SKEE 2073
Signals & Systems

Chapter 5 (Part I):
Frequency Response and Filter

Outline

- Introduction
- Transfer Function
- Frequency Response Plot
- Bode Plot
- Real Poles and Zeros

Introduction

- The frequency response of a circuit describes the relationships between the amplitudes and phase angles of the input and output sinusoids which are frequency dependant.
- Bode plots is a generalizations of obtaining the frequency response of a network based on the transfer functions.
- Bode plots allow us to approximate the frequency response of a circuit using straight line approximations and becomes the industry standard method of presenting frequency response information.

Application

- Most networks behave or act as filters.
- Many applications need to retain components in a given range of frequencies and discard the components in another range.
- This can be accomplished by the use of electrical networks called filters.

Transfer Function

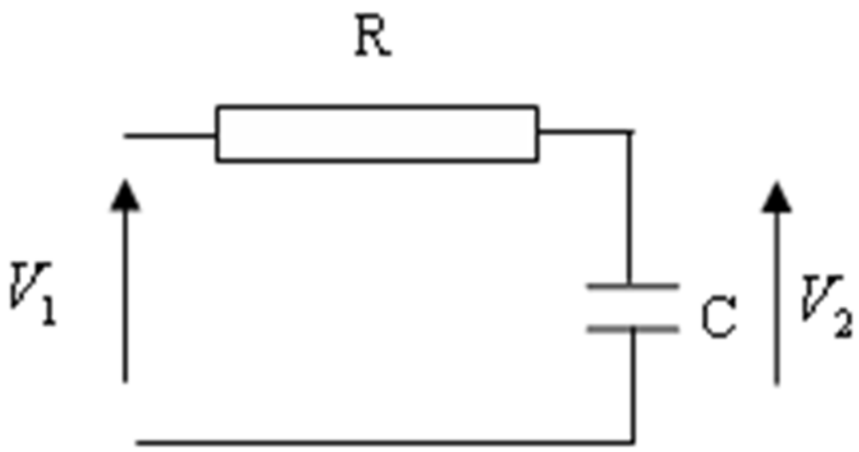
- Defined as the ratio of the phasor output voltage to the phasor input voltage as a function of frequency.

$$H(s) = \frac{V_{out}}{V_{in}}$$

- Magnitude of transfer function shows how the amplitude of each frequency component is affected by the network.
- Phase of transfer function shows how the phase of each frequency component is affected by the network.

Example 1

- Find the transfer function of the network below given as $H(s)=\frac{V_2}{V_1}$



Example 1: Solution

- The transfer function is found by voltage division

$$H(s)=\frac{1/sC}{R+1/sC}=\frac{1}{sRC+1}$$

where $s=\sigma+j\omega$. At steady state $\sigma=0$, $s = j\omega$.

Therefore the magnitude and angle of H(s) are

$$|H(jw)|=\frac{1}{\sqrt{1+(wRC)^2}} \qquad \theta(w)=-\tan^{-1}(wRC)$$

Example 2

- Given the transfer function of a network as

$$H(s) = \frac{s10^4}{s^2 + 2000\,s + 5\times10^6}$$

- Evaluate the transfer function at $\omega = 1000$ rad/s and $\omega = 3000$ rad/s.

Example 2: Solution

- At $\omega = 1000$ rad/s, the value of H(s) is:

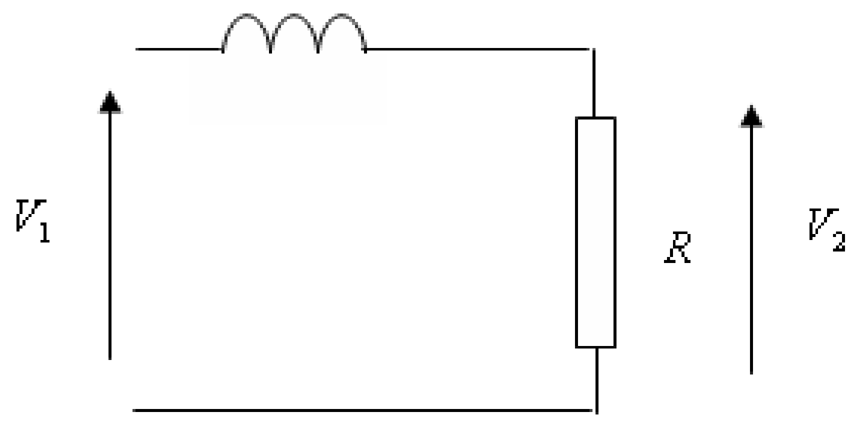
$$H(j1000) = \frac{(j1000)10^4}{(j1000)^2 + 2000(j1000) + 5\times10^6}$$

$$= \frac{j10^7}{(5\times10^6 - 10^6) + j2\times10^6} = \frac{j10}{4+j2}$$

$$= \frac{10e^{j90^0}}{\sqrt{20}e^{j26.6^0}} = 2.24e^{j63.4^0}$$

Example 3

- Find the frequency response of the system shown below at frequency ω1 = 1000 π, ω2 = 3000 π, ω3 = 5000 π, and ω4 = 7000 π. Given the value of R = 200 π Ω and L = 0.1 H.



Example 3: Solution

- To find the frequency response of the system for each frequency, we need to evaluate the transfer function.

$$H(s)=\frac{V_2}{V_1}=\frac{R}{R+sL}$$

- For $s = j\omega$

$$H(j\omega)=\frac{R}{R+j\omega L}=\frac{1}{1+j(\omega L/R)}$$

Example 3: Solution (cont.)

- The algebraic expression for the transfer function may be simplified in form by defining $\omega_p = R/L = 2000\,\pi$. ω_p is sometimes referred as “corner frequency”. Then:

$$H(j\omega) = \frac{\angle -\tan^{-1}(\omega/\omega_p)}{\sqrt{1+(\omega/\omega_p)^2}} = |H(j\omega| \angle \theta(\omega)$$

- At ω_1 , ω_2 , ω_3 , ω_4 :

$$H(j1000\,\pi) = \frac{\angle -\tan^{-1} 0.5}{(1+0.25)^{1/2}} = 0.89\angle -26.6$$

$$H(j3000\pi) = \frac{\angle -\tan^{-1} 1.5}{(1+2.25)^{1/2}} = 0.55\angle -56.3$$

$$H(j5000\pi) = \frac{\angle -\tan^{-1} 2.5}{(1+6.25)^{1/2}} = 0.37\angle -68.2$$

$$H(j7000\pi) = \frac{\angle -\tan^{-1} 3.5}{(1+12.25)^{1/2}} = 0.27\angle -74.1$$

FREQUENCY RESPONSE

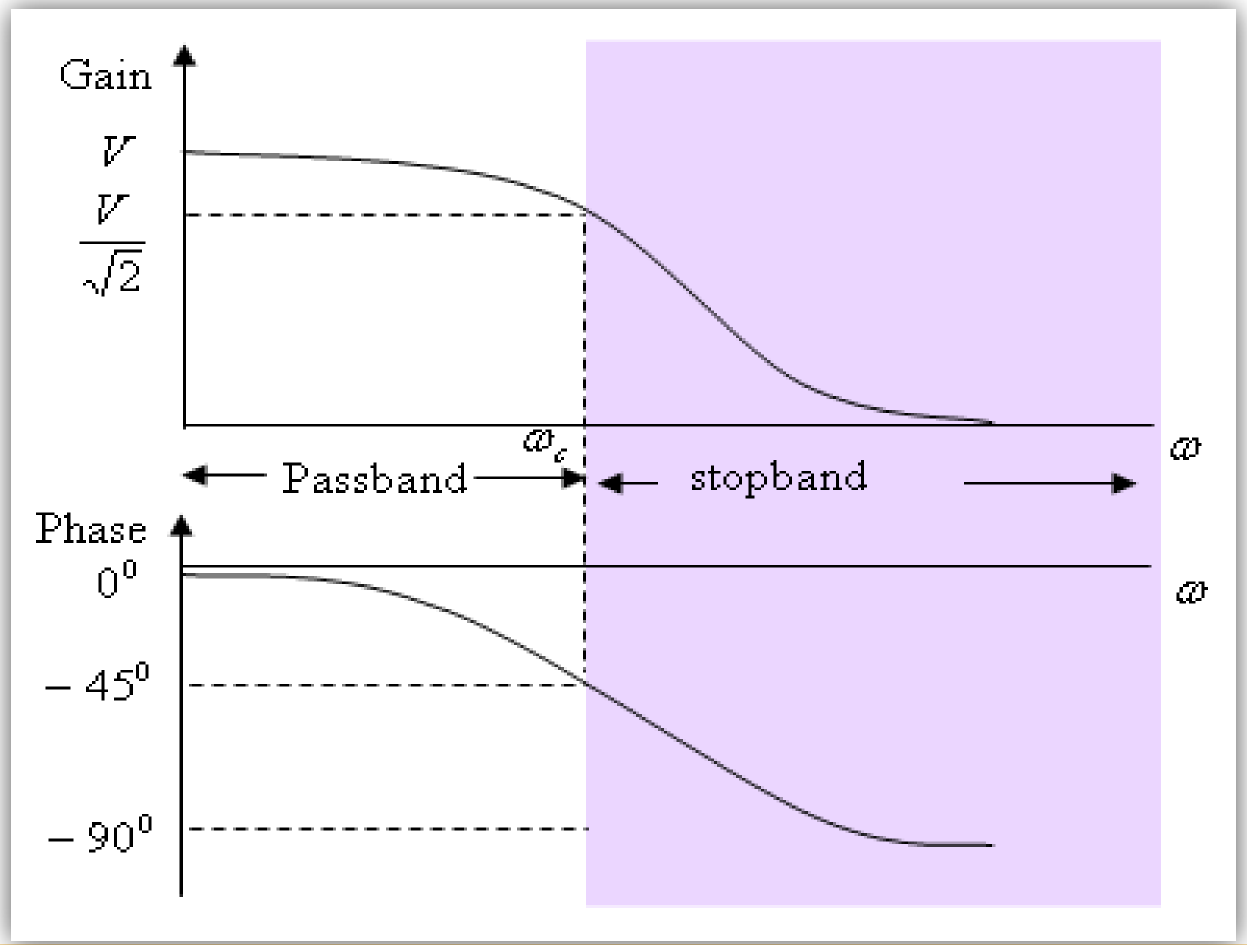
- Frequency response describes the output amplitude and the output phase of the network.
- Generally it is in a form of a filter function.

Output amplitude = $|H(j\omega)|$ x input amplitude (GAIN)

Output phase = input phase + $\theta(\omega)$ (PHASE)

Frequency Response Plot

- Frequency response: Gain and phase response.



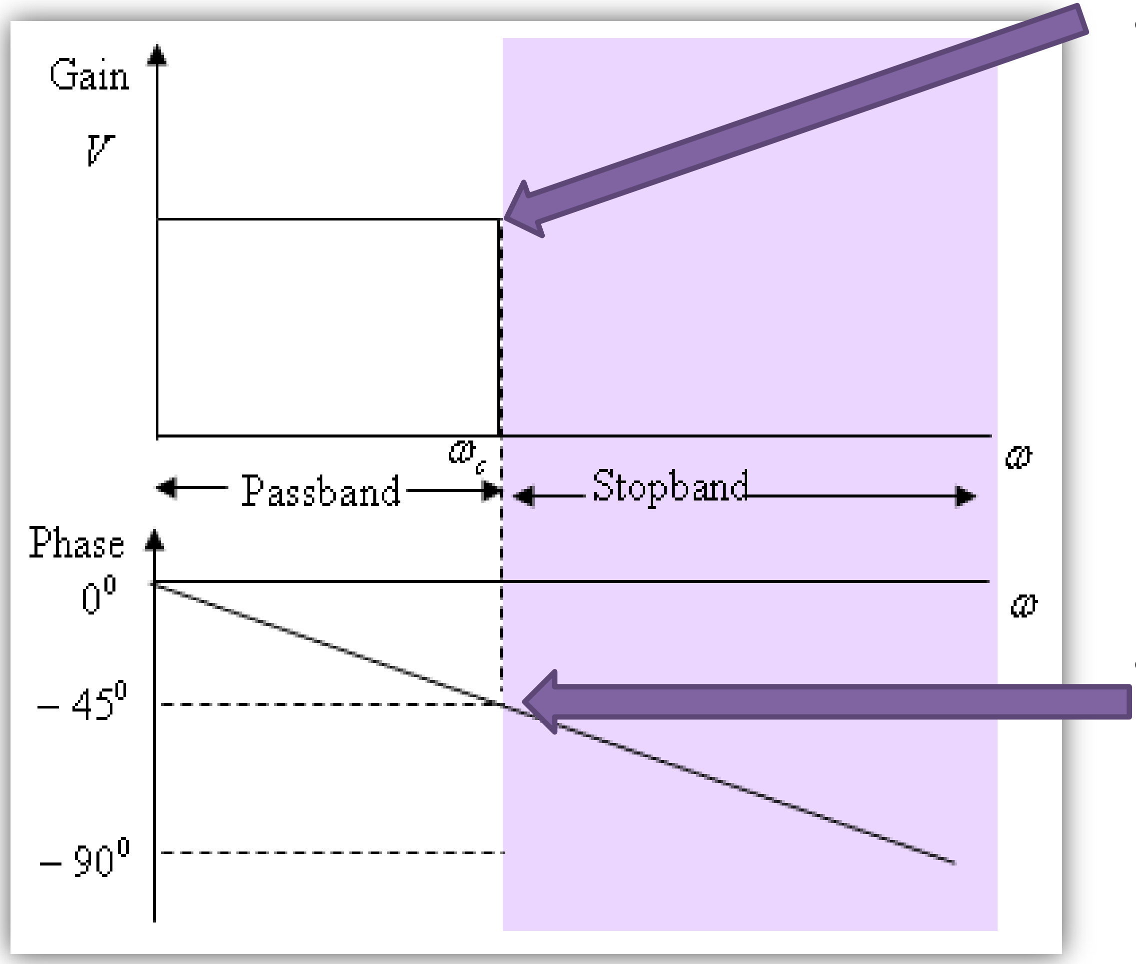
Filter

- ☐ Special network purposely designed to block unwanted frequencies.
- ☐ **Passive filter network** consists of passive elements such as L, C, and R.
- ☐ **Active filter network** consists of active element such as op-amp and transistors.

Filter (cont.)

- The frequency range over which the output is significantly attenuated is called *stopband*.
- The frequency range over which there is little attenuation is called *passband*.
- The *cutoff frequency,* ω_c is defined as the boundary between passband and stopband.

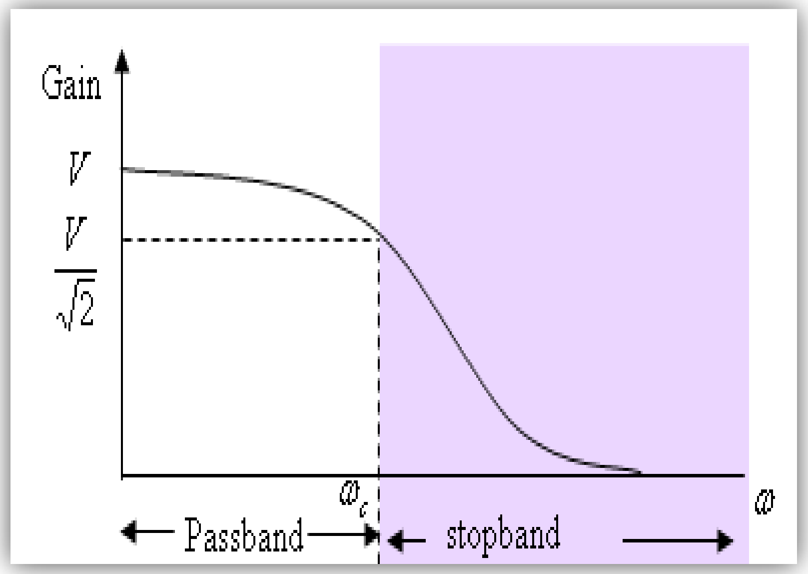
Ideal Filter



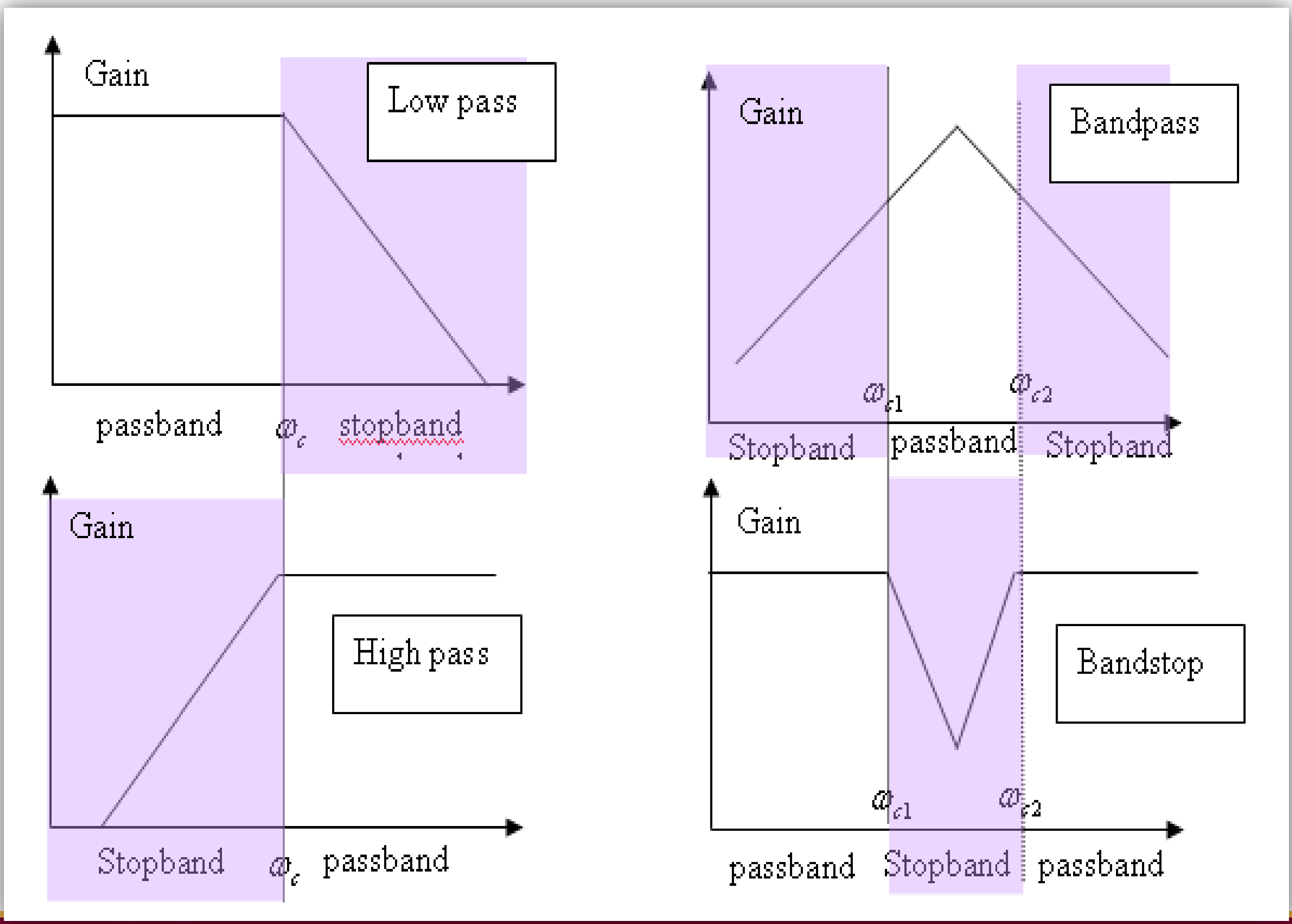
- Sharp cutoff frequency.
- Linear phase change.

Practical Filter

- The transition from passband to stopband is gradual.
- The *cutoff frequency* is at which the gain has decreased by a factor of *0.707 from its maximum* value passband gain.
- In terms of power, the cutoff frequency is at half the maximum power of -3dB point.



Types of Filter



Bandwidth of the Filter

- The bandwidth of a filter is defined as the range spanned by its passband.
- E.g. for bandpass filter, the bandwidth is the difference in the two cutoff frequencies:

$$BW = \omega_{c2} - \omega_{c1}$$