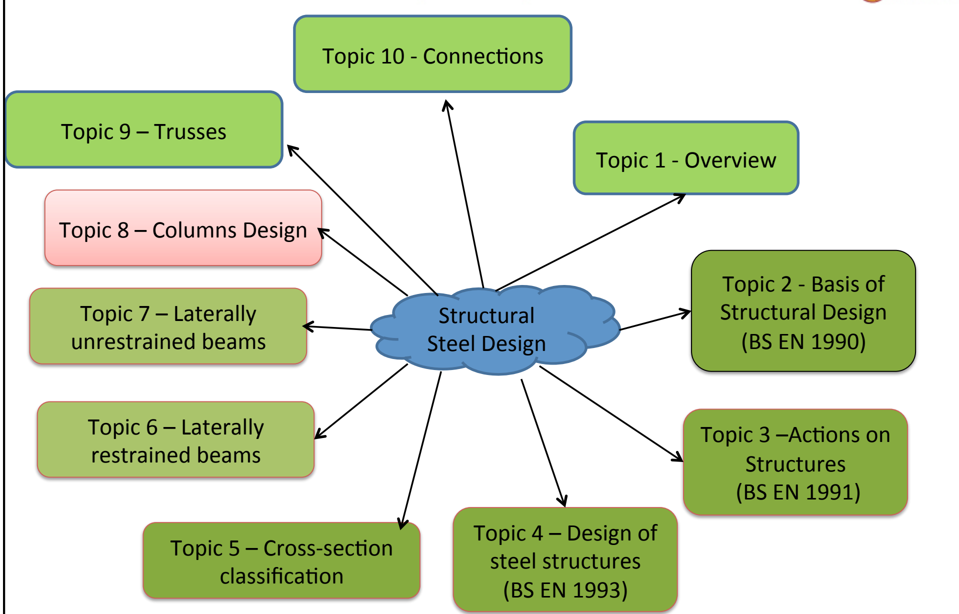
Structural Steel and Timber Design SAB3233

Topic 8
Columns Design

Prof Dr Shahrin Mohammad



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Columns subjected to axial load

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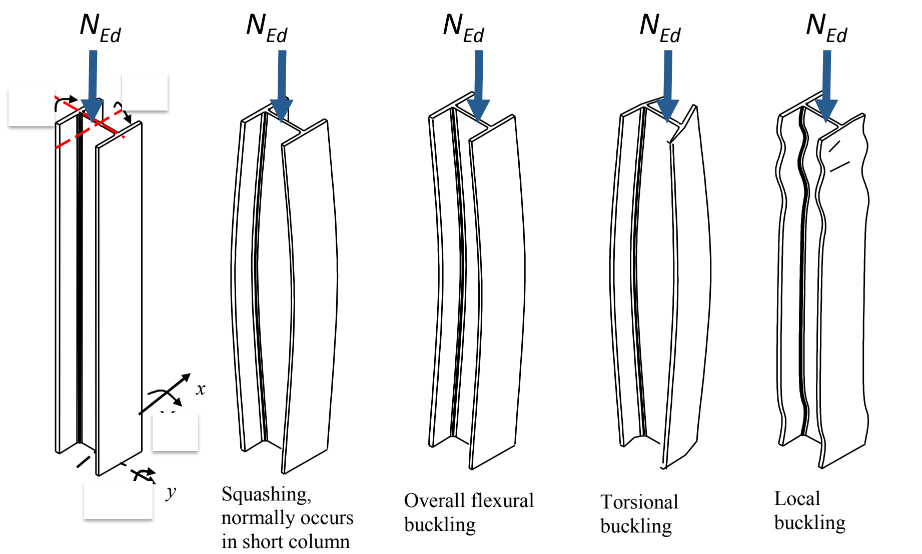
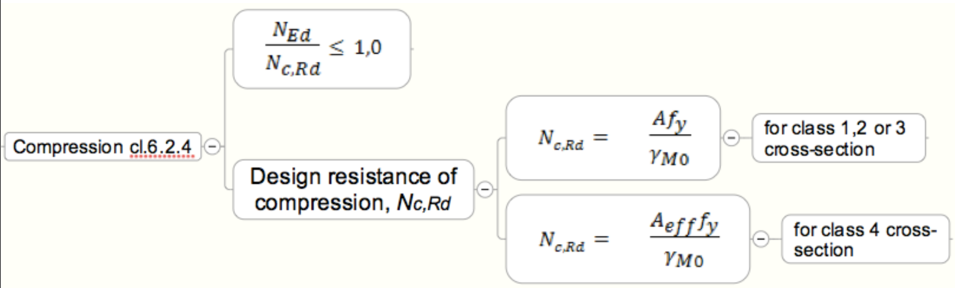


N_{Ed} = *Design value of compression force*

- compression members subject to
  - axial compression only
  - no bending
- however in practically real columns are subject to
  - eccentricities of axial loads
  - transverse forces
- the treatment distinguishes between
  - stocky columns, and
  - slender columns

Stocky columns

- The characteristics of stocky columns are
  - very low slenderness
  - unaffected by overall buckling
- The compressive strength of stocky columns is
  - dictated by the cross-section
  - a function of the section classification



Slender Steel Columns

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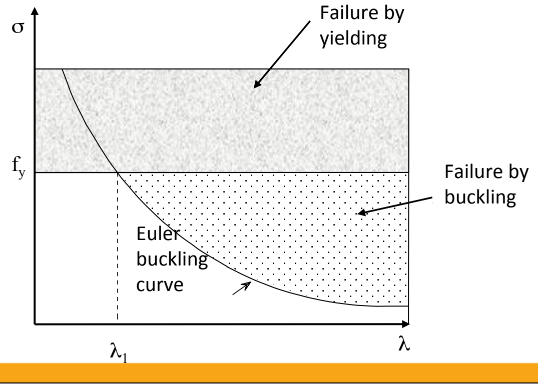
- Slender columns present a quasi elastic buckling behaviour
- Euler critical stress $\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$

$\lambda = L_{cr} / r$, where r is radius of gyration
L_{cr} is the buckling length

Euler buckling curve and modes of failure



Behaviour of real steel columns

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- columns of medium slenderness are very sensitive to the effects of imperfections
- inelastic buckling occurs before the Euler buckling load due to various imperfections
  - initial out-of-straightness
  - residual stresses
  - eccentricity of axial applied loads
  - strain-hardening

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Effects of imperfections

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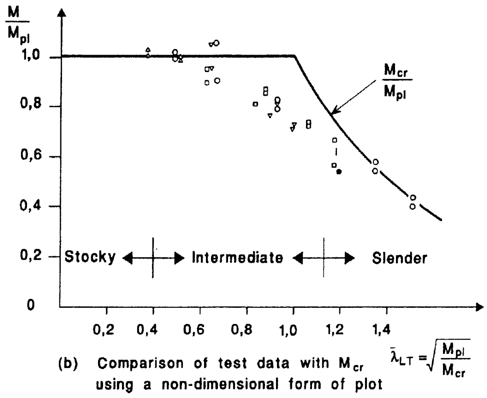
- structural imperfections most important for intermediated columns
- this represents most practical columns
- lower bound curve is obtained from a statistical analysis of test results



(b) Comparison of test data with M_{cr} using a non-dimensional form of plot

Effect of imperfections in relation to slenderness

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- Slender column
  - largely unaffected by imperfections
  - ultimate failure load ≈ Euler load (N_{cr})
  - independent of the yield stress
- Intermmediate column
  - imperfections important
  - failure load less than Euler load
  - out-of-straightness and residual stresses are the most significant imperfections

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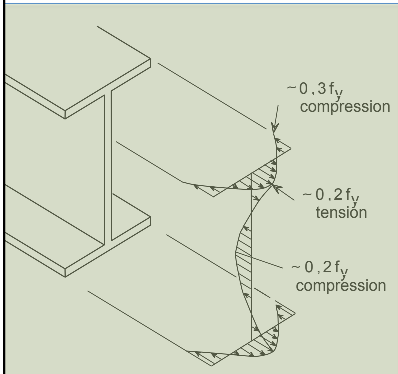
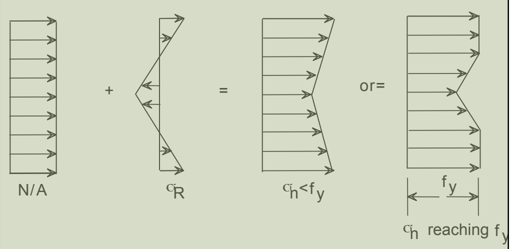
Residual stresses patterns



Typical residual stress pattern



Combination with axial stresses

- combined with axial stresses cause yielding
- effective area reduced

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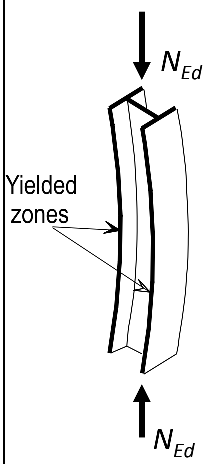
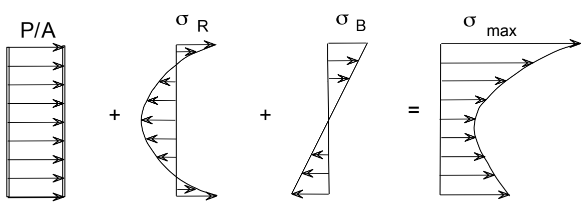
Initial out-of-straightness

- induces bending moments



Combined effect of imperfections and axial load

- bending stress σ_B
- residual stress, σ_R
- applied axial stress, N_{Ed}/A



Buckling resistance in axial compression

The design buckling resistance of a compression member

for Class 1, 2 and 3 cross section	$N_{b.Rd} = \chi \dfrac{Af_y}{\gamma_{M1}}$
for Class 4 cross section	$N_{b.Rd} = \chi \dfrac{A_{eff} f_y}{\gamma_{M1}}$

where χ a reduction factor and is related to the reference slenderness
Buckling curves plotted as χ versus reference slenderness ratio

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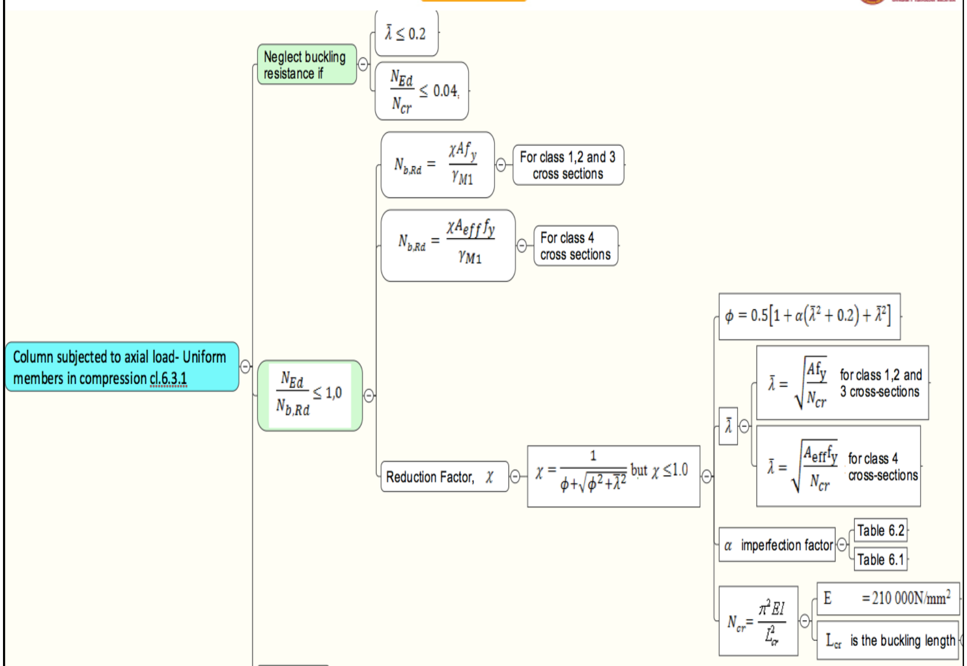

European buckling curves (Cl 6.3.1.2)

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$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi \le 1{,}0$$

where $\Phi = 0{,}5\left[1 + \alpha\left(\bar{\lambda} - 0{,}2\right) + \bar{\lambda}^2\right]$

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 cross-sections}$$

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} \quad \text{for Class 4 cross-sections}$$

α is an imperfection factor

N_{cr} is the elastic critical force for the relevant buckling mode based on the gross cross sectional properties.

European buckling curves

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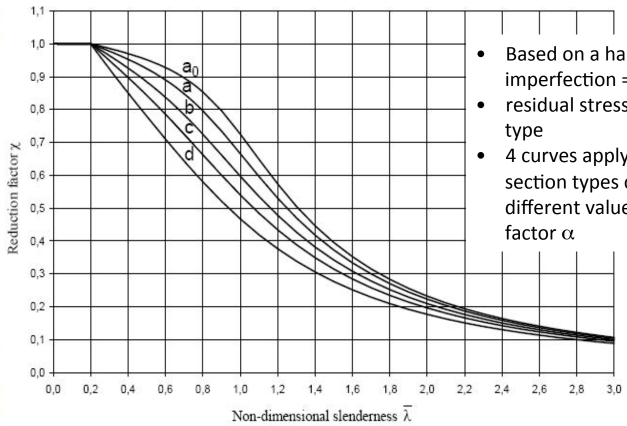
Based on experiment more than 1000 tests section
Range of slenderness ratios between 55 and 160



- Based on a half sine-wave geometric imperfection = L/1000
- residual stresses related to section type
- 4 curves apply to different cross-section types corresponding to different values of the imperfection factor α

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Imperfection factor a

- α depends on
  - the shape of the column cross-section
  - the direction of buckling (y or z axis)
  - the fabrication process (hot-rolled, welded or cold-formed
- imperfection factors given in *Table 6.1*

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

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Non dimensional slenderness, $\bar{\lambda}$

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{1}{\lambda_1} \quad \text{for Class 1, 2 and 3 cross-sections}$$

$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{\sqrt{\frac{A_{eff}}{A}}}{\lambda_1} \quad \text{for Class 4 cross-sections}$$

where L_{cr} is the buckling length in the buckling plane considered

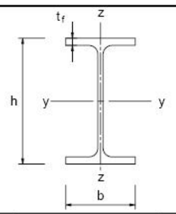
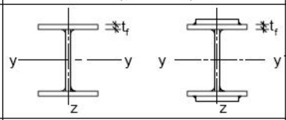
i is the radius of gyration about the relevant axis, determined using the properties of the gross cross-section

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93{,}9\varepsilon$$

$$\varepsilon = \sqrt{\frac{235}{f_y}} \quad (f_y \text{ in N/mm}^2)$$

Selection of appropriate buckling curve

Table 6.2 helps with the selection of the appropriate buckling curve

Cross section			Limits	Buckling about axis	Buckling curve: S 235, S 275, S 355, S 420	Buckling curve: S 460
Rolled sections		$h/b > 1{,}2$	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
			40 mm < $t_f \le 100$	y – y z – z	b c	a a
		$h/b \le 1{,}2$	$t_f \le 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections			$t_f \le 40$ mm	y – y z – z	b c	b c
			$t_f > 40$ mm	y – y z – z	c d	c d
Hollow sections			hot finished	any	a	a_0
			cold formed	any	c	c

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Example 1 : Design of an axially loaded column

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Columns subjected to combined bending and axial load

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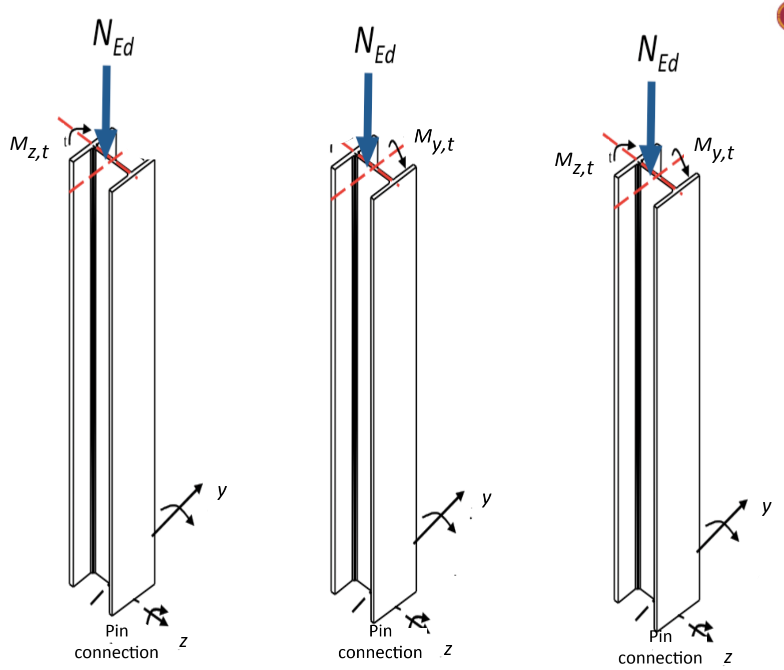
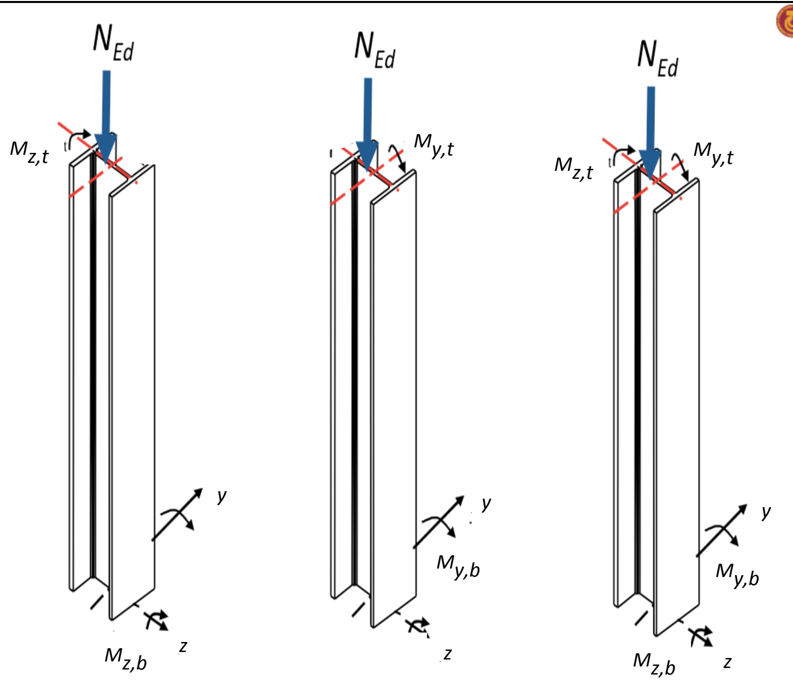


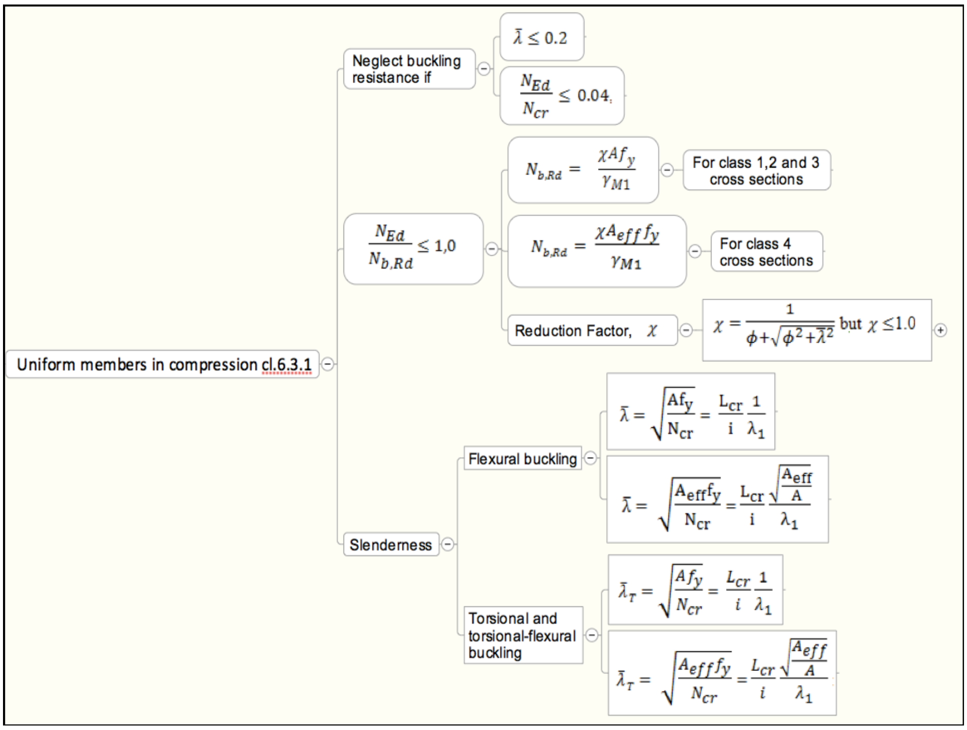


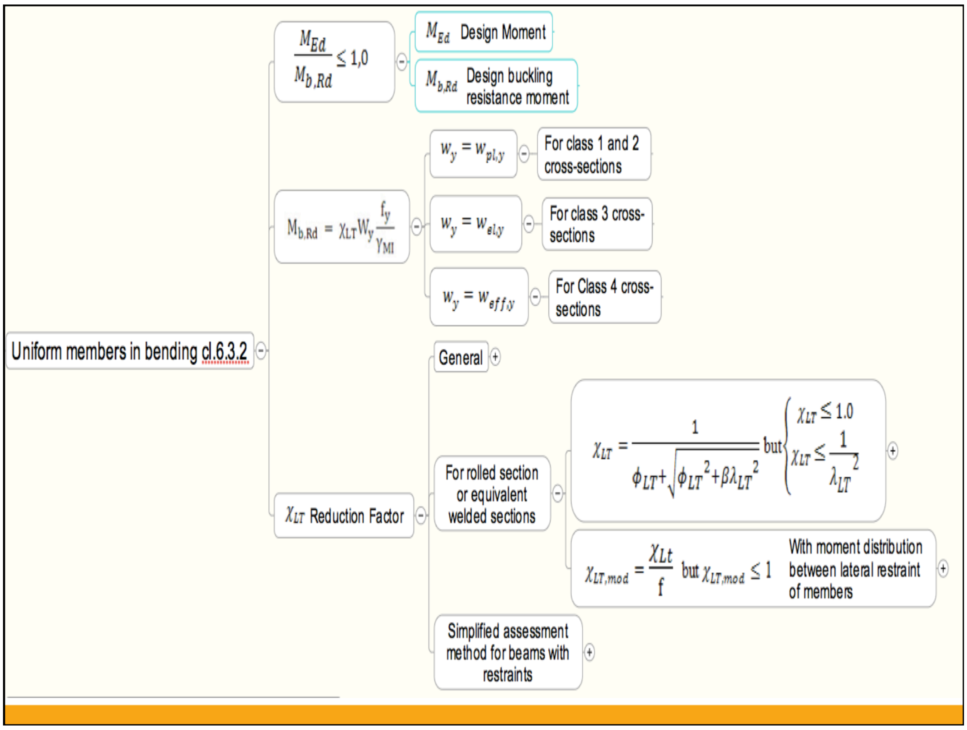
Compression cl.6.2.4

- $\frac{N_{Ed}}{N_{c,Rd}} \leq 1,0$
- Design resistance of compression, $N_{c,Rd}$
  - $N_{c,Rd} = \frac{A f_y}{\gamma_{M0}}$ — for class 1,2 or 3 cross-section
  - $N_{c,Rd} = \frac{A_{eff} f_y}{\gamma_{M0}}$ — for class 4 cross-section

- Bending moment cl.6.2.5
  - $$\frac{M_{Ed}}{M_{c,Rd}} \leq 1{,}0$$
  - Design resistance of bending, Mc,Rd
    - $$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}}$$
    - $$M_{c,Rd} = M_{el,Rd} = \frac{W_{el,min} f_y}{\gamma_{M0}}$$
      - For class 3 cross-sections
      - Wel,min elastic section modulus
    - $$M_{c,Rd} = \frac{W_{eff,min} f_y}{\gamma_{M0}}$$
      - For class 4 cross-sections
      - Weff,min effective section modulus

- Bending and axial force cl.6.2.9
  - Class 1 and 2 cross-sections
    - $M_{Ed} \leq M_{N,Rd}$
  - Class 3 cross-sections
    - The Longitudinal stress should satisfy
      - $$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}}$$
  - Class 4 cross-sections
    - The Longitudinal stress should satisfy
      - $$\sigma_{x,Ed} \leq \frac{f_y}{\gamma_{M0}}$$
      - Using effective cross sections
    - $$\frac{N_{Ed}}{A_{eff} f_y/\gamma_{M0}} + \frac{M_{y,Ed} + N_{Ed} e_{Ny}}{W_{eff,y,min} f_y/\gamma_{M0}} + \frac{M_{z,Ed} + N_{Ed} e_{Nz}}{W_{eff,z,min} f_y/\gamma_{M0}} \leq 1$$
  - Design Plastic moment reduced due to axial force. $M_{N,Rd}$
    - Doubly symmetrical I and H sections (Rolled or welded)
      - $$M_{N,y,Rd} = \frac{M_{pl,y,Rd}(1-n)}{(1-0.5a)}$$
    - Rectangular hollow and welded box sections (equal flanges and equal webs)
      - $$M_{N,y,Rd} = \frac{M_{pl,y,Rd}(1-n)}{(1-0.5a_w)}$$ Major axis y-y
      - $$M_{N,z,Rd} = \frac{M_{pl,y,Rd}(1-n)}{(1-0.5a_f)}$$ Minor axis x-x
  - Bi-axial bending
    - $$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,y,Rd}}\right]^{\beta} \leq 1$$
  - If both equations are satisfied, no reduction in the y-y axis plastic moment resistance is necessary
  - if equation is satisfied, no reduction in the z-z axis plastic moment resistance is necessary
  - $a_w = \frac{(A-2bt)}{A}$ but $a_w \leq 0.5$ for hollow sections
    $a_w = \frac{(A-2bt_f)}{A}$ but $a_w \leq 0.5$ for welded box sections
    $a_f = \frac{(A-2ht)}{A}$ but $a_f \leq 0.5$ for hollow sections
    $a_f = \frac{(A-2ht_w)}{A}$ but $a_f \leq 0.5$ for hollow sections
  - $n = N_{Ed}/N_{pl,Rd}$


Uniform members in compression cl.6.3.1
Neglect buckling resistance if
$\bar{\lambda} \leq 0.2$
$\frac{N_{Ed}}{N_{cr}} \leq 0.04,$
$\frac{N_{Ed}}{N_{b,Rd}} \leq 1{,}0$
$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}}$
For class 1,2 and 3 cross sections
$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}}$
For class 4 cross sections
Reduction Factor, χ
$\chi = \frac{1}{\phi + \sqrt{\phi^2 + \bar{\lambda}^2}}$ but $\chi \leq 1.0$
Slenderness
Flexural buckling
$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{1}{\lambda_1}$
$\bar{\lambda} = \sqrt{\frac{A_{eff}f_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{\sqrt{\frac{A_{eff}}{A}}}{\lambda_1}$
Torsional and torsional-flexural buckling
$\bar{\lambda}_T = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{1}{\lambda_1}$
$\bar{\lambda}_T = \sqrt{\frac{A_{eff}f_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{\sqrt{\frac{A_{eff}}{A}}}{\lambda_1}$


Uniform members in bending cl.6.3.2
$\frac{M_{Ed}}{M_{b,Rd}} \leq 1{,}0$
M_{Ed} Design Moment
$M_{b,Rd}$ Design buckling resistance moment
$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}$
$W_y = W_{pl,y}$
For class 1 and 2 cross-sections
$W_y = W_{el,y}$
For class 3 cross-sections
$W_y = W_{eff,y}$
For Class 4 cross-sections
χ_{LT} Reduction Factor
General
For rolled section or equivalent welded sections
$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 + \beta \bar{\lambda}_{LT}^2}}$ but $\chi_{LT} \leq 1.0$, $\chi_{LT} \leq \frac{1}{\bar{\lambda}_{LT}^2}$
$\chi_{LT,mod} = \frac{\chi_{Lt}}{f}$ but $\chi_{LT,mod} \leq 1$
With moment distribution between lateral restraint of members
Simplified assessment method for beams with restraints

Column subjected to axial load and bending -Uniform members in bending and axial compression cl.6.3.3

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1$$

where

- N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ — Design values of compression and maximum moments about y-y and z-z axis along members.
- $\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$ — Moments due the shift of centroidal axis for class 4.
- χ_y and χ_z — Reduction factor due to flexural buckling
- χ_{LT} — Reduction factor due to Lateral torsional buckling
  - χ_{LT} = 1.0 if not susceptible to torsional deformation
- k_{yy}, k_{yz}, k_{zy}, k_{zz} — Interaction factors
  - Annex A (alternatives method 1)
  - Annex B (alternatives method 2)

Table 6.7 Values for $N_{Rk} = f_y A_i$, $M_{i,Rk} = f_y W_i$ and $\Delta M_{i,Ed}$

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	0	0	0	$e_{N,y}N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z}N_{Ed}$

simple construction

'Simple construction' is commonly used for the design of multi-storey buildings

- Beams are designed as simply supported
- Columns are designed for nominal moments arising from the eccentricity at the beam-to-column connection.

The moment components are small for simple construction, the interaction factors can be conservatively simplified to :

$$\frac{N_{Ed}}{N_{b,z,Rd}} + \frac{M_{y,Ed}}{M_{b,Rd}} + 1.5\frac{M_{z,Ed}}{M_{c,z,Rd}} \leq 1$$

Example 1 : Design of column with combined bending and axial load

Thank You