{B}C + AB\overline{C} + ABC$$
$$= \sum m(1,5,6,7)$$

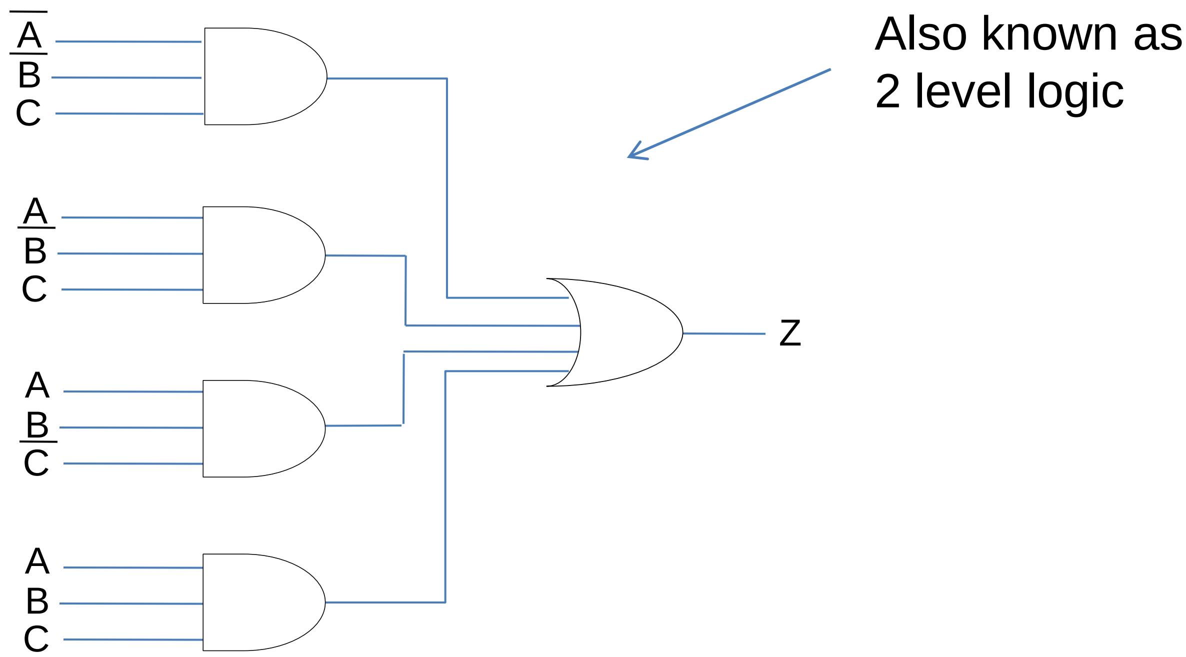
For POS form,

$$Z = (A+B+C)(A+\overline{B}+C)$$
$$(A+\overline{B}+\overline{C})(\overline{A}+B+C)$$
$$= \prod M(0,2,3,4)$$

Minterm and Maxterm

- How to design minterms – AND-OR logic

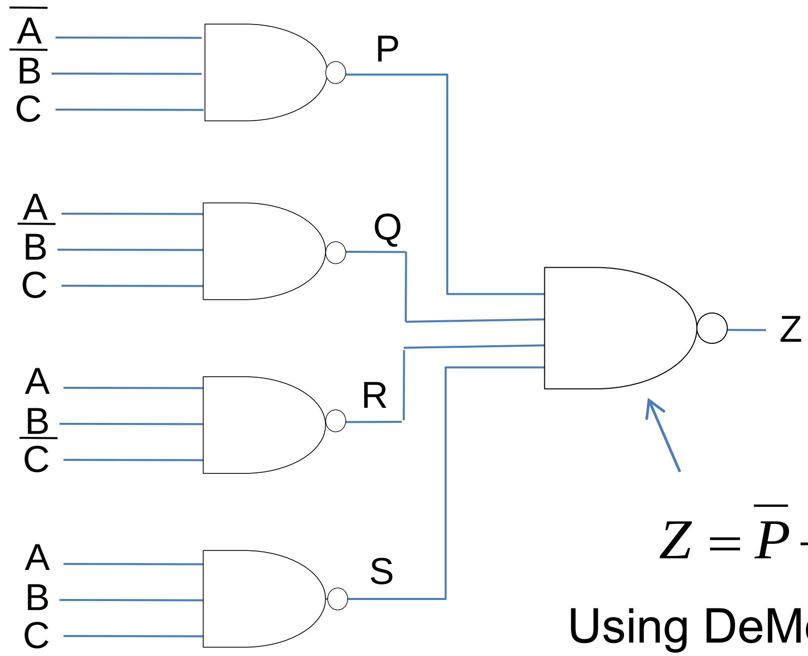
$$Z = \overline{A}\,\overline{B}C + A\overline{B}C + AB\overline{C} + ABC$$



Minterm and Maxterm

- How to design minterms – NAND-NAND Logic

$$Z = \overline{A}\,\overline{B}C + A\overline{B}C + AB\overline{C} + ABC$$



$$Z = \overline{P} + \overline{Q} + \overline{R} + \overline{S}$$

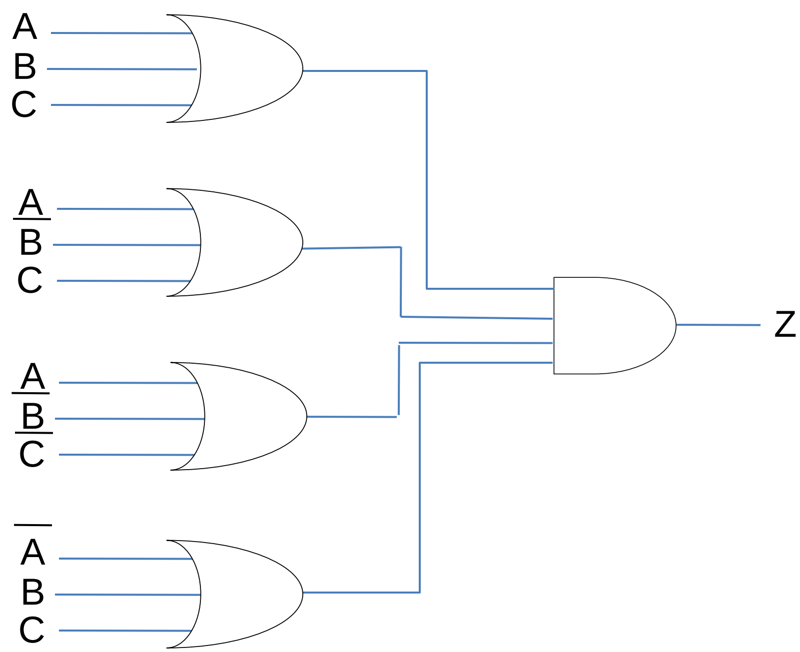
Using DeMorgan's Theorem

$$Z = \overline{P \cdot Q \cdot R \cdot S}$$

Minterm and Maxterm

- How to design maxterms – OR-AND Logic

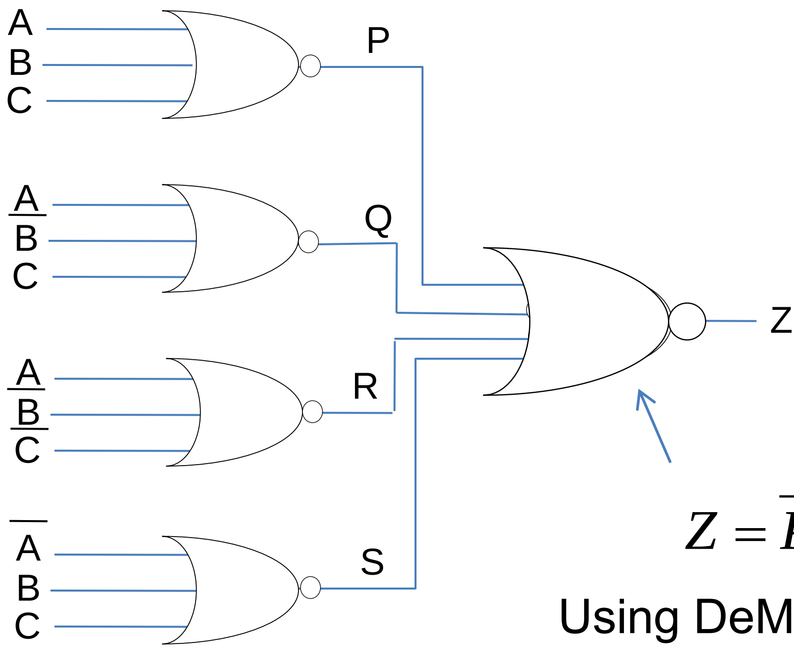
$$Z = (A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+C)$$



Minterm and Maxterm

- How to design maxterms – NOR-NOR Logic

$$Z = (A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+C)$$



$$Z = \overline{P} \cdot \overline{Q} \cdot \overline{R} \cdot \overline{S}$$

Using DeMorgan's Theorem

$$Z = \overline{P+Q+R+S}$$

Minterm and Maxterm

- Can the minterm and maxterm logic be optimized?
  - Yes, using Boolean algebra – explore yourself
  - Yes, using Karnaugh maps – next lecture