Prestressed Concrete Design (SAB 4323)

Preliminary Design for Flexure

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Analysis or Design?

Analysis

- Check if the specified design criteria at every section along the member are satisfied
- Beam's description and characteristics given (loading, span, cross sectional dimensions, material properties etc)

Design :

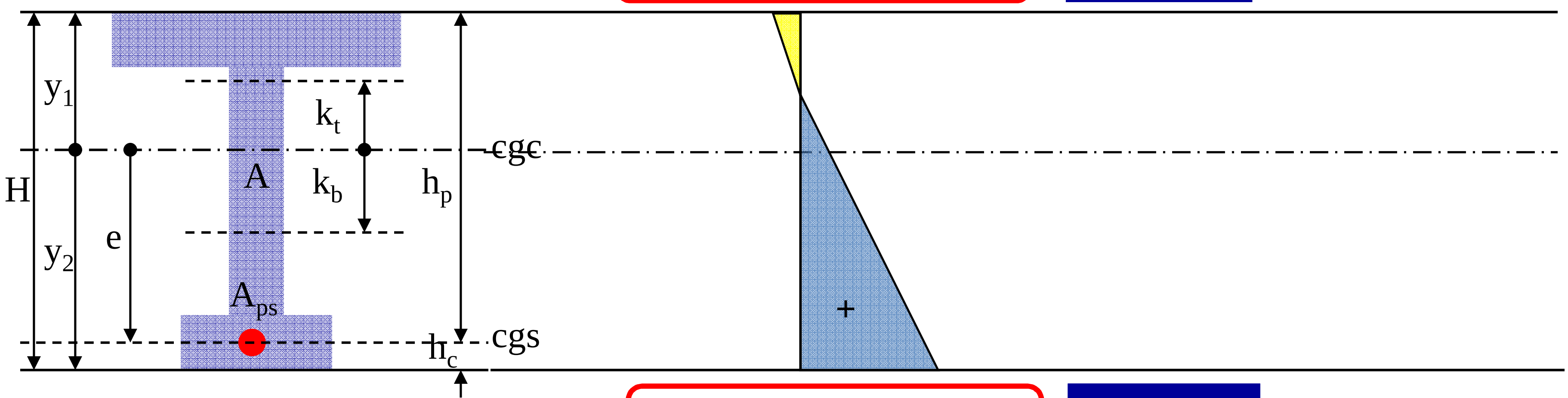
- Reverse process of analysis
- Involves finding of member size required and details of prestressing force and tendon profile

Basic Inequalities

Inequalities At Transfer

- Consider at mid span of a simply supported beam

$$\alpha P_i/A - \alpha P_i e/z_1 + M_i/z_1 \quad >= f_{tt}$$

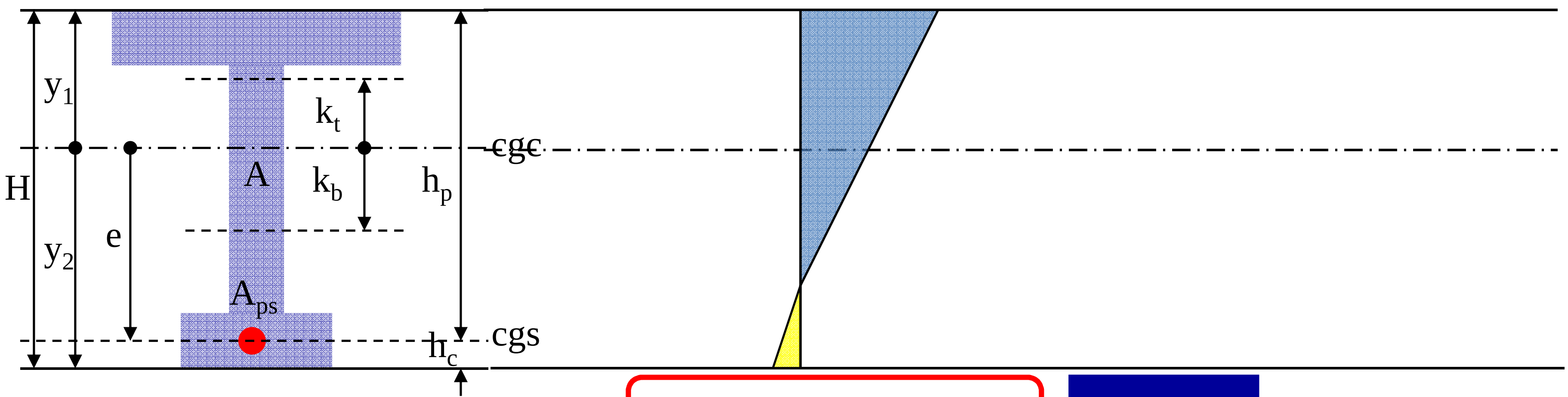


$$\alpha P_i/A + \alpha P_i e/z_2 - M_i/z_2 \quad <= f_{ct}$$

Inequalities At Service

- Consider at mid span of a simply supported beam

$$\beta P_i/A - \beta P_i e/z_1 + M_s/z_1 \quad <= \; f_{cs}$$



$$\beta P_i/A + \beta P_i e/z_2 - M_s/z_2 \quad >= \; f_{ts}$$

Inequalities At Service

Writing down all the inequalities:

$\alpha P_i/A \; - \; \alpha P_i e/z_1 \; + \; M_i/z_1 >= \boldsymbol{f}_{tt}$ ....(1)

$\alpha P_i/A \; + \; \alpha P_i e/z_2 \; - \; M_i/z_2 <= \boldsymbol{f}_{ct}$ ....(2)

$\beta P_i/A \; - \; \beta P_i e/z_1 \; + \; M_s/z_1 <= \boldsymbol{f}_{cs}$ ....(3)

$\beta P_i/A \; + \; \beta P_i e/z_2 \; - \; M_s/z_2 >= \boldsymbol{f}_{ts}$ ....(4)

By combining inequalities (1) & (3) and (2) & (4)

$z_1 >= (\alpha M_s - \beta M_i) \,/\, (\alpha \boldsymbol{f}_{cs} - \beta \boldsymbol{f}_{tt})$....................(5)

$z_2 >= (\alpha M_s - \beta M_i) \,/\, (\beta \boldsymbol{f}_{ct} - \alpha \boldsymbol{f}_{ts})$....................(6)

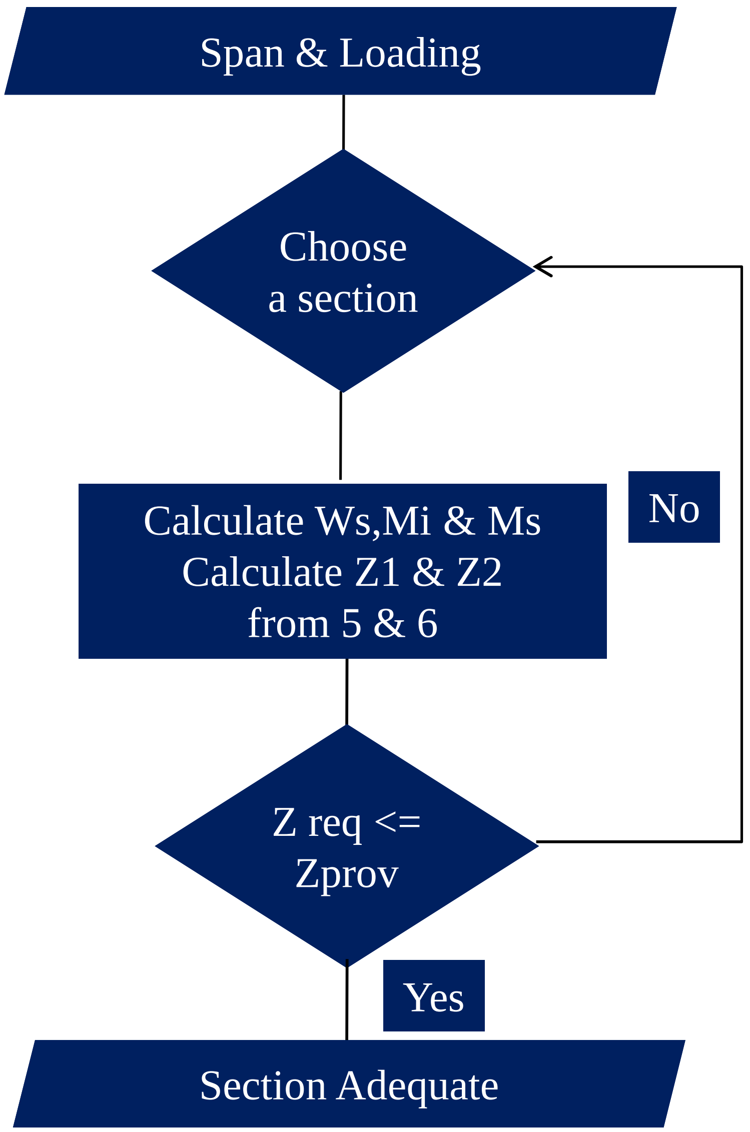
Beware of +ve and –ve values!

Derive (5) & (6)!

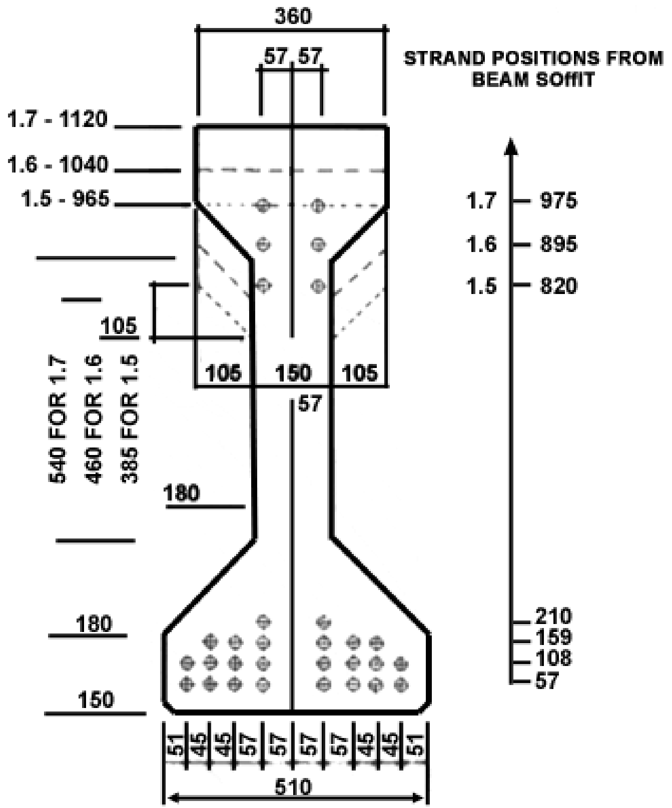
Section Selection

- From (5) & (6), a suitable section can be selected
- Both z_1 and z_2 depend on M_i and M_s
- M_i and M_s can be determined if the member self weight is known
- However, the self weight can only be determined if the section size (hence z_1 and z_2) is known
- In general, the solution can be obtained using trial and error method or using standard section

Section Adequacy Flowchart

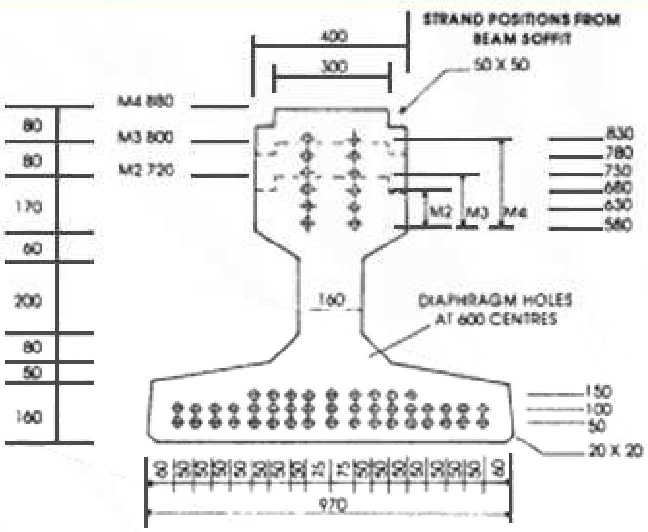


I BEAMS

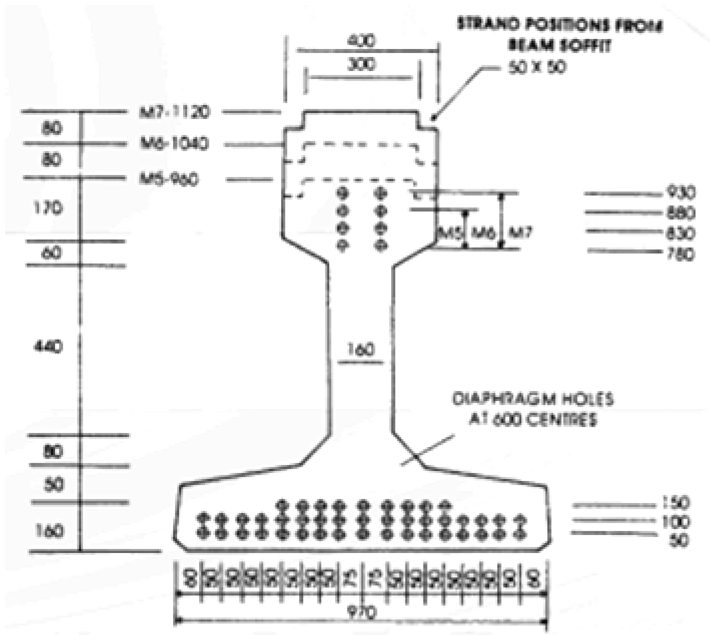


DIMENSIONS AND SECTION PROPERTIES OF 1-5, 1-6 AND 1-7 BEAMS				
DESCRIPTION \ I BEAM TYPE		1.5	1.6	1.7
MAX LENGTH	L (m)	18.3	19.8	21.3
DEPTH	D (mm)	965	1040	1120
WEIGHT	W (kN/m)	6.63	6.91	7.20
SECTIONAL AREA	A (mm^2)	272625	283875	295875
NEUTRAL AXIS	Yt (mm)	538	579	623
	Yb (mm)	427	461	497
MOMENT OF NERTIA	Ixx (mm^4)	28.15×10^9	34.46×10^9	42.09×10^9
SECTION MODULII	Zt (mm^3)	52.32×10^6	59.56×10^6	67.67×10^6
	Zb (mm^3)	65.80×10^6	74.76×10^6	84.68×10^6

M BEAMS



DIMENSIONS AND SECTION PROPERTIES OF M2, M3, M4					
DESCRIPTION \ M BEAM TYPE			M2	M3	M4
SPAN RANGE	L	(m)	16.0 - 18.0	17.5 - 19.5	19.0 - 21.5
DEPTH	D	(mm)	720	800	880
WEIGHT	W	(kN/m)	7.71	8.49	9.26
SECTIONAL AREA	A	(mm^2)	316650	348650	380650
NEUTRAL AXIS	Yt	(mm)	455	490	527
	Yb	(mm)	265	310	353
MOMENT OF NERTIA	Ixx	(mm^4)	16.20×10^9	23.02×10^9	30.94×10^9
SECTION MODUL II	Zt	(mm^3)	35.64×10^6	46.96×10^6	58.77×10^6
	Zb	(mm^3)	61.04×10^6	74.31×10^6	87.57×10^6



DIMENSIONS AND SECTION PROPERTIES OF M5, M6, M7					
DESCRIPTION \ M BEAM TYPE			M5	M6	M7
SPAN RANGE	L	(m)	20.5 - 22.5	22.0 - 24.0	23.5 - 26.0
DEPTH	D	(mm)	960	1040	1120
WEIGHT	W	(kN/m)	8.64	9.42	10.20
SECTIONAL AREA	A	(mm^2)	355050	387050	419050
NEUTRAL	Yt	(mm)	603	631	660
AXIS	Yb	(mm)	357	409	460
MOMENT OF NERTIA	Ixx	(mm^4)	35.81×10^9	47.56×10^9	60.46×10^9
SECTION	Zt	(mm^3)	59.39×10^6	75.39×10^6	91.53×10^6
MODUL II	Zb	(mm^3)	100.33×10^6	116.23×10^6	131.54×10^6

Example 3-1

A 20m span simply supported beam for a bridge construction is to be designed using class 1 post-tensioned prestressed concrete. The beam is subjected to a characteristic live load of 20kN/m in addition to its own self weight. The initial prestressing force is 2000kN with an eccentricity of 500mm. The short and long term losses of prestress are estimated to be 10% and 20% respectively. With f_{ci} = 30 N/mm^2 and f_{cu} = 50 N/mm^2 select a suitable section for the beam using,

1. Rectangular section
2. Standard M beams

Solution

Given:

Span = 20m; f_{ci} = 30 N/mm^2 ; f_{cu} = 50 N/mm^2 and class 1 category

P_i = 2000kN and e = 500 mm

$\alpha = 0.9$, $\beta = 0.8$

Stress Limits:

At transfer

$f_{ct} = 0.5 f_{ci} = 15$ N/mm^2 and $f_{tt} = 1.0$ N/mm^2

At service

$f_{cs} = 0.33 f_{cu} = 16.5$ N/mm^2 and $f_{ts} = 0$ N/mm^2

Solution

1) Rectangular Section

try: b = 300mm and h = 1300 mm

$A = 390000\ mm^2$; $z_1 = z_2 = bh^2/6 = 84.5 \times 10^6\ mm^3$

Self wt, $W_{sw} = 24 \times 0.39 = 9.36$ kN/m

$M_i = 9.36 \times 20^2/8 = 468$ kNm

Total service load, $W_s = 20 + 9.36 = 29.36$ kN/m

$M_s = 29.36 \times 20^2/8 = 1468$ kNm

Design as RC
Size: 200 x 2500
2 layers of 3T25
l/d actual = 8.2
l/d all = 11.9

Required Section Modulus

from (5): $z_1 \geq (0.9 \times 1468 – 0.8 \times 468) \times 10^6 / (0.9 \times 16.5 – 0.8(-1))$

$\geq 60.50 \times 10^6\ mm^3$

z_1 provided = $84.5 \times 10^6\ mm^3$ → Ok

from (6): $z_2 \geq (0.9 \times 1468 – 0.8 \times 468) \times 10^6 / (0.8 \times 15.0 – 0.9(0))$

$\geq 78.90 \times 10^6\ mm^3$

z_2 provided = $84.5 \times 10^6\ mm^3$ → Ok

Solution

2) Standard Section – M beams

try: M6 beams

$A = 387050\ mm^2$; $z_1 = 75.39 \times 10^6\ mm^3$; $z_2 = 116.23 \times 10^6\ mm^3$

Self wt, $W_{sw} = 9.42\ kN/m$

$M_i = 9.42 \times 20^2/8 = 471\ kNm$

Total service load, $W_s = 20 + 9.43 = 29.42\ kN/m$

$M_s = 29.42 \times 20^2/8 = 1471\ kNm$

Required Section Modulus

from (5): $z_1 >= (0.9x1471–0.8x471)x10^6/(0.9x16.5–0.8(-1))$

$>= 60.52 \times 10^6\ mm^3$

z_1 provided $= 75.39 \times 10^6\ mm^3$ → Ok

from (6): $z_2 >= (0.9x1471–0.8x471)x10^6/(0.8x15.0–0.9(0))$

$>= 78.93 \times 10^6\ mm^3$

z_2 provided $= 116.23 \times 10^6\ mm^3$ → Ok

Design of Prestress Force

- Rearranging inequalities (1) to (4) will yield inequalities for the required prestress force, for a given value of eccentricity
- Thus the new inequalities are:

$$P_i >= (z_1 f_{tt} - M_i) \,/\, \alpha(z_1/A - e) \ldots\ldots (7)$$

$$P_i <= (z_2 f_{ct} + M_i) \,/\, \alpha(z_2/A + e) \ldots\ldots (8)$$

$$P_i <= (z_1 f_{cs} - M_s) \,/\, \beta(z_1/A - e) \ldots\ldots (9)$$

$$P_i >= (z_2 f_{ts} + M_s) \,/\, \beta(z_2/A + e) \ldots\ldots (10)$$

- The inequalities sign in (7) & (9) will be reversed if the denominator becomes -ve

Example 3-2

A post-tensioned prestressed concrete bridge deck is in the form of a solid slab with a depth of 525 mm and is simply supported over 20 m. It carries a service load of 10.3 kN/m^2. If the maximum eccentricity of the tendons at midspan is 75 mm above the soffit, find the minimum value of the prestress force required. Use the following information:

$$f_{ct} = 20.0 \text{ N/mm}^2 \text{ and } f_{tt} = 1.0 \text{ N/mm}^2$$

$$f_{cs} = 16.7 \text{ N/mm}^2 \text{ and } f_{ts} = 0 \text{ N/mm}^2$$

$$\alpha = 0.9 , \beta = 0.8$$

Solution

$z_1 = z_2 = 525^2 \times 10^3 / 6 = 45.94 \times 10^6 \text{ mm}^3/\text{m}$

$A = 525 \times 1000 = 5.25 \times 10^5 \text{ mm}^2/\text{m}$; $e = 525/2 – 75 = 188 \text{ mm}$

$M_i = 24 \times 0.525 \times 20^2 / 8 = 630 \text{ kNm/m}$

$M_s = 630 + (10.32 \times 20^2 / 8) = 1145 \text{ kNm/m}$

$P_i <= 7473.4$ kN ....(7)

$P_i <= 6426.31$ kN......................(8)

$P_i >= 4699.25$ kN......................(9)

$P_i >= 5195.01$ kN.....................(10)

Inequalities sign reversed

The minimum value of P_i which lies within the limits is 5195.01kN

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Solution

Minimum Prestressing Force

Sectional Properties

A =	5.25E+05	mm2	Z_1/A =	87.50	mm	Pi	>=	(z1 ftt – Mi) / α(z1/A – e)	(7)
I =	1.34E+11	mm4	Z_2/A =	87.50	mm	Pi	<=	(z2 fct + Mi) / α(z2/A + e)	(8)
y_1 =	588	mm	Z_1ftt =	-4.59E+07	Nmm	Pi	<=	(z1 fcs – Mi) / β(z1/A – e)	(9)
y_2 =	912	mm	Z_2fct =	9.19E+08	Nmm	Pi	>=	(z2 fts + Mi) / β(z2/A + e)	(10)
Z_1 =	4.594E+07	mm3	Z_1fcs =	7.67E+08	Nmm				
Z_2 =	4.594E+07	mm3	Z_2fts =	0.00E+00	Nmm				

Limiting Stresses

ftt =	-1.00	N/mm2	$\alpha(Z_1/A - e)$ =	-90.45	mm	===> Ineq (7) change from >= to <=
fct =	20.00	N/mm2	$\alpha(Z_2/A + e)$ =	247.95	mm	
fts =	0.00	N/mm2	$\beta(Z_1/A - e)$ =	-80.40	mm	===> Ineq (9) change from <= to >=
fcs =	16.70	N/mm2	$\beta(Z_2/A + e)$ =	220.40	mm	

Prestressing Properties

α =	0.9		Z_1ftt - Mi =	-6.76E+08	Nmm	Pi	<=	7473.43	kN	(7)
β =	0.8		Z_2fct + Mi =	1.55E+09	Nmm	Pi	<=	6246.31	kN	(8)
M_i =	630	kNm	Z_1fcs - Ms =	-3.78E+08	Nmm	Pi	>=	4699.25	kN	(9)
M_s =	1145	kNm	Z_2fts + Ms =	1.15E+09	Nmm	Pi	>=	5195.01	kN	(10)
e =	188	mm				Choose Pi min	=	5195.01	kN	

Magnel Diagram

- First explored by Magnel, a Belgian engineer
- Plot of e versus P_i produced a hyperbolic curve
- Plot of e versus $1/P_i$ produced a straight line
- Therefore, we will use e versus $1/P_i$
- Sign convention:
  - X-axis represents $1/P_i$
  - Y-axis represents e
  - +ve Y-axis (e values) pointing downwards (if possible)
  - +ve X-axis ($1/P_i$ values) pointing to the right

Magnel Diagram

- Rearranging inequalities (7) to (10):
- $\mathbf{e <= (M_i - z_1 f_{tt})/\alpha P_i + z_1/A}$............(11 $\mathbf{m = (M_i - z_1 f_{tt})/\alpha \text{ , } c = z_1/A}$
- $\mathbf{e <= (M_i + z_2 f_{ct})/\alpha P_i - z_2/A}$............(12 $\mathbf{m = (M_i - z_2 f_{ct})/\alpha \text{ , } c = -z_2/A}$
- $\mathbf{e >= (M_s - z_1 f_{cs})/\beta P_i + z_1/A}$...........(13 $\mathbf{m = (M_s - z_1 f_{cs})/\alpha \text{ , } c = z_1/A}$
- $\mathbf{e >= (M_s + z_2 f_{ts})/\beta P_i - z_2/A}$............(14 $\mathbf{m = (M_s - z_2 f_{ts})/\alpha \text{ , } c = -z_2/A}$
- Note that $z_1/A = k_b$ and $z_2/A = k_t$ i.e lower and upper limits of the central kern respectively

The above inequalities can be written as:
e <= mx + c or e >= mx + c
where m is the gradient and c is the vertical axis intercept

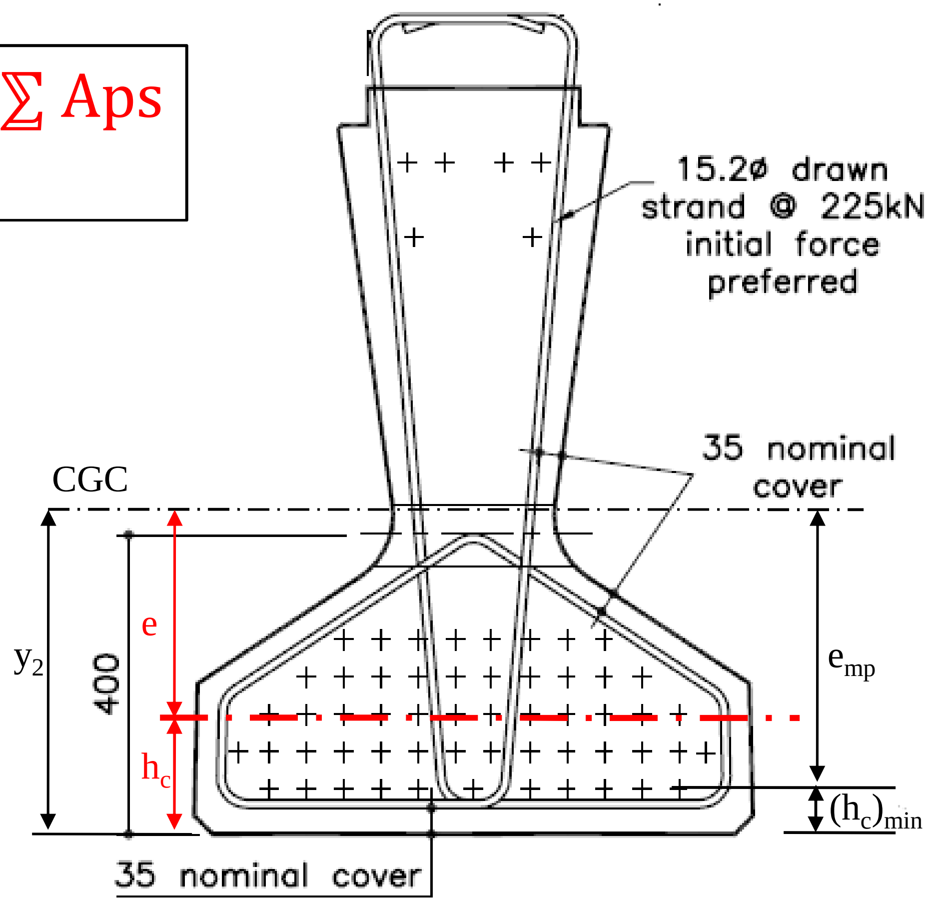
Magnel Diagram

- The maximum permissible eccentricity,

  $e_{mp} = y_2 - (h_c)_{min}$..........................(15)
- Where $(h_c)_{min}$ is the minimum concrete cover to c.g.s. which must conform to durability and fire protection requirements
- Therefore, $e <= e_{mp}$

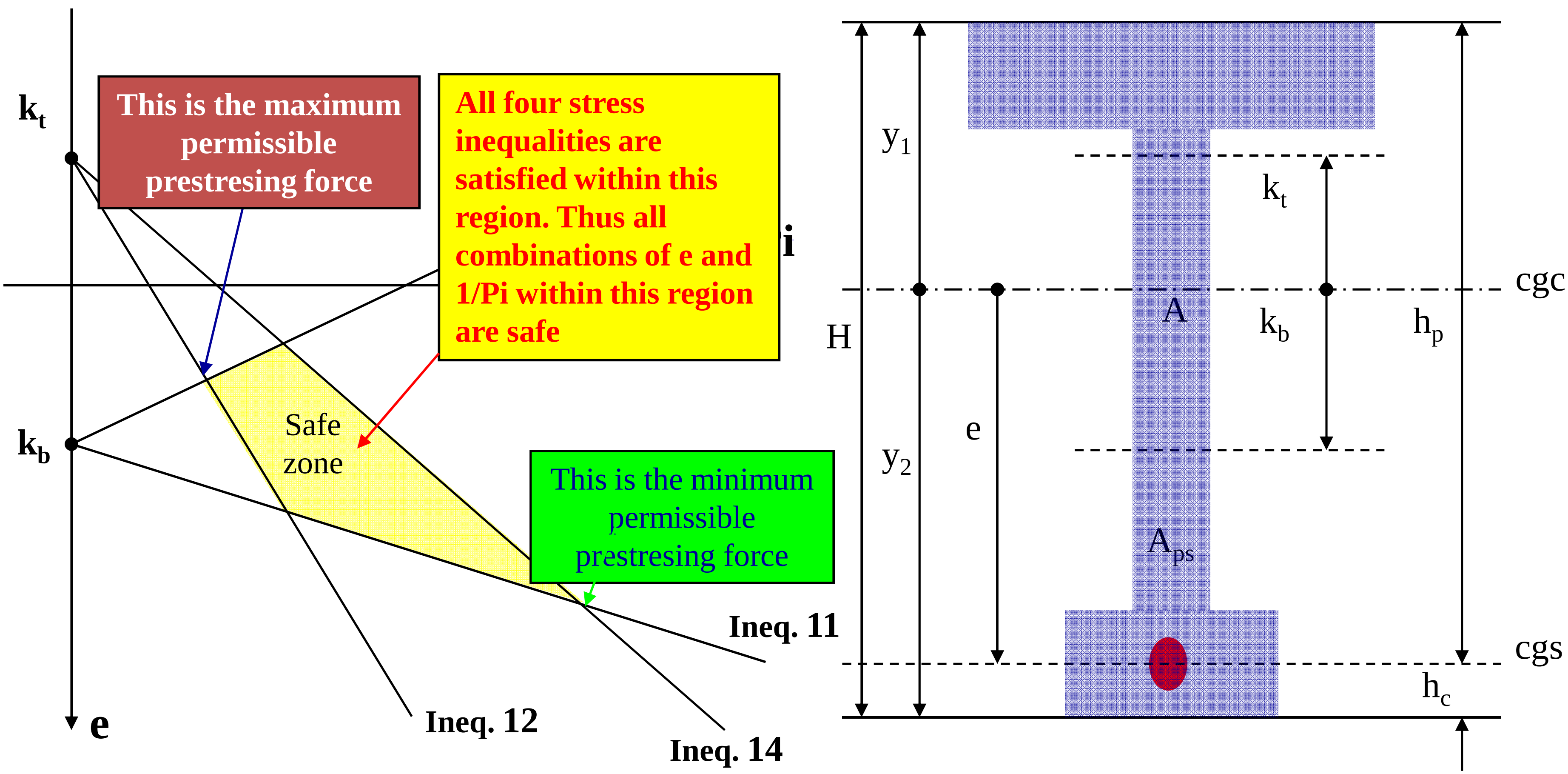
Cover & Eccentricity

$$hc = \sum Aps * y / \sum Aps$$
$$e = y_2 - hc$$

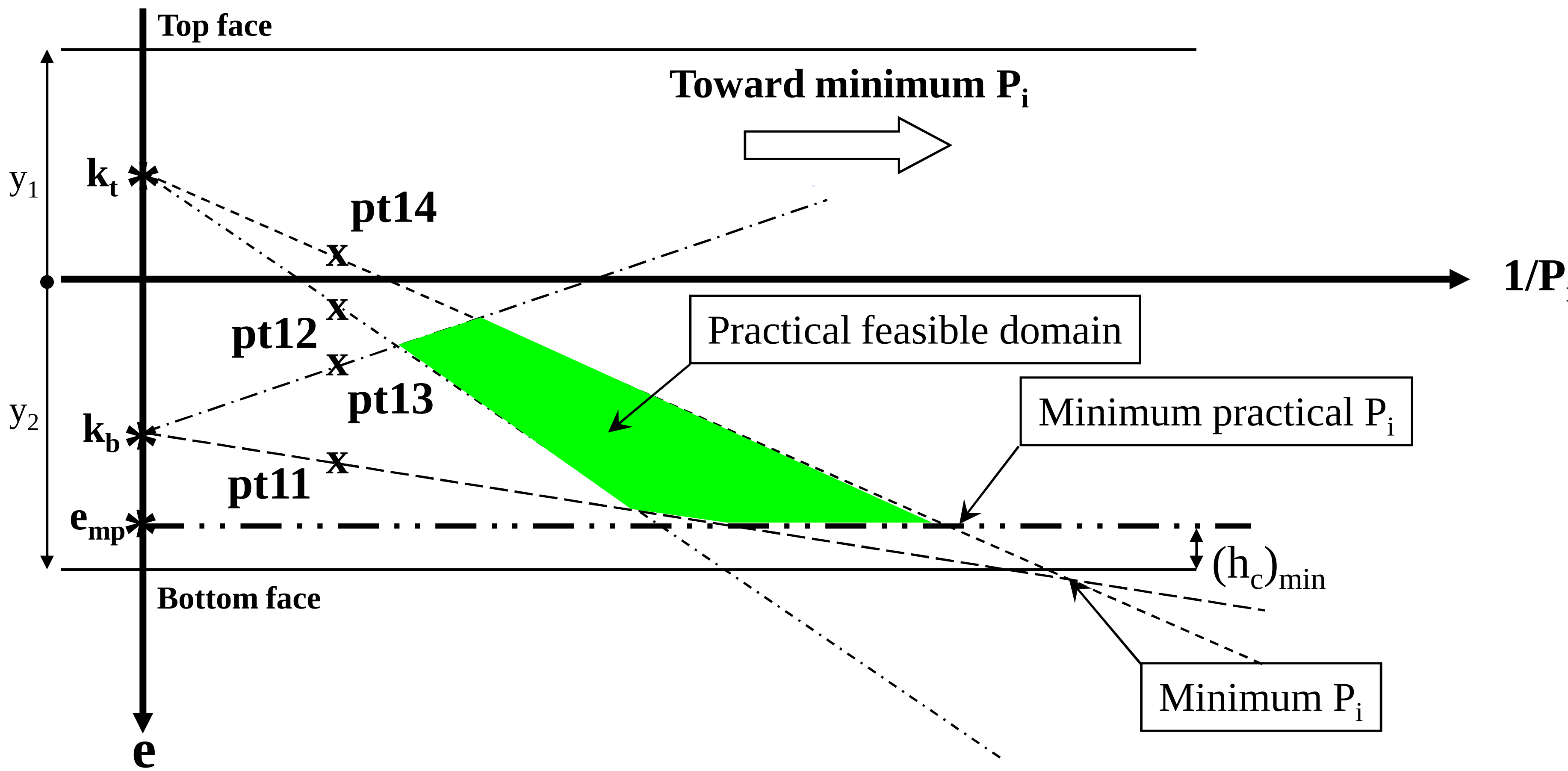


TYPICAL SECTION (Y4)

Magnel Diagram



Magnel Diagram



Example 3-3

It is required to construct a building floor using standard precast, pre-tensioned units of double T-section (Class 2) as shown on next slide. Given the following information:

f_{cu} = 50 N/mm^2; f_{ci} = 36 N/mm^2

Span = 10m (simply supported)

Dead load due to floor finish = 1.5 kN/m^2

Live load = 3.0 kN/m^2

(a) Choose a suitable double T-section

(b) Construct a Magnel diagram to determine the minimum prestressing force for the tendon.


3

Example 3-3

SECTION PROPERTIES 2400 WIDE MODULES

SECTION	AREA x 10^3 mm^2 (x 10^3)	Y_t mm	I x 10^9 mm^4 (x 10^9)	SELF WEIGHT kPa	SELF WEIGHT kg/m
200 x 2400	184	142	0.52	1.92	468
250 x 2400	202	177	0.99	2.11	515
300 x 2400	220	211	1.67	2.29	561
350 x 2400	237	244	2.55	2.47	605
400 x 2400	254	276	3.67	2.64	647
450 x 2400	269	308	5.03	2.81	687
500 x 2400	285	340	6.40	2.97	725

Try this section (250 x 2400)

Solution

Section Adequacy

Try Section 250 x 2400

Sectional Properties

A =	2.02E+05	mm2
I =	9.90E+08	mm4
y_1 =	73	mm
y_2 =	177	mm
Z_1 =	1.356E+07	mm3
Z_2 =	5.593E+06	mm3

Limiting Stresses

ftt =	-2.70	N/mm2
fct =	18.00	N/mm2
fts =	-3.18	N/mm2
fcs =	16.50	N/mm2

Prestressing Properties

α =	0.9
β =	0.8

M_i =	60.6	kNm
M_s =	195.6	kNm

αM_s =	176.04	kNm
βM_i =	48.48	kNm
αf_{cs} =	14.85	N/mm^2
αf_{ts} =	-2.86	N/mm^2
βf_{tt} =	-2.16	N/mm^2
βf_{ct} =	18.00	N/mm^2

$z1 >= (\alpha Ms - \beta Mi) / (\alpha f cs - \beta f tt)$ ....(5)

$z2 >= (\alpha Ms - \beta Mi) / (\beta f ct - \alpha f ts)$ ....(6)

z1 >=	7.50E+06
z2 >=	6.11E+06

Wsw = 0.202 x 24 = 4.85 kN/m; Mi = 4.85 x 10 x 10 / 8 = 60.6 kNm
Ws = (1.5+3.0) x 2.4 + 4.85 = 15.65 kN/m
Ms = 15.65 x 100/8 = 195.6 kNm

Solution

Section Adequacy

Try Section 300 x 2400

Sectional Properties

A =	2.20E+05	mm2
I =	1.67E+09	mm4
y_1 =	89	mm
y_2 =	211	mm
Z_1 =	1.876E+07	mm3
Z_2 =	7.915E+06	mm3

Limiting Stresses

ftt =	-2.70	N/mm2
fct =	18.00	N/mm2
fts =	-3.18	N/mm2
fcs =	16.50	N/mm2

Prestressing Properties

α =	0.9	
β =	0.8	
M_i =	68.793	kNm
M_s =	203.793	kNm
e =	176	mm

αM_s =	183.41	kNm
βM_i =	55.03	kNm
αf_{cs} =	14.85	N/mm^2
αf_{ts} =	-2.86	N/mm^2
βf_{tt} =	-2.16	N/mm^2
βf_{ct} =	18.00	N/mm^2

$z1 >= (\alpha Ms - \beta Mi) / (\alpha f cs - \beta f tt)$ ....(5)

$z2 >= (\alpha Ms - \beta Mi) / (\beta f ct - \alpha f ts)$ ....(6)

z1 >=	7.55E+06	ok
z2 >=	6.15E+06	ok

Solution

Sectional Properties		
A =	2.20E+05	mm^2
I =	1.67E+09	mm^4
y_1 =	89	mm
y_2 =	211	mm
Z_1 =	1.876E+07	mm^3
Z_2 =	7.915E+06	mm^3
Prestressing Properties		
α =	0.9	
β =	0.8	
M_i =	68.793	kNm
M_s =	203.793	kNm
cover =	35	mm
e_{max} =	176	mm
Limiting Stresses		
ftt =	-2.70	N/mm^2
fct =	18.00	N/mm^2
fts =	-3.18	N/mm^2
fcs =	16.50	N/mm^2

L =	10.000	m
Gk =	3.600	kN/m
Qk =	7.200	kN/m
W_{sw} =	5.503	kN/m
slab L =	2.400	mm
M_i =	68.793	kNm
M_s =	203.793	kNm

$(-z_1 f_{tt} + M_i)/\alpha$ =	1.33E+08
$(z_2 f_{ct} + M_i)/\alpha$ =	2.35E+08
$(M_s - z1 f_{cs})/\beta$ =	-1.32E+08
$(M_s + z2 f_{ts})/\beta$ =	2.23E+08
$k_b = Z_1/A$ =	85
$k_t = Z_2/A$ =	36

$e <= (Mi - z1 f tt)/\alpha Pi + z1/A$............(11)

$e <= (Mi + z2 f ct)/\alpha Pi - z2/A$............(12)

$e >= (Ms - z1 f cs)/\beta Pi + z1/A$............(13)

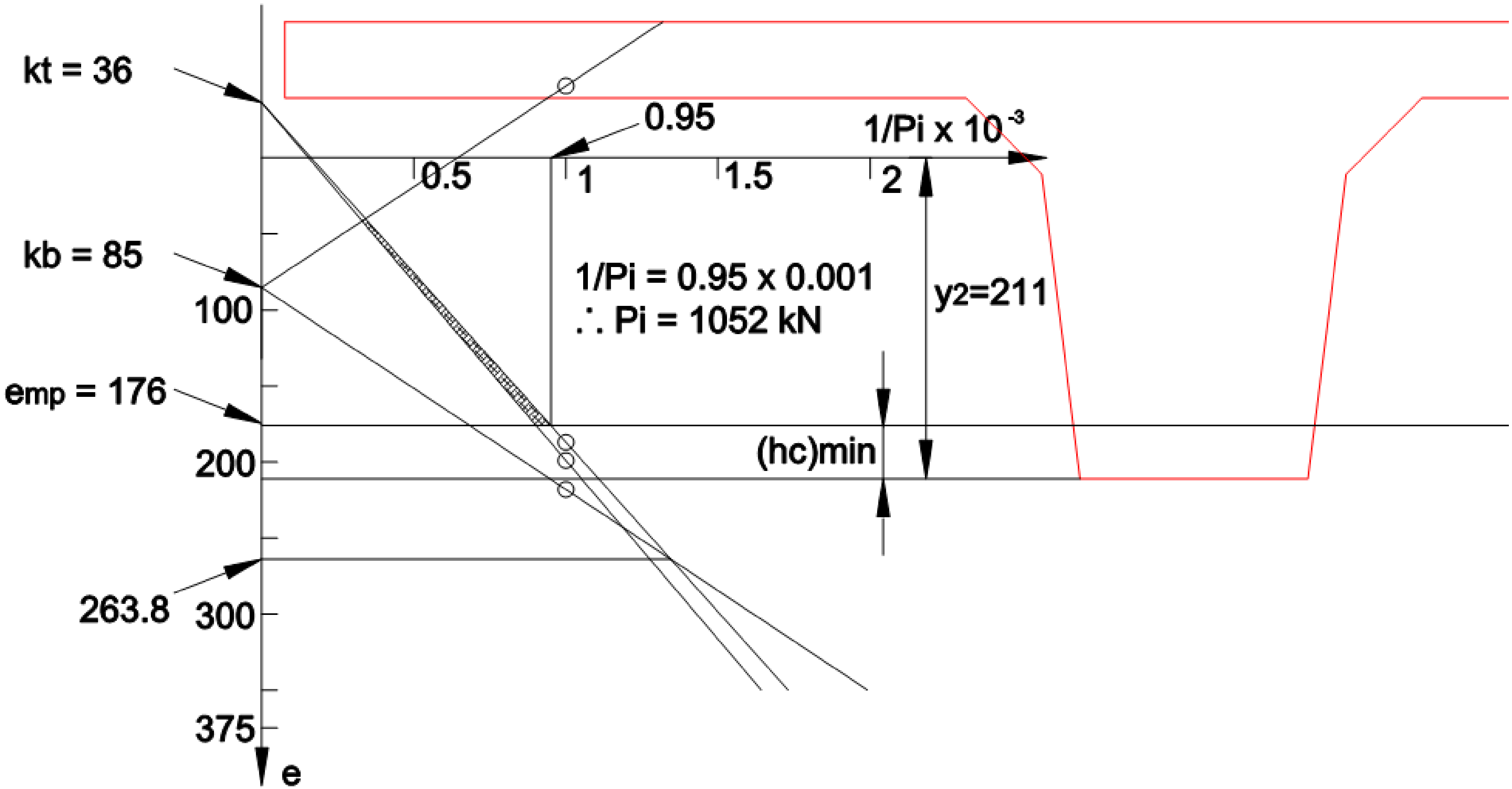
$e >= (Ms + z2 f ts)/\beta Pi - z2/A$............(14)

Pi (kN)	1/Pi	$1/Pi \times 10^3$	e	Ineq.
1000	0.001	1	218	(11)
1000	0.001	1	199	(12)
1000	0.001	1	-47	(13)
1000	0.001	1	187	(14)

Points on Graph

	x	y
pt12	1	199
pt14	1	187
pt11	1	218
pt13	1	-47
emp	0	176
kb	0	85
kt	0	36

Solution – Manual Plotting



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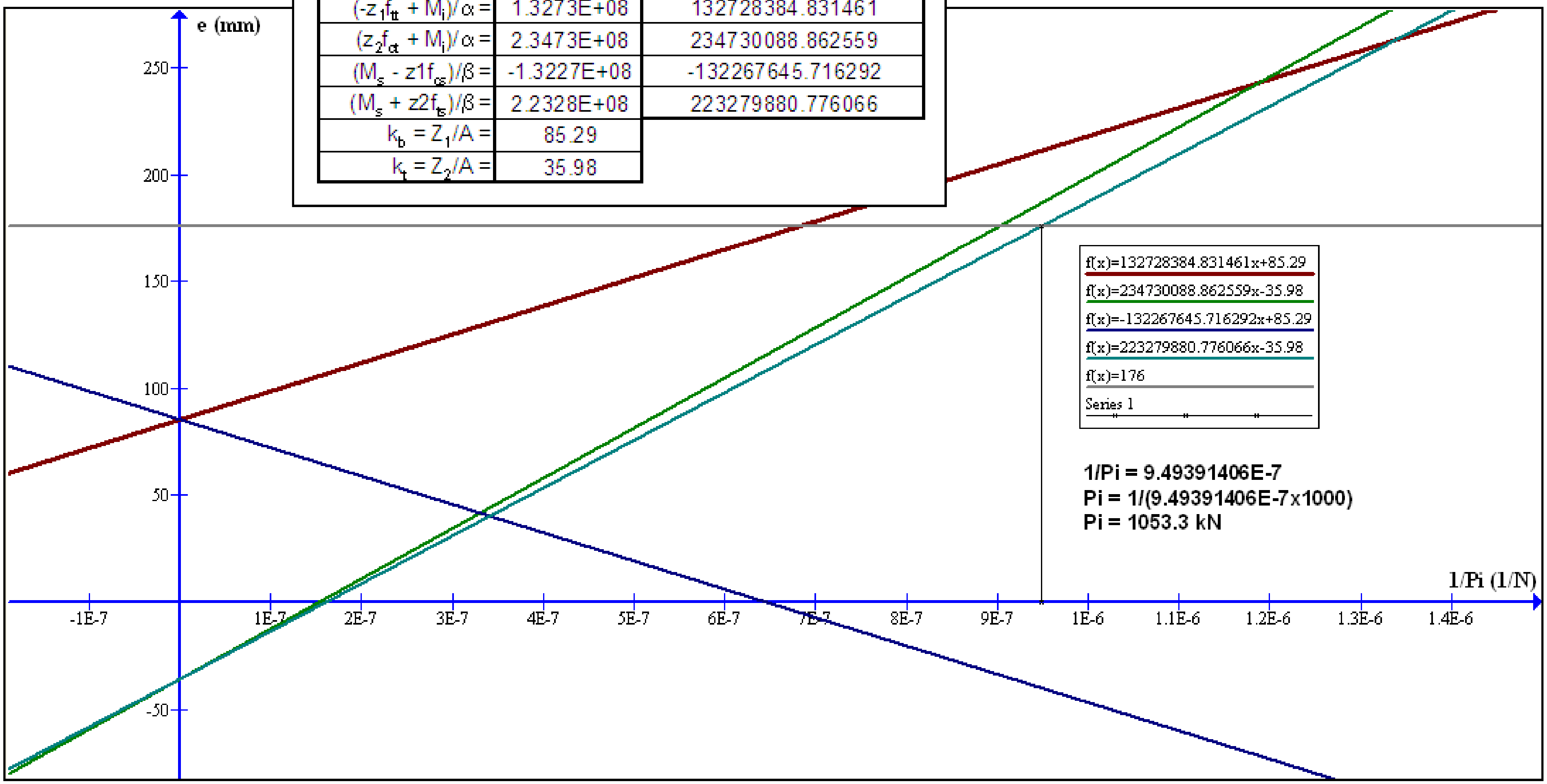
Solution – Using Graph v4.3

$(-z_1 f_{tt} + M_i)/\alpha =$	1.3273E+08	132728384.831461
$(z_2 f_{ct} + M_i)/\alpha =$	2.3473E+08	234730088.862559
$(M_s - z1 f_{cs})/\beta =$	-1.3227E+08	-132267645.716292
$(M_s + z2 f_{ts})/\beta =$	2.2328E+08	223279880.776066
$k_b = Z_1/A =$	85.29	
$k_t = Z_2/A =$	35.98	



1/Pi = 9.49391406E-7
Pi = 1/(9.49391406E-7x1000)
Pi = 1053.3 kN

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Solution – Using Inequalities

Minimum Prestressing Force

Sectional Properties

A =	2.20E+05	mm2	Z_1/A =	85.29	mm
I =	1.67E+09	mm4	Z_2/A =	35.98	mm
y_1 =	89	mm	Z_1ftt =	-5.07E+07	Nmm
y_2 =	211	mm	Z_2fct =	1.42E+08	Nmm
Z_1 =	1.876E+07	mm3	Z_1fcs =	3.10E+08	Nmm
Z_2 =	7.915E+06	mm3	Z_2fts =	-2.52E+07	Nmm

Limiting Stresses

ftt =	-2.70	N/mm2	Z_1ftt - Mi =	-1.19E+08	Nmm
fct =	18.00	N/mm2	Z_2fct + Mi =	2.11E+08	Nmm
fts =	-3.18	N/mm2	Z_1fcs - Ms =	1.06E+08	Nmm
fcs =	16.50	N/mm2	Z_2fts + Ms =	1.79E+08	Nmm

Prestressing Properties

α =	0.9		$\alpha(Z_1/A - e)$ =	-81.64	mm
β =	0.8		$\alpha(Z_2/A + e)$ =	190.78	mm
M_i =	68.793	kNm	$\beta(Z_1/A - e)$ =	-72.57	mm
M_s =	203.793	kNm	$\beta(Z_2/A + e)$ =	169.58	mm
e =	176	mm			

Pi >= (z1 ftt – Mi) / α(z1/A – e) (7)

Pi <= (z2 fct + Mi) / α(z2/A + e) (8)

Pi <= (z1 fcs – Ms) / β(z1/A – e) (9)

Pi >= (z2 fts + Ms) / β(z2/A + e) (10)

Pi	<=	1463.24	kN	(7)
Pi	<=	1107.35	kN	(8)
Pi	>=	-1458.15	kN	(9)
Pi	>=	1053.33	kN	(10)
Choose Pi min	=	1053.33	kN	

===> Ineq (7) change from >= to <=

===> Ineq (9) change from <= to >=

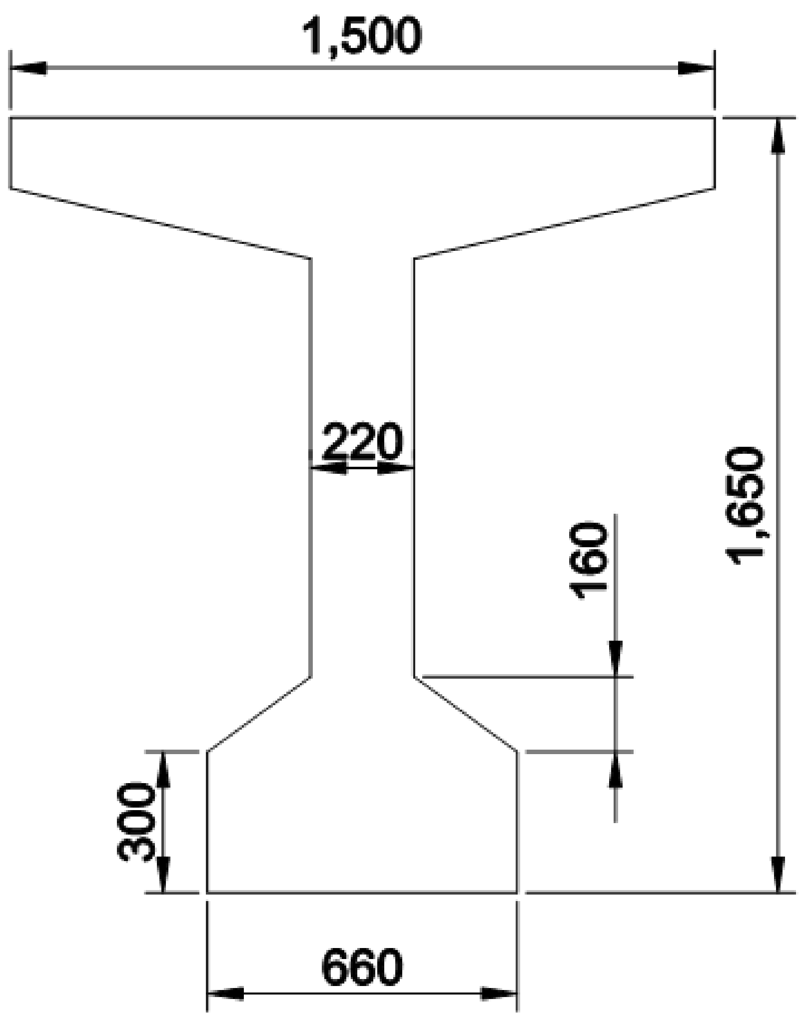
Example 3-4

A post-tensioned precast concrete beam (shown in next slide), simply supported over a span of 29.4m carries a total uniformly distributed service load of 35.8 kN/m in addition to its own self weight. The following information is given:

- Class 1 category; fci = 45 N/mm^2; fcu = 50 N/mm^2
- A = 723700 mm^2; y2= 876 mm
- I = 255.34 x $10^9 mm^4$; cover to tendon = 152 mm
- Take unit weight of concrete, g as 25 kN/m^3

Construct a Magnel diagram and find the minimum prestress force. Compare your results with those obtained using the inequalities.

Example 3-4



UTM

Solution

Sectional Properties		
A =	7.24E+05	mm^2
I =	2.55E+11	mm^4
y_1 =	774	mm
y_2 =	876	mm
Z_1 =	3.299E+08	mm^3
Z_2 =	2.915E+08	mm^3
Prestressing Properties		
α =	0.9	
β =	0.8	
M_i =	1954.804	kNm
M_s =	5822.815	kNm
cover =	152	mm
e_{max} =	724	mm
Limiting Stresses		
ftt =	-1.00	N/mm^2
fct =	22.50	N/mm^2
fts =	0.00	N/mm^2
fcs =	16.50	N/mm^2

L =	29.400	m
Gk =	35.800	kN/m
Qk =	0.000	kN/m
W_{sw} =	18.093	kN/m
Ws =	53.893	kN/m
M_i =	1954.80	kNm
M_s =	5822.82	kNm

Solution

Section Adequacy

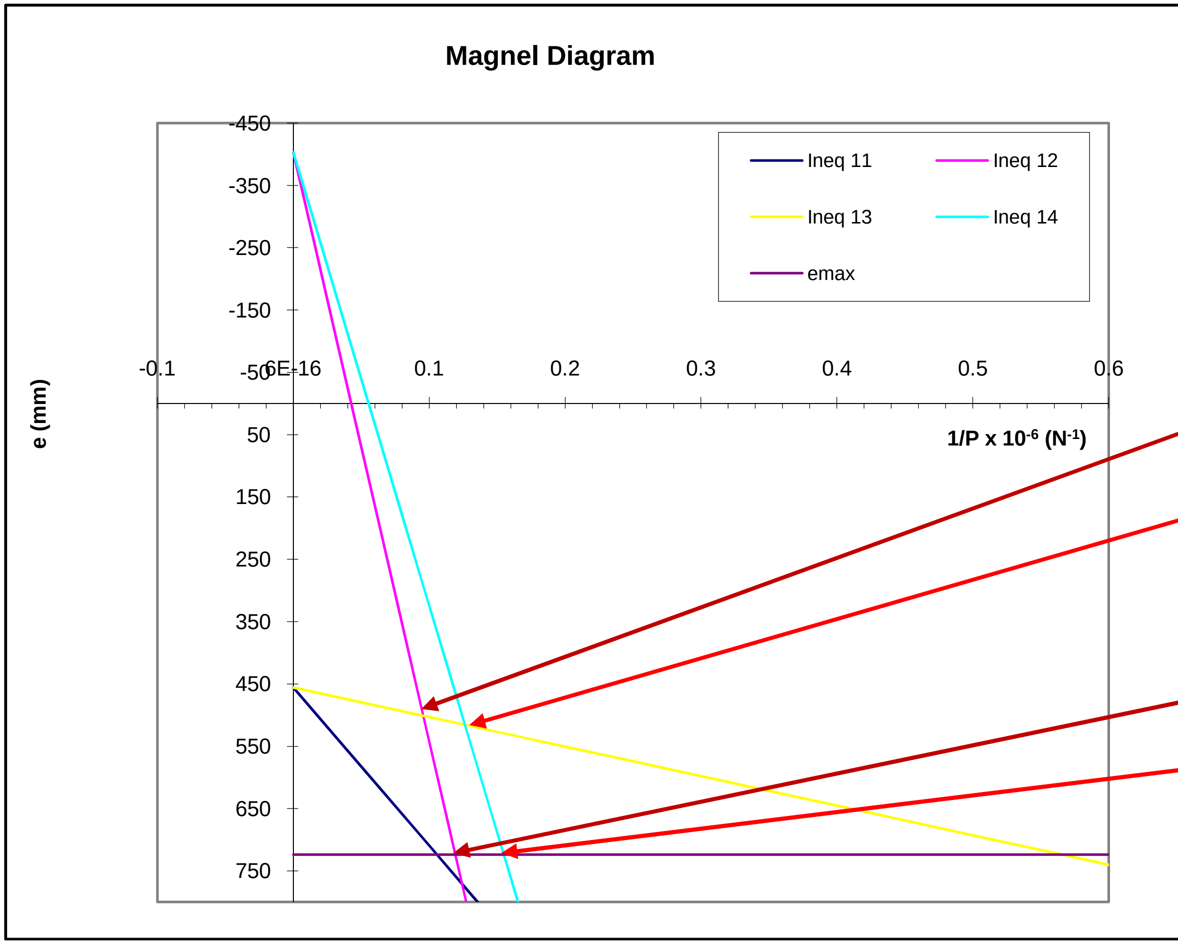
αM_s =	5240.53	kNm	z1 > =	$(\alpha Ms - \beta Mi) / (\alpha f cs - \beta f tt)$………(5)	
βM_i =	1563.84	kNm	z2 > =	$(\alpha Ms - \beta Mi) / (\beta f ct - \alpha f ts)$………(6)	
αf_{cs} =	14.85	N/mm^2	z1 >=	2.35E+08	ok
αf_{ts} =	0.00	N/mm^2	z2 >=	2.04E+08	ok
βf_{tt} =	-0.80	N/mm^2			
βf_{ct} =	18.00	N/mm^2			

Magnel Diagram

$(-z_1 f_{tt} + M_i)/\alpha$ =	2.54E+09
$(z_2 f_{ct} + M_i)/\alpha$ =	9.46E+09
$(M_s - z1 f_{cs})/\beta$ =	4.74E+08
$(M_s + z2 f_{ts})/\beta$ =	7.28E+09
$k_b = Z_1/A$ =	456
$k_t = Z_2/A$ =	403

P(kN)	1/P	e_{11}	e_{12}	e_{13}	e_{14}	e_{max}	$1/P \times 10^{-6}$
infiniti	0	456	-403	456	-403	724	0
1000	0.000001	2994	9056	930	6876	724	1.00
750	1.33E-06	3841	12209	1088	9302	724	1.33
500	0.000002	5533	18515	1405	14154	724	2.00
250	0.000004	10610	37434	2353	28711	724	4.00
100	0.00001	25841	94188	5200	72382	724	10.00

Solution – Using MS Excel



Points of Intersection

1. Between Inequalities

			$1/P \times 10^{-6}$
$P_{11\&12}$ =	8060.12	kN	0.12
$e_{11\&12}$ =	770.80	mm	
$P_{11\&14}$ =	5520.47	kN	0.18
$e_{11\&14}$ =	915.69	mm	
$P_{12\&13}$ =	10464.17	kN	0.10
$e_{12\&13}$ =	501.18	mm	
$P_{13\&14}$ =	7924.52	kN	0.13
$e_{13\&14}$ =	515.71	mm	
Pmin =	5520.47	kN	

2. Between Inequalities & emax

			$1/P \times 10^{-6}$
$P_{11\&e}$ =	9466.83	kN	0.11
$P_{12\&e}$ =	8394.89	kN	0.12
$P_{13\&e}$ =	1769.14	kN	0.57
$P_{14\&e}$ =	6459.64	kN	0.155
$P_{min\ practical}$ =	6459.64	kN	

Solution – Using Inequalities

Using Inequalities

Z_1/A =	455.85	mm
Z_2/A =	402.77	mm
Z_1ftt =	-3.30E+08	Nmm
Z_2fct =	6.56E+09	Nmm
Z_1fcs =	5.44E+09	Nmm
Z_2fts =	0.00E+00	Nmm

Z_1ftt - Mi =	-2.28E+09	Nmm
Z_2fct + Mi =	8.51E+09	Nmm
Z_1fcs - Ms =	-3.80E+08	Nmm
Z_2fts + Ms =	5.82E+09	Nmm

$\alpha(Z_1/A - e)$ =	-241.34	mm	====> Ineq (7) change from >= to <=
$\alpha(Z_2/A + e)$ =	1014.09	mm	
$\beta(Z_1/A - e)$ =	-214.52	mm	====> Ineq (9) change from <= to >=
$\beta(Z_2/A + e)$ =	901.42	mm	

Pi	>=	(z1 ftt – Mi) / α(z1/A – e)	(7)
Pi	<=	(z2 fct + Mi) / α(z2/A + e)	(8)
Pi	<=	(z1 fcs – Ms) / β(z1/A – e)	(9)
Pi	>=	(z2 fts + Ms) / β(z2/A + e)	(10)

Pi	<=	9466.83	kN	(7)
Pi	<=	8394.89	kN	(8)
Pi	>=	1769.14	kN	(9)
Pi	>=	6459.64	kN	(10)

Choose Pi min = 6459.64 kN

Solution Using Graph V4.3