CHAPTER 3

System Representation

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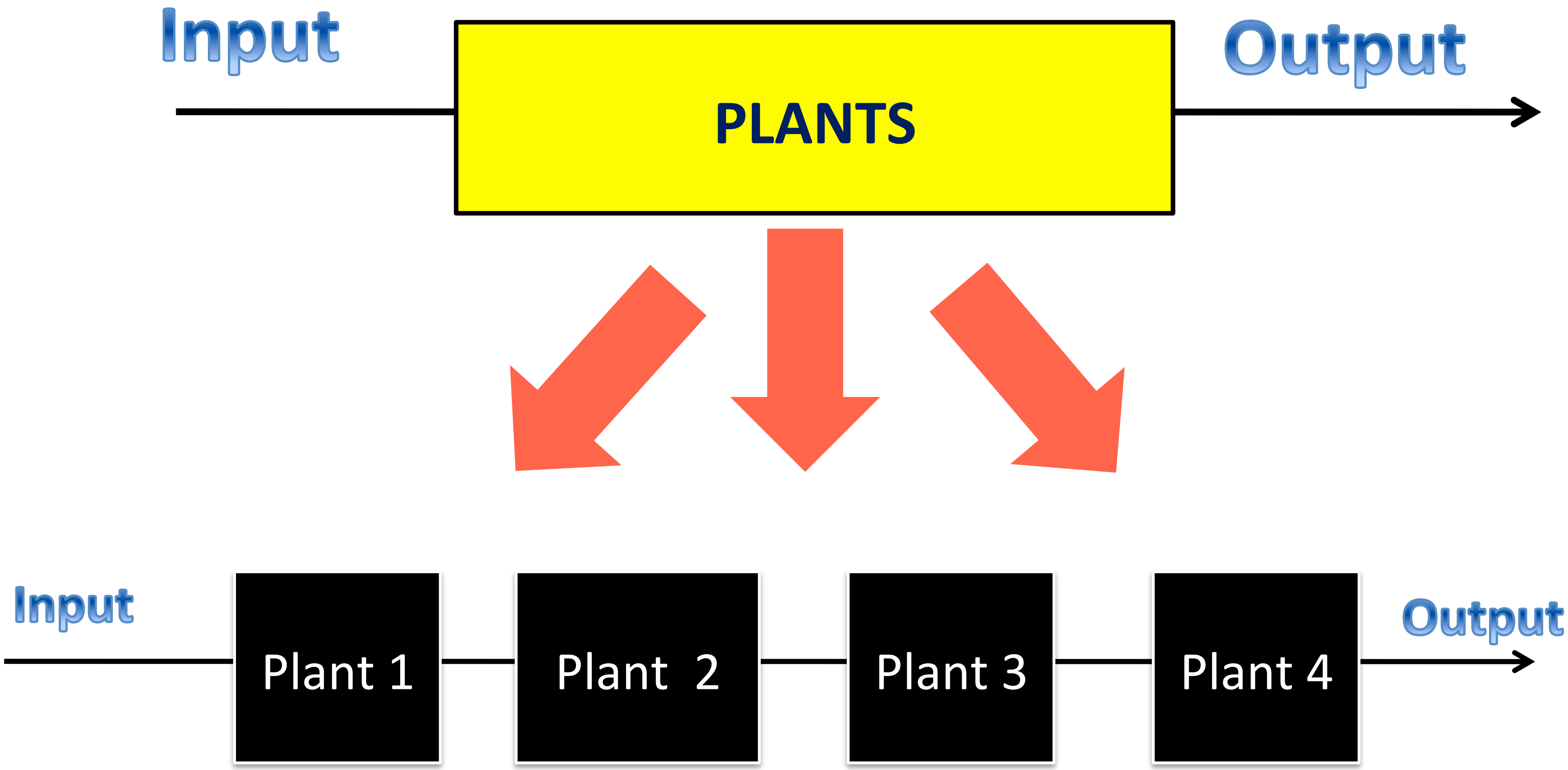
3.1	• Important definitions
3.2	• Techniques of simplifying block diagrams.
3.3	• Signal flow graphs
3.4	• Changing block diagrams to signal flow graphs and vice versa
3.5	• Mason's Rule and example questions

3.1 Important Definitions

Control System Definitions



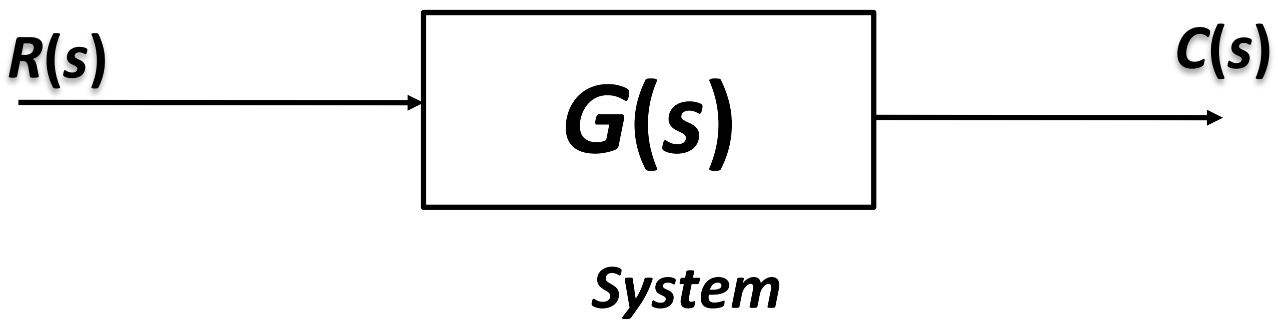
INTRODUCTION

- A control system consists of the inter-connection of subsystems.
- A more complicated system will have many inter-connected subsystems.
- For the analysis purposes → we present multiple subsystems as a single transfer function.

BLOCK DIAGRAMS

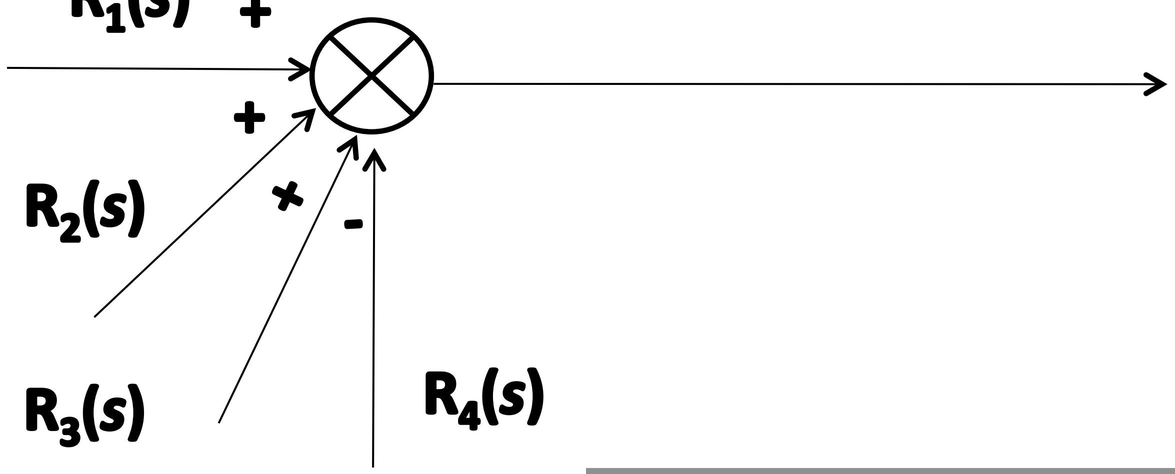
Signal: The direction of signal flow is shown by the arrow

Block Diagram: The system block represented by a transfer function



1. Summing Junction

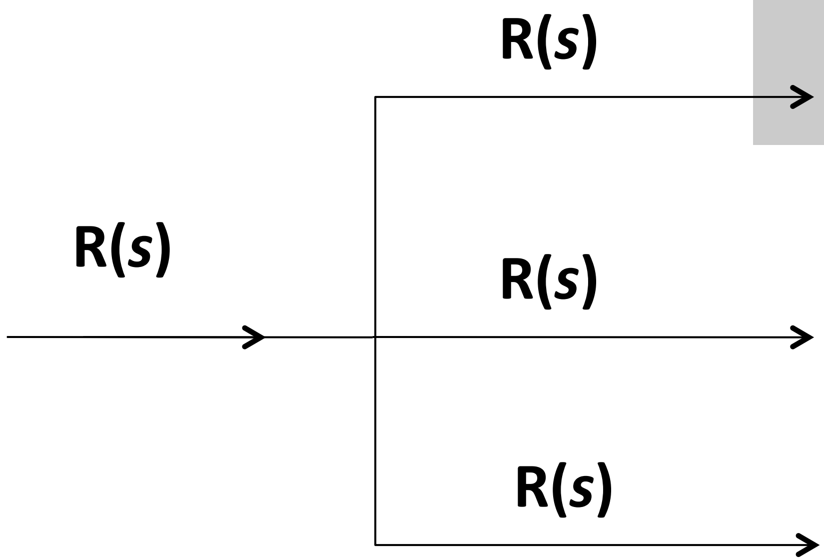
$$C(s) = R_1(s)+R_2(s)+R_3(s)-R_4(s)$$



Allows 2 or more signals to be added/subtracted

2. Take-Off Point

Signal from a block



Goes to other block/ summing junction

Signal can be taken from any component's output

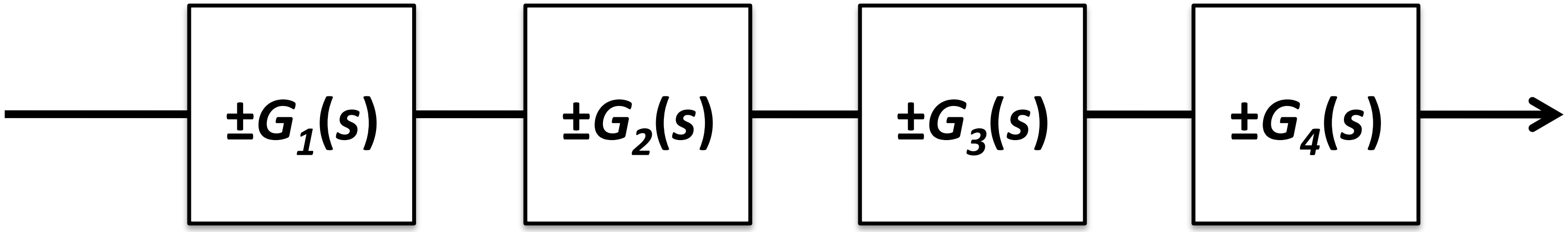
Assume constant signal

The subsystems in a block diagram are normally connected in three forms:

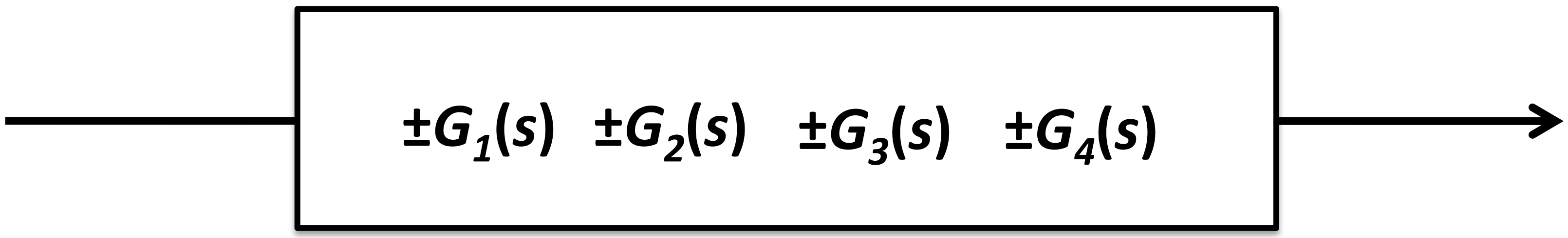
1. **Cascade form**

2. **Parallel form**

3. **Feedback form**



* **The block diagram can be reduced into a single block by multiplying every block to give:**



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2. **Parallel form**

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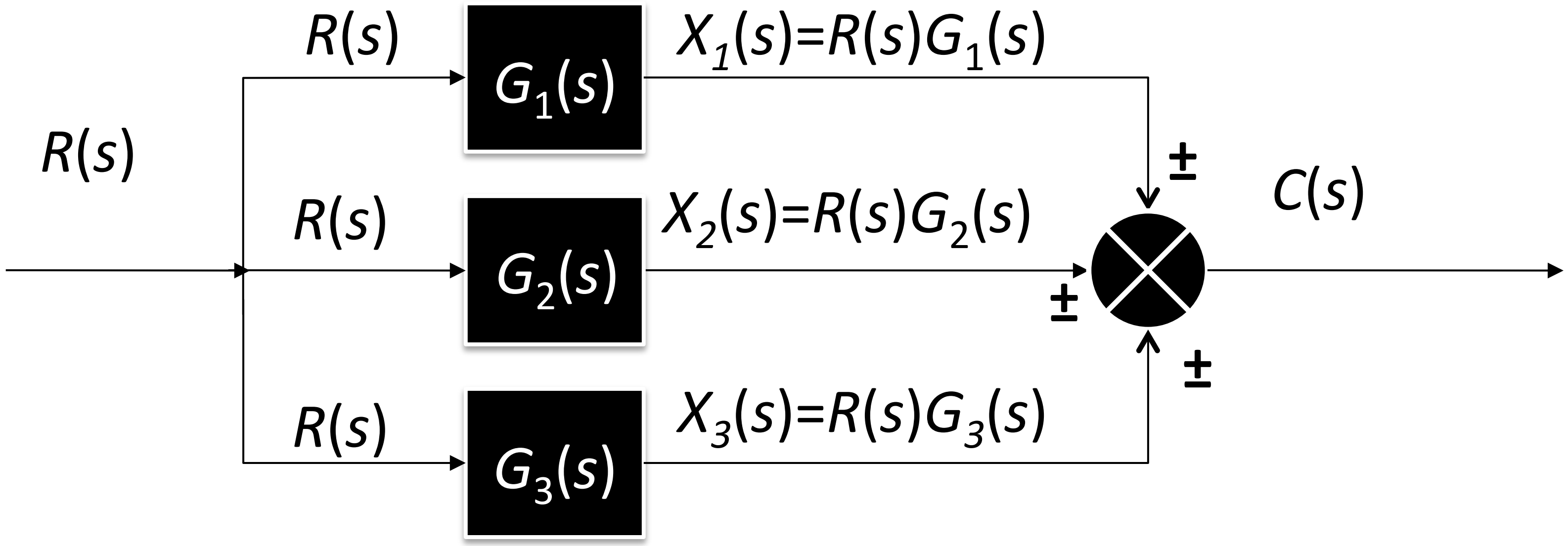
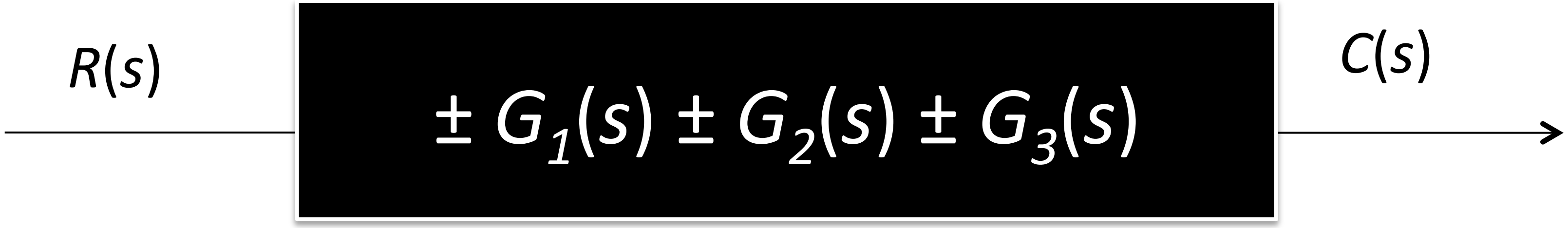


$$C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]\, R(s)$$

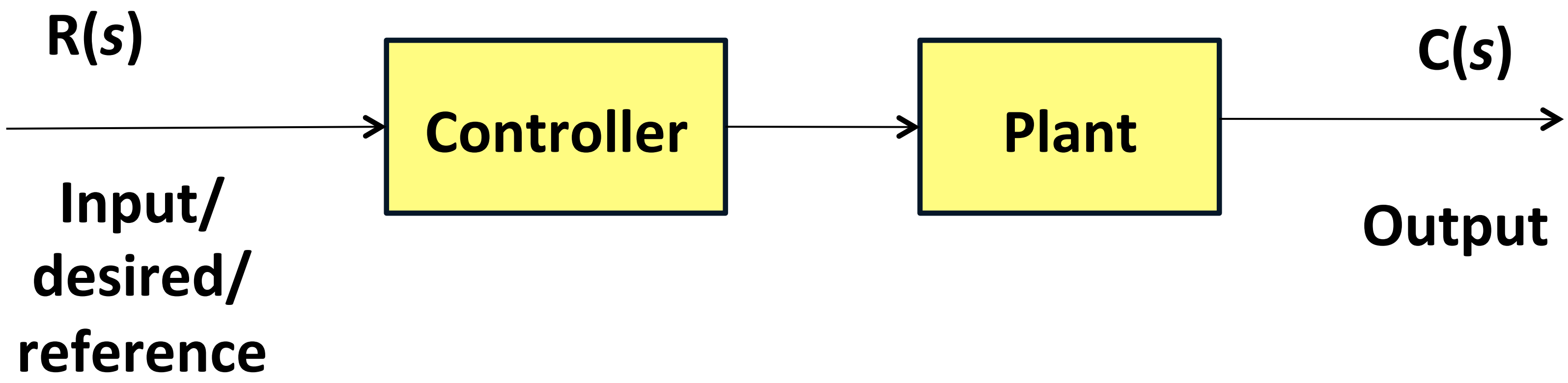
*** The block diagram can be reduced into a single block by summing every block to give:**



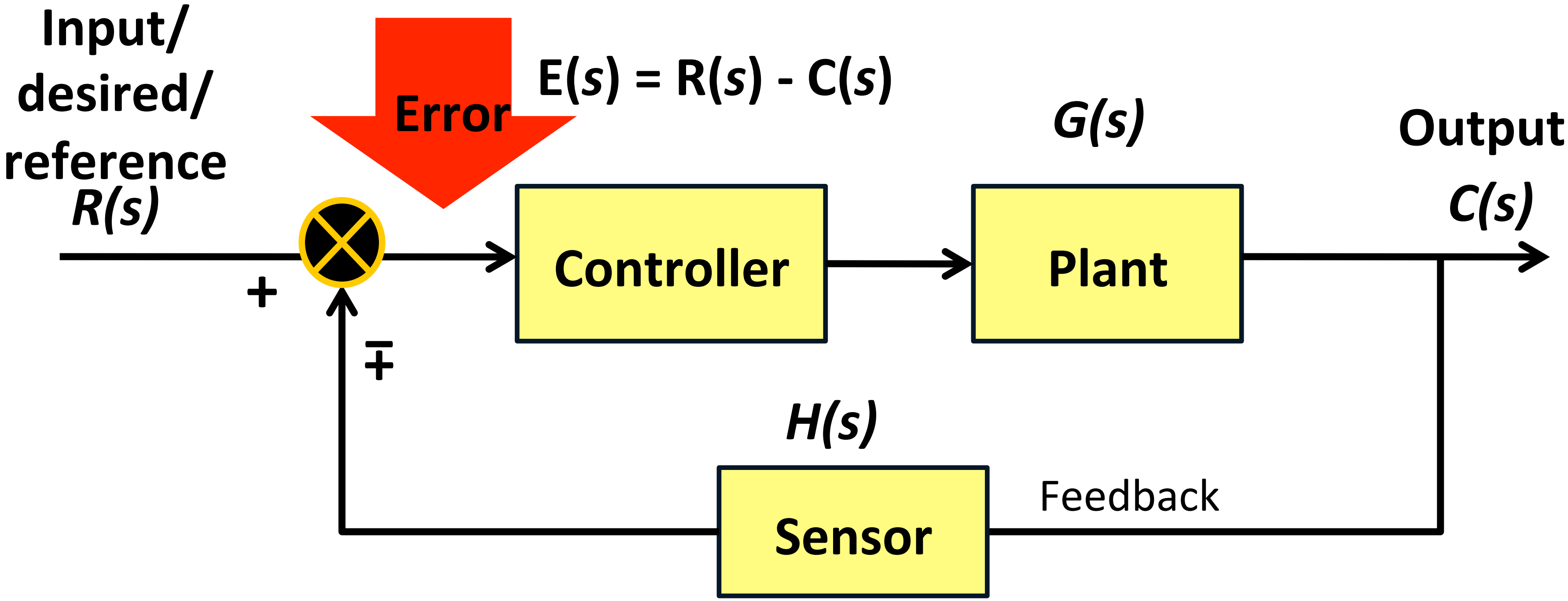
The subsystems in a block diagram are normally connected in three forms:

1. **Cascade form**
2. **Parallel form**
3. **Feedback form**

Open loop System



Closed-loop System



R(s)

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

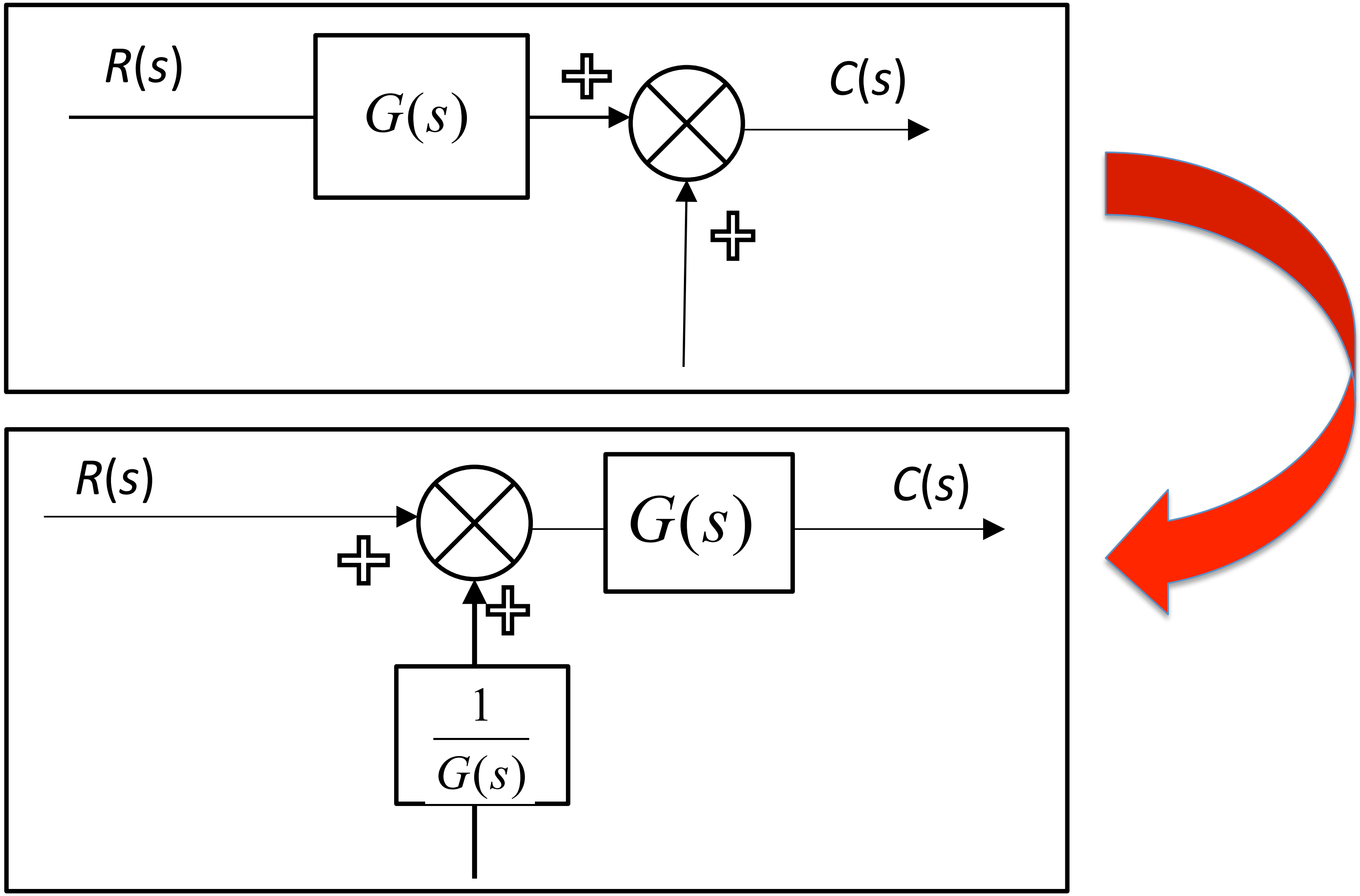
C(s)

3.2 Techniques of Simplifying Block Diagrams

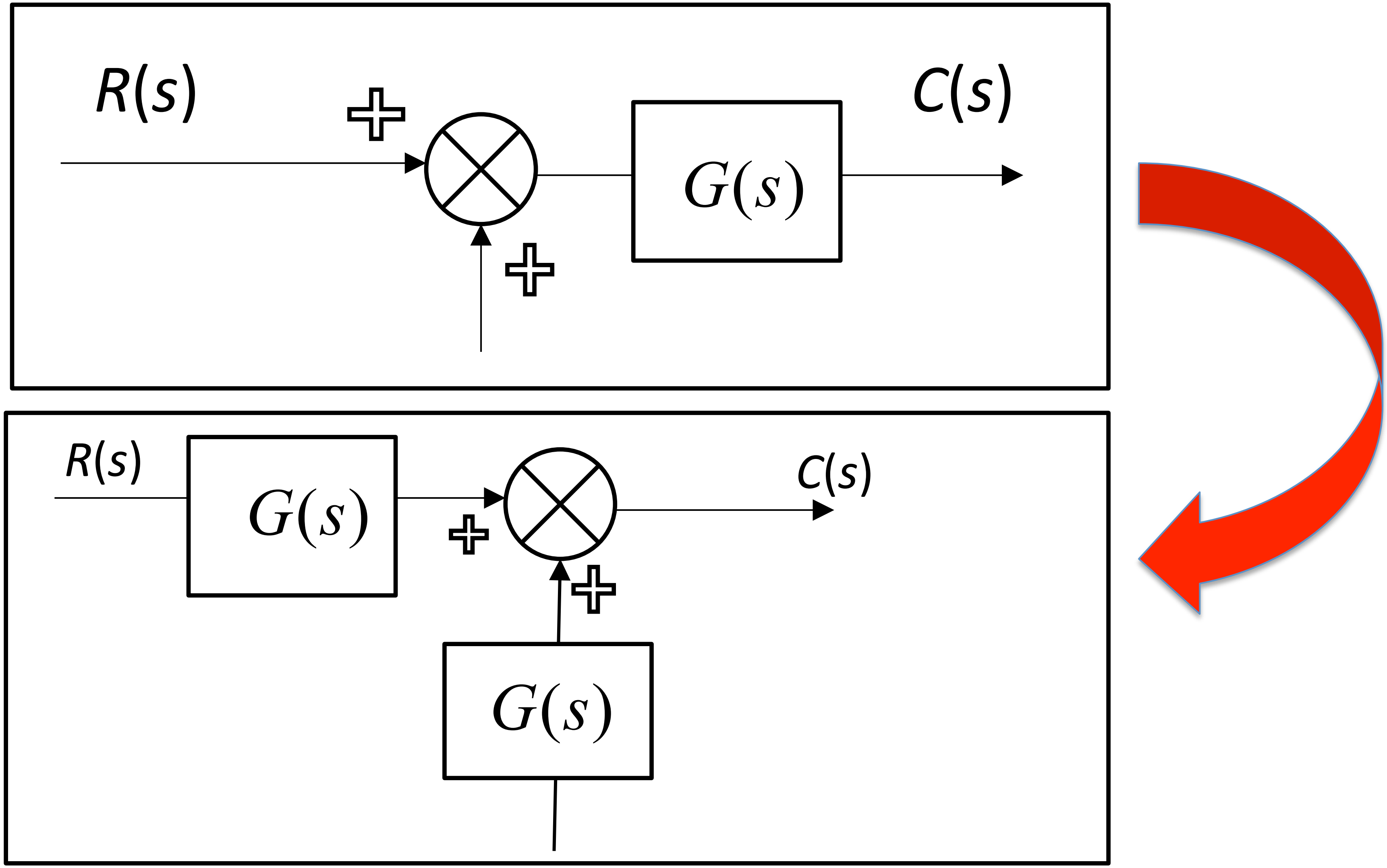
Moving blocks to create familiar forms

- It is not always apparent to get block diagrams in the familiar forms
- We have to move blocks to get the familiar forms in order
- To be able to reduce the block diagram into single transfer function.

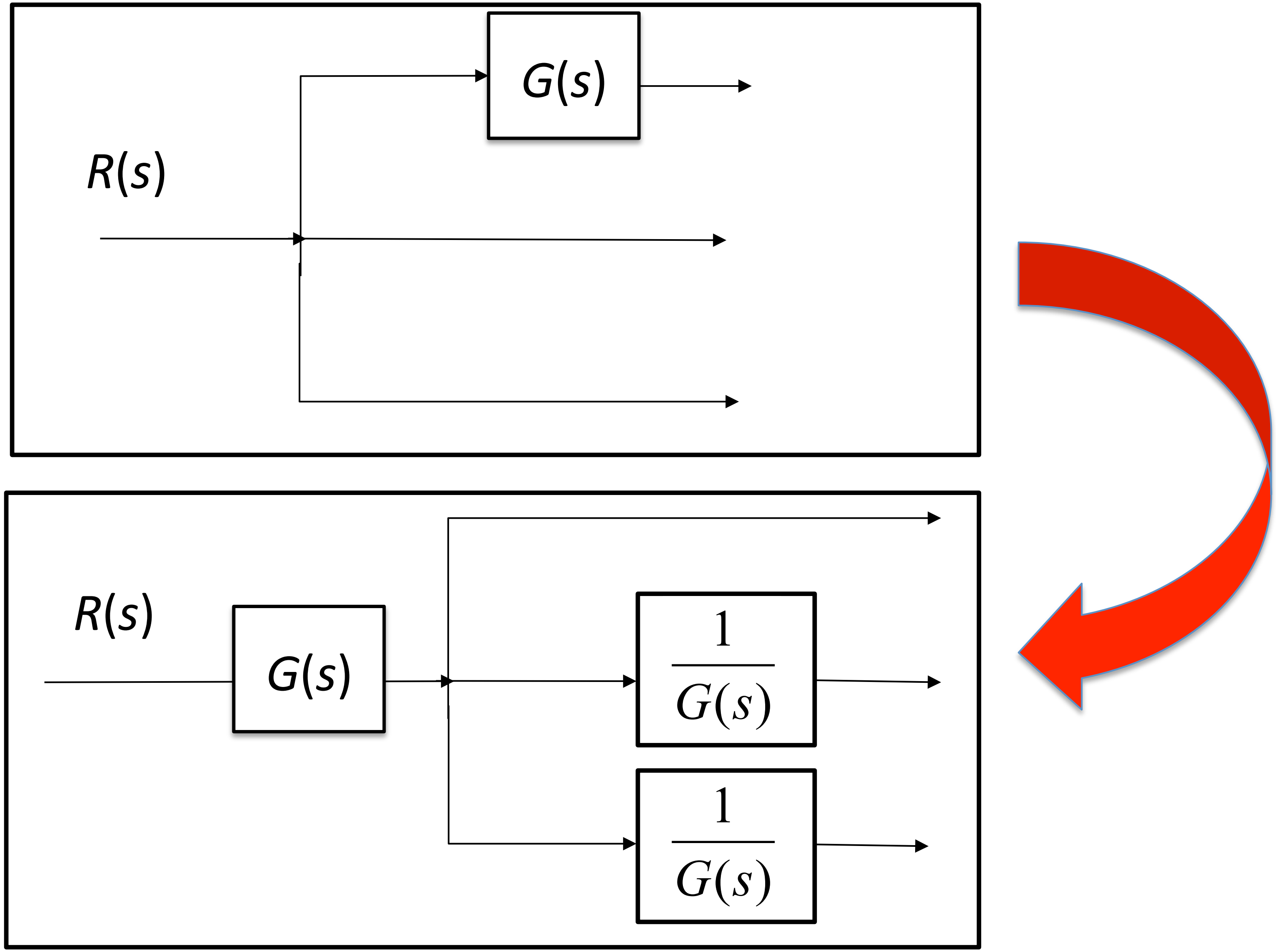
(i) Summing Junction



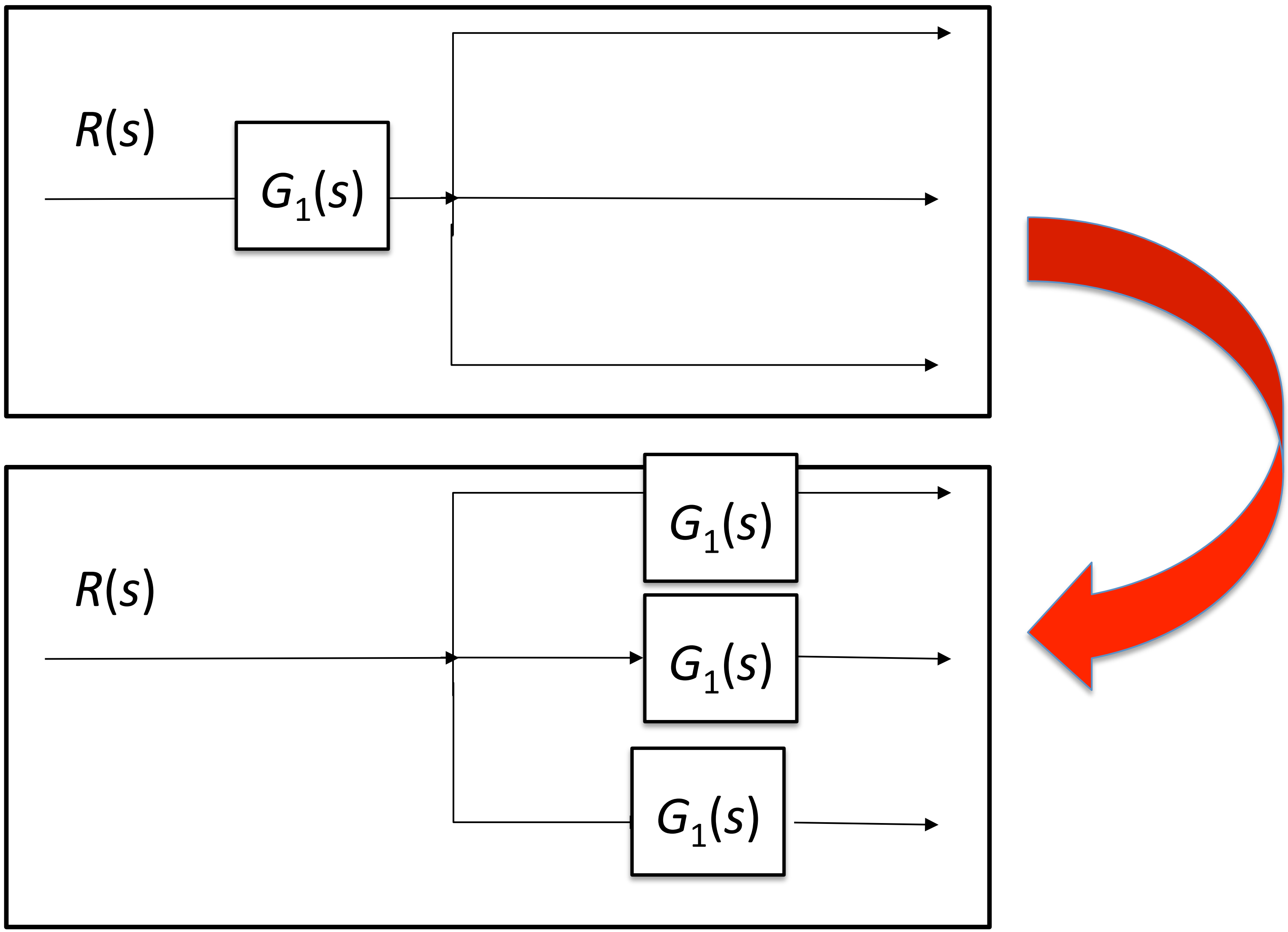
(i) Summing Junction



(ii) Pick-off point



(ii) Pick-off point



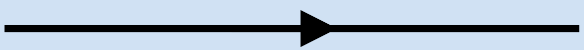
3.3
Signal Flow Graphs

COMPONENTS OF SIGNAL FLOW GRAPH

branches

(represent system)

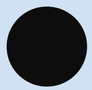
represented by a line with arrow showing the direction of signal flow through the system



nodes

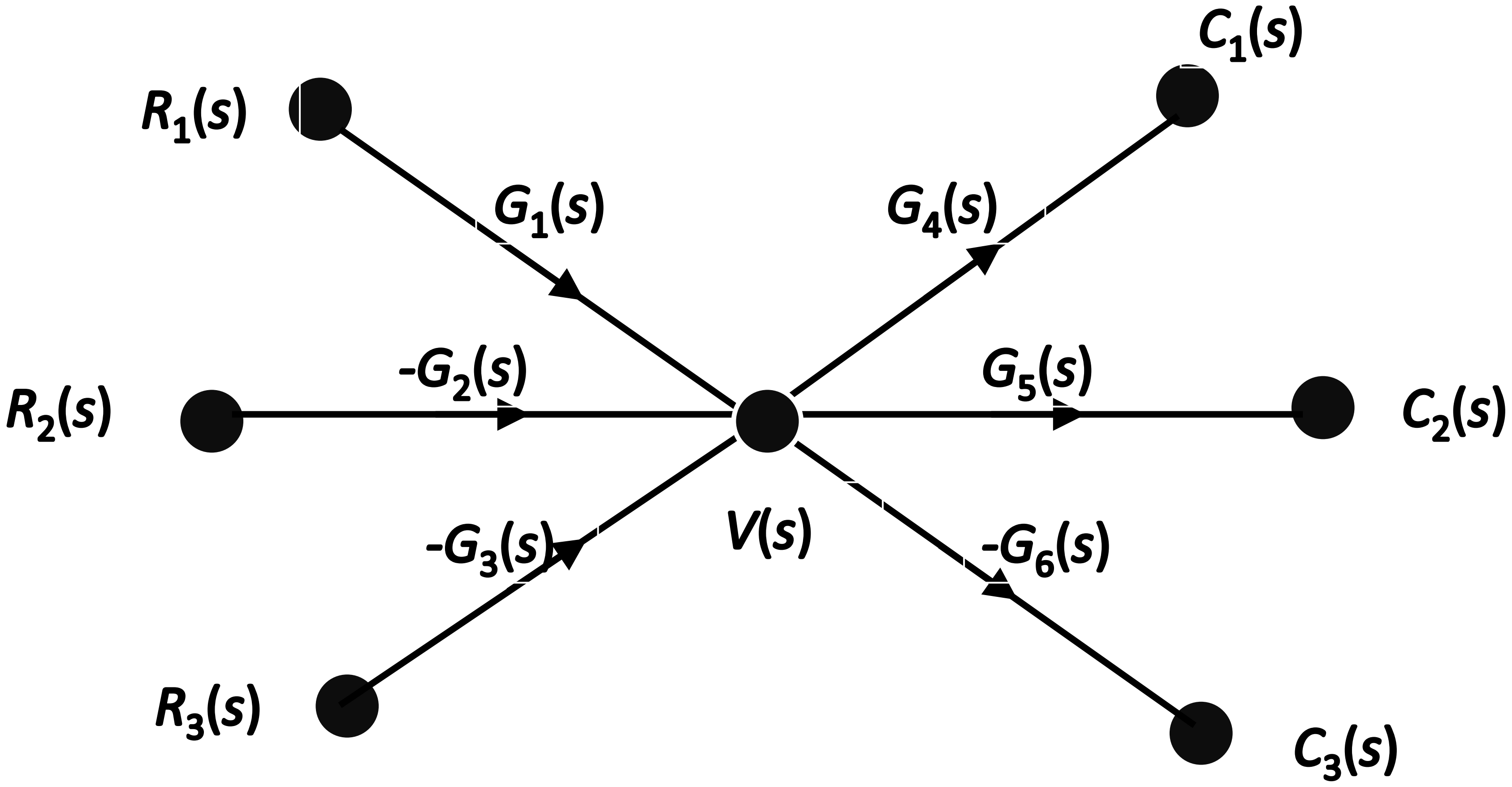
(represents signals)

represented by a small circle with the signal's name is written near the node

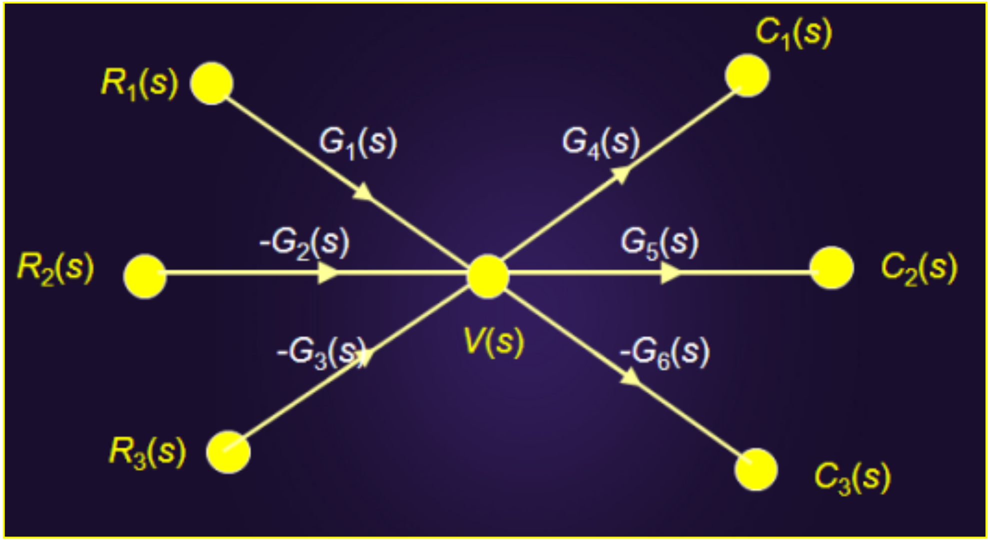


Interconnection of systems & signals

Each signal is the sum of signals flowing into it the connection



Each signal is the sum of signals flowing into it



- $V(s) = R_1(s)G_1(s) – R_2(s)G_2(s) - R_3(s)G_3(s)$

- $C_2(s) = V(s)G_5(s)$

  $= R_1(s)G_1(s)G_5(s) – R_2(s)G_2(s)G_5(s)\ G_3(s)G_3(s)G_5(s)$

- $C_3(s) = - V(s)G_6(s)$

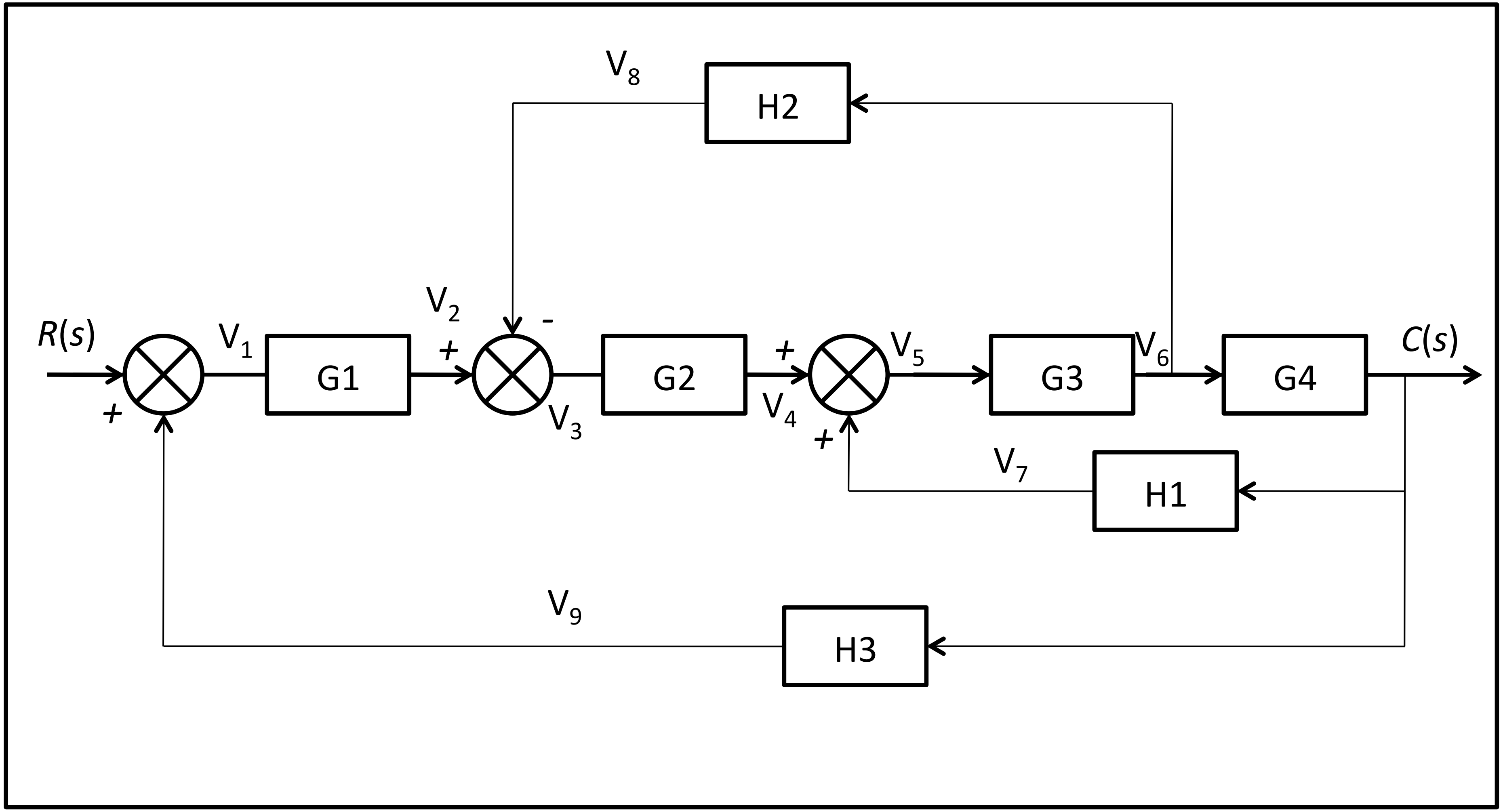
  $= - R_1(s)G_1(s)G_6(s)-R_2(s)G_2(s)G_6(s) + R_3(s)G_3(s)G_6(s)$

3.4 Changing Block Diagrams to Signal Flow Graphs and vice versa

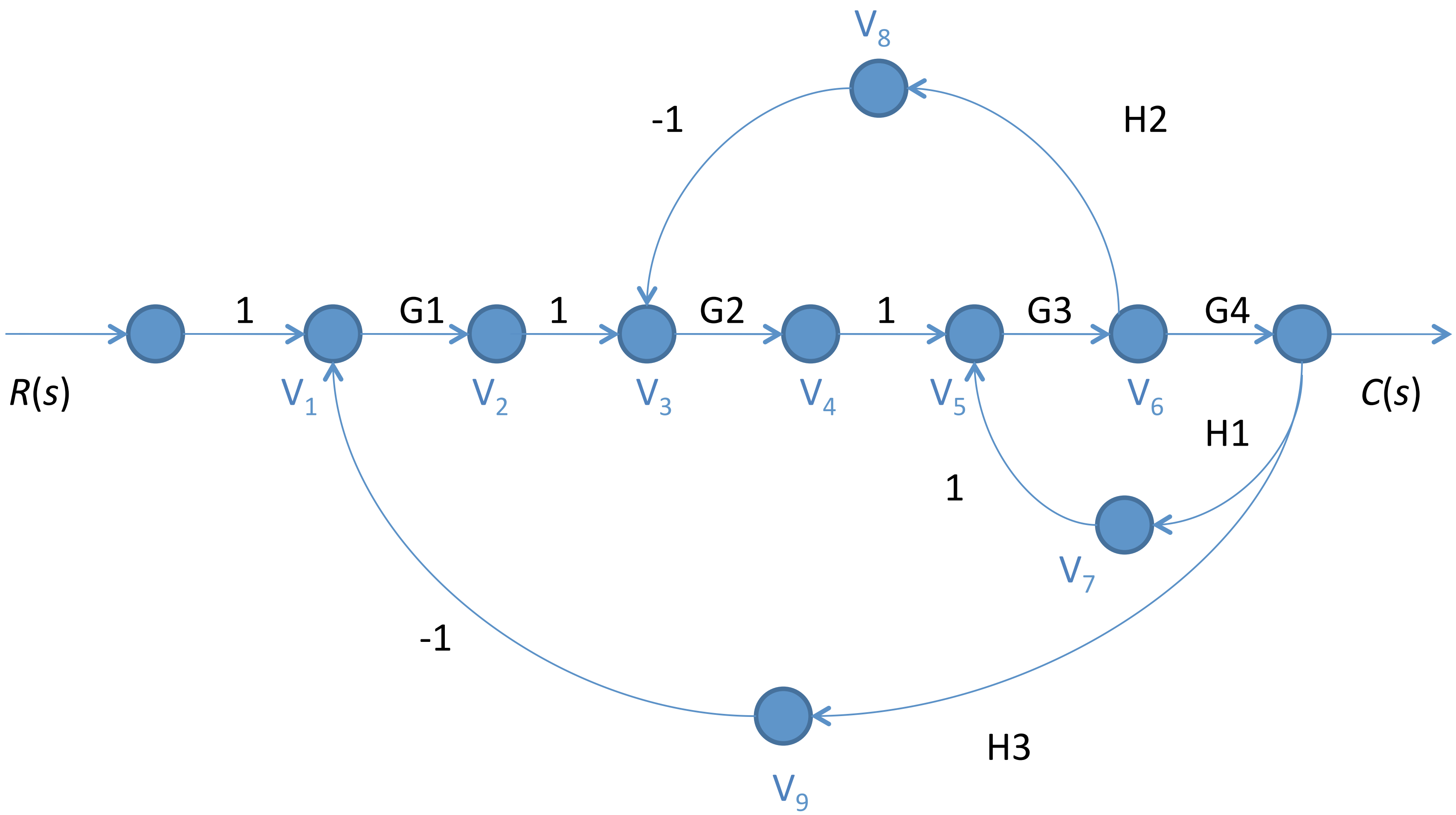
Converting Common Block Diagrams into Signal-flow Graphs

- We can convert the block diagrams in *cascade*, *parallel* and *feedback forms* into signal-flow diagrams
- We can start with drawing the signal nodes, and then interconnect the signal nodes with system branches.

Example



Answer



3.5
Mason's Rule

MASON'S RULE

Mason's rule is used to get the single transfer function in the signal-flow graph using a formula

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

Mason's Formula

Forward Path

Loop gains related too

$$\frac{\sum_k T_k \Delta_k}{\Delta}$$

Something to do with loop gains

Mason’s Formula

Forward Path

A path from the input node to the output node in the direction of signal flow.

$$\frac{\sum_k T_k \Delta_k}{\Delta}$$

$$\frac{\sum_k T_k \Delta_k}{\Delta}$$

Something to do with loop gains

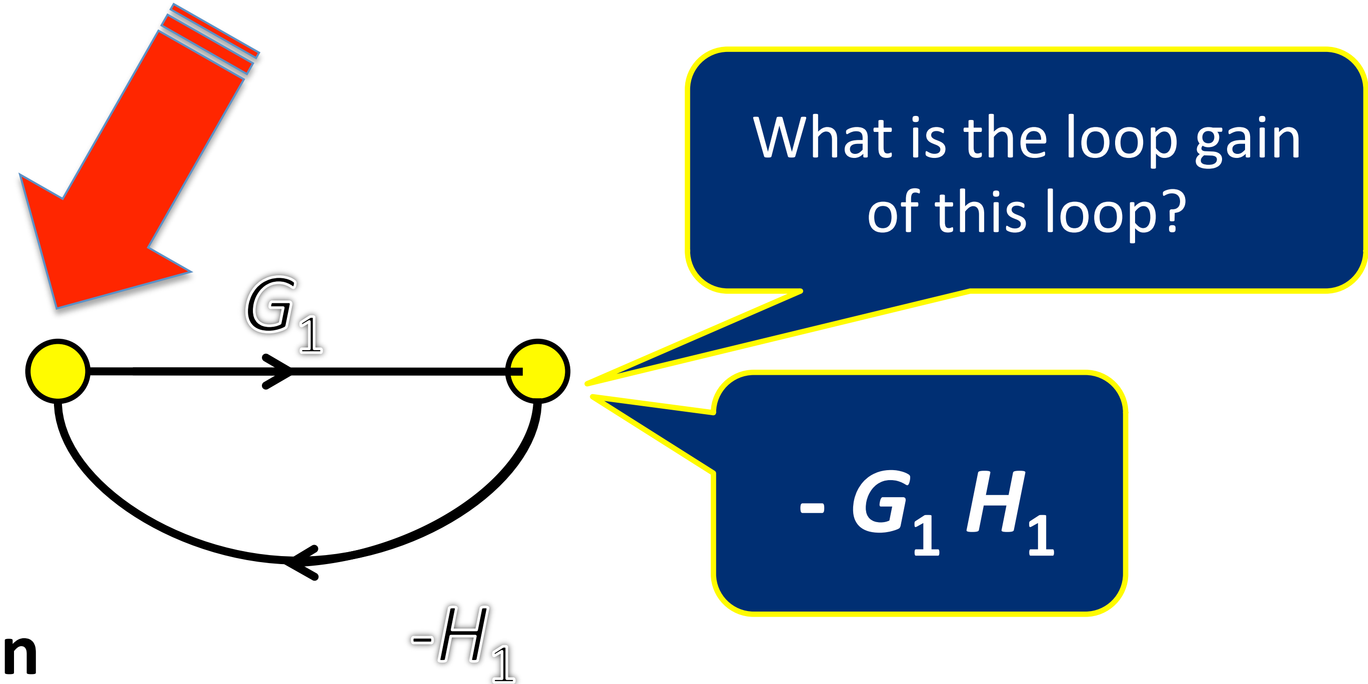
What are loop gains?

Δ =

1 - ∑ loop gains
+ ∑ 2 non-touching loop gains (at one time)
- ∑ 3 non-touching loop gains (at one time)
+ ∑ 4 non-touching loop gains (at one time)

Define Loop

A closed path which starts and ends at the same node



Loop Gain

the product of branch gains (value) found by going around a loop

Non-touching Loops

loops that do not have any nodes in common.

Non-touching loops gain

the product of loop gains from non-touching loops taken two, three, four, or more at a time.

Mason's Formula

$\Delta_k = \Delta$ - $\sum$ loop gain terms in **Δ** that touch the k^{th} forward path

Loop gains related too

$$\frac{\sum_k T_k \Delta_k}{\Delta}$$

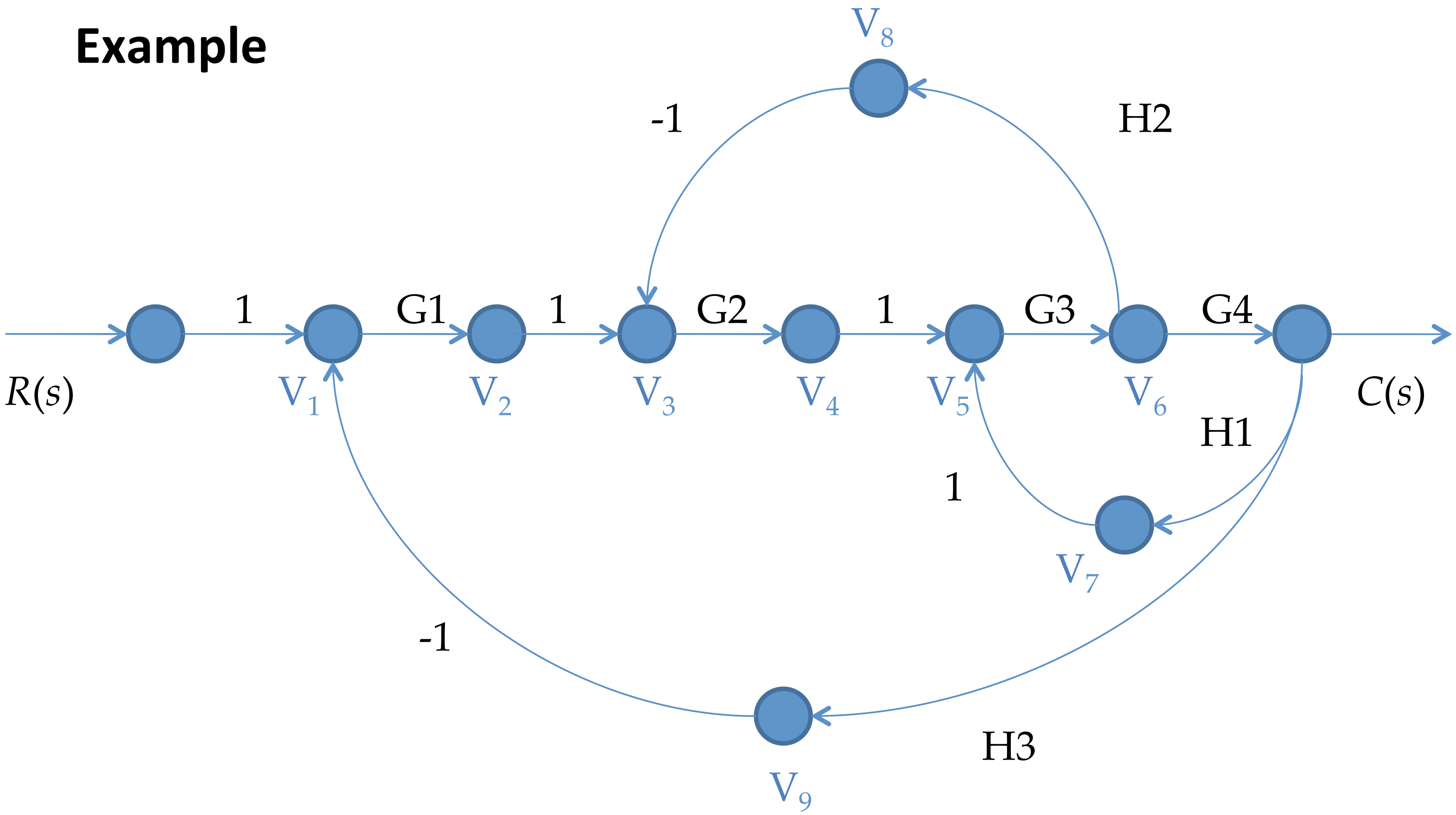
$$G(s) = \frac{\sum_k T_k \Delta_k}{\Delta}$$

$\Delta_k = \Delta -$ ∑ loop gain terms in Δ that touch the k^{th} forward path

$\Delta = 1 -$ ∑ loop gains
+ ∑ 2 non-touching loop gains at one time
- ∑ 3 non-touching loop gains at one time
+ ∑ 4 non-touching loop gains at one time

Sum of loop gains in Δ that touches the forward paths

Example



Answer

- Identify forward path gains

  G1G2G3G4

- Identify Loop gains

  -G2G3H2, G3G4H1, -G1G2G3G4H3

- Non touching Loops → NONE → 0
- Form Δ

  $\Delta = 1 - [-G2G3H2 + G3G4H1 - G1G2G3G4H3] + 0$

  $= 1 - G3G4H1 + G2G3H2 + G1G2G3G4H3$

- Form Δ_k

  $\Delta_k = \Delta - [-G2G3H2 + G3G4H1 - G1G2G3G4H3] = 1$

Put into formula yields

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$

Compare the answer when you do block order reduction → SAME!!

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