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ONLINE LEARNING

Digital Electronics (SKEE1223)

Boolean Algebra

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Introduction

- **Boolean algebra consists of:**
  - **Boolean Theorems**
  - **DeMorgan's Theorem**

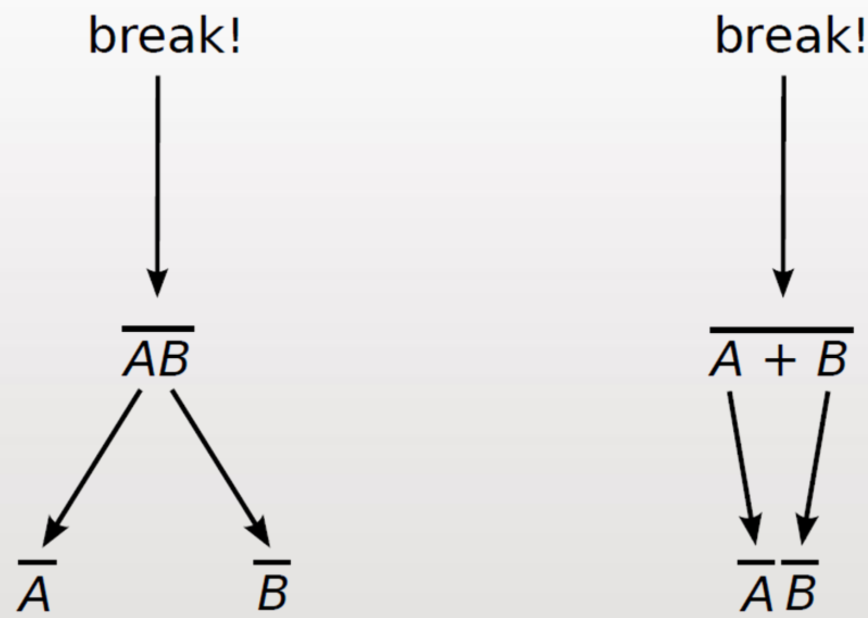
Boolean Theorems

Theorem 1	$A + 0 = A$	$A.1 = A$
Theorem 2	$A + \overline{A} = 1$	$A.\overline{A} = 0$
Theorem 3	$A + A = A$	$A.A = A$
Theorem 4	$A + 1 = 1$	$A.0 = 0$
Theorem 5	$\overline{\overline{A}} = A$	
Theorem 6	$A + B = B + A$	$A.B = B.A$
Theorem 7	$A + (B + C) = (A + B) + C$	$A.(B.C) = (A.B).C$
Theorem 8	$A.(B + C) = A.B + A.C$	$A + B.C = (A + B)(A + C)$
Theorem 9	$A + A.B = A$	$A.(A + B) = A$
Theorem 10	$A + \overline{A}.B = A + B$	$A.(\overline{A} + B) = A.B$

DeMorgan’s Theorem

$\overline{A.B} = \bar{A} + \bar{B}$

$\overline{A + B} = \bar{A}\bar{B}$



Example of Boolean Proof

$$A + \bar{A}B = A + B$$

$$= A(\bar{B}+B) + \bar{A}.B \qquad \bar{B}+B=1$$

$$= A\bar{B}+AB + \bar{A}.B$$

$$= A\bar{B}+AB+AB + \bar{A}.B \qquad AB+AB=AB$$

$$= A(\bar{B}+B) +B(\bar{A}+A)$$

$$= A+B$$

Boolean Expressions

SOP	POS
Sum-of-Products	Product-of-Sums

Canonical SOP

- **Each AND term must have all the input variables (or their complements).**

Simplified or Minimized SOP

$$F(A,B,C) = A.B + \bar{B}.C$$

Canonical SOP

$$F(A,B,C) = \bar{A}.\bar{B}.C + A.\bar{B}.C + A.B.\bar{C} + A.B.C$$

Canonical POS

- **Each OR term must have all the input variables (or their complements).**

Simplified or Minimized SOP

$$F(A,B,C) = (A + \bar{B})(B + C)$$

Canonical SOP

$$F(A,B,C) = (\bar{A} + B + C).(A + \bar{B} + \bar{C})(A + \bar{B} + C)(A + B + C)$$

Minterms & Maxterms

Decimal Equivalent		Minterm		Maxterm	
	ABC	Expression	Symbol	Expression	Symbol
0	000	$\bar{A}.\bar{B}.\bar{C}$	m_0	$A+B+C$	M_0
1	001	$\bar{A}.\bar{B}.C$	m_1	$A+B+\bar{C}$	M_1
2	010	$\bar{A}.B.\bar{C}$	m_2	$A+\bar{B}+C$	M_2
3	011	$\bar{A}.B.C$	m_3	$A+\bar{B}+\bar{C}$	M_3
4	100	$A.\bar{B}.\bar{C}$	m_4	$\bar{A}+B+C$	M_4
5	101	$A.\bar{B}.C$	m_5	$\bar{A}+B+\bar{C}$	M_5
6	110	$A.B.\bar{C}$	m_6	$\bar{A}+\bar{B}+C$	M_6
7	111	$A.B.C$	m_7	$\bar{A}+\bar{B}+\bar{C}$	m_7

Deriving SOP Canonical Expressions

A	B	C	F	Minterm
0	0	0	0	
0	0	1	1	$\overline{A}.\overline{B}.C$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	$A.\overline{B}.C$
1	1	0	1	$A.B.\overline{C}$
1	1	1	1	$A.\overline{B}.\overline{C}$

$$F = \overline{A}.\overline{B}.C + A.\overline{B}.C + A.B.\overline{C} + A.\overline{B}.\overline{C}$$
$$= \Sigma(m_1 + m_5 + m_6 + m_7)$$
$$= \Sigma m(1,5,6,7)$$

Deriving POS Canonical Expressions

A	B	C	F	Maxterm
0	0	0	0	$A + B + C$
0	0	1	1	
0	1	0	0	$A + \overline{B} + C$
0	1	1	0	$A + \overline{B} + \overline{C}$
1	0	0	0	$\overline{A} + B + C$
1	0	1	1	
1	1	0	1	
1	1	1	1	

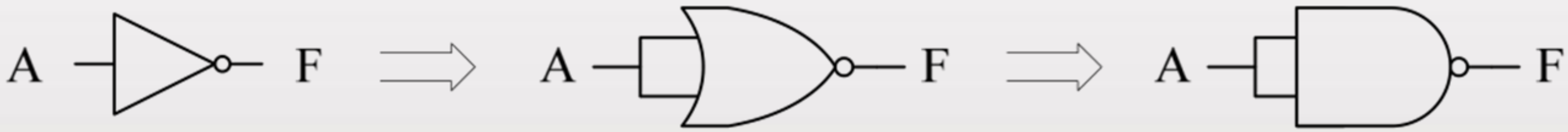
$$F = (A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)$$

$$= \prod (M_0 + M_2 + M_3 + M_4)$$

$$= \prod M(0,2,3,4)$$

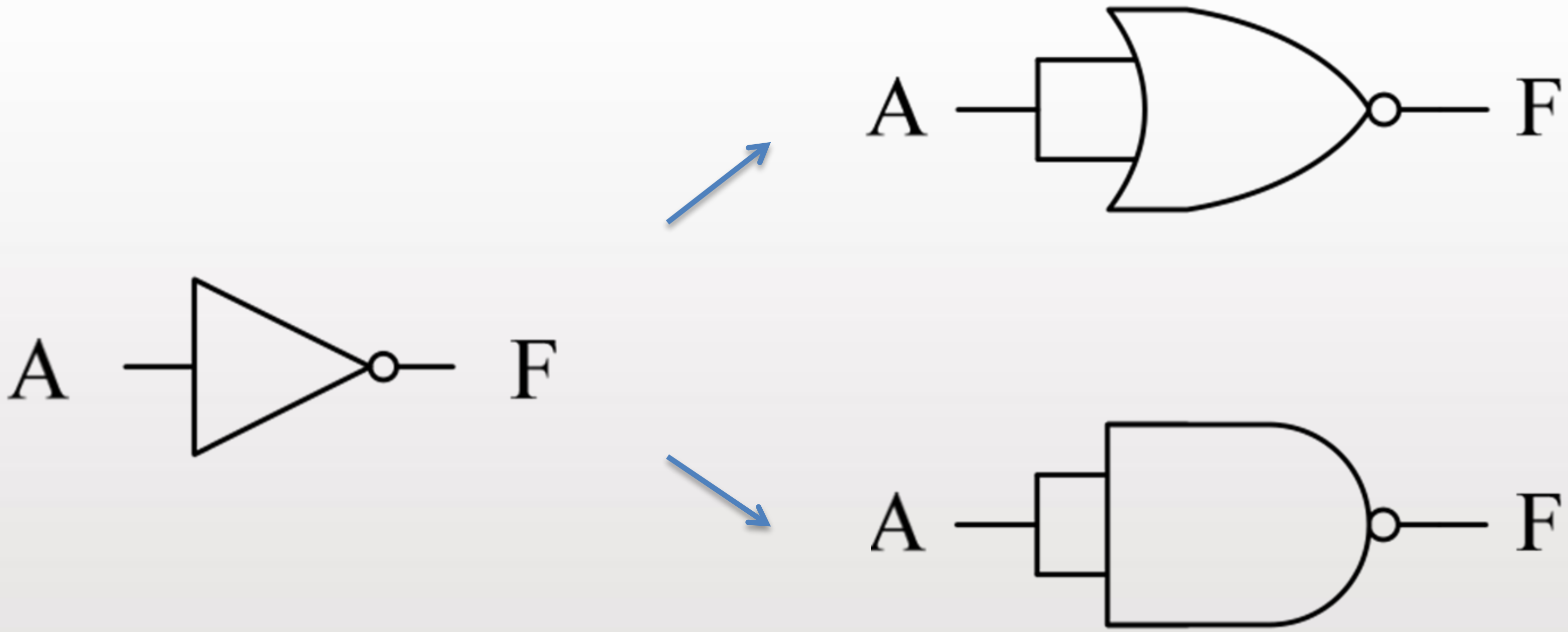
Universal Gates

- NAND and NOR gates are universal
- Any Boolean function can be implemented using either of these gates only.

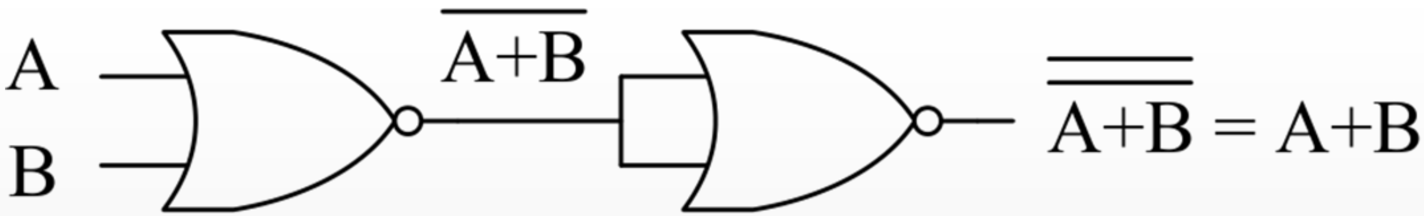


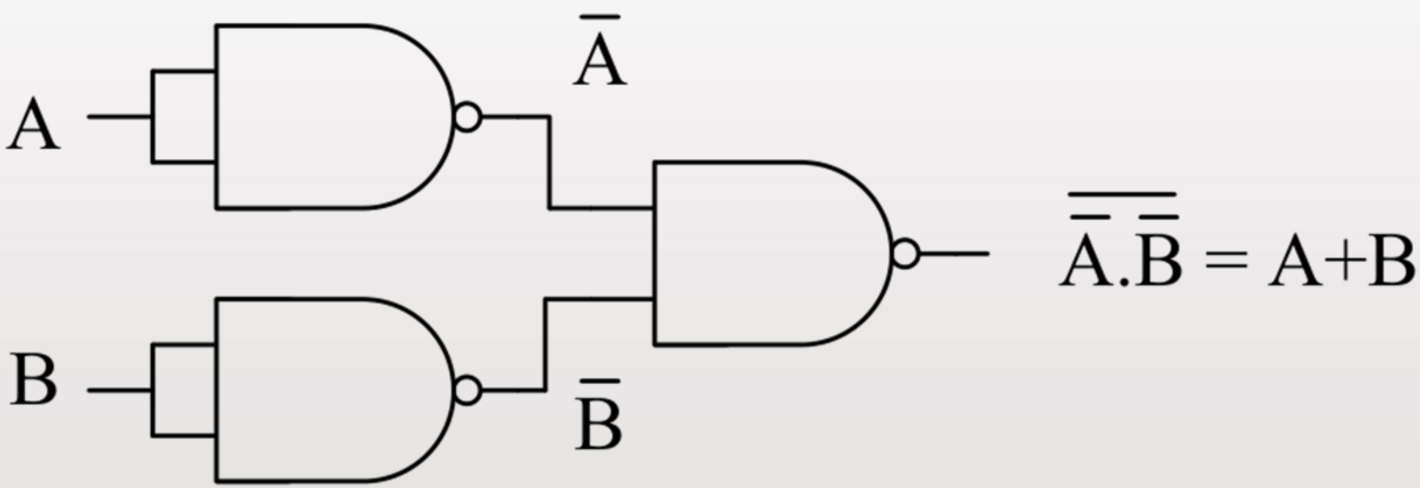
NOT gate implemented using NOR or NAND gates

Implementing the NOT gate



Implementing the OR gate

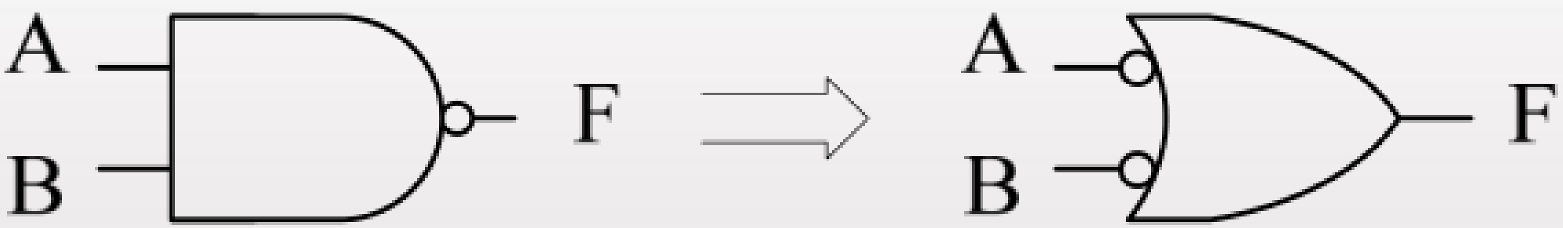




Alternative Logic Symbols

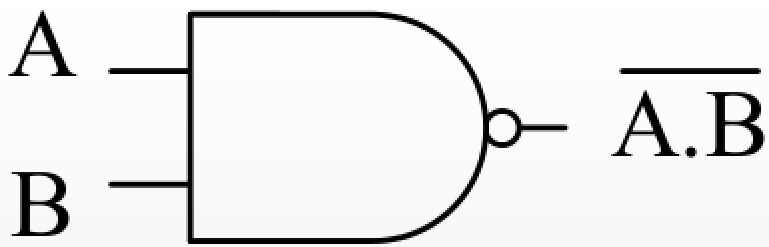
- All basic gates (except XOR and XNOR) have alternative symbols which can be obtained as follows :
  1. Add bubbles to the inputs. Add a bubble at the output if there is no bubble in the standard symbol and remove it if there is one.
  2. Change the gate from AND to OR or vice-versa

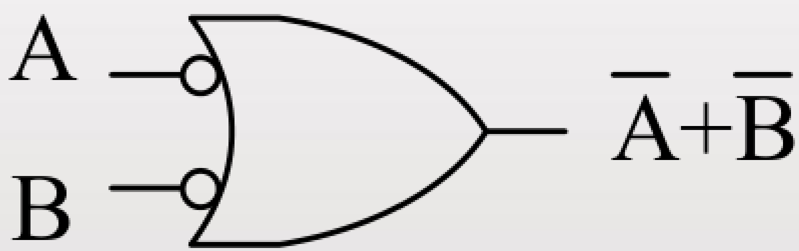
Alternative Symbol for NAND



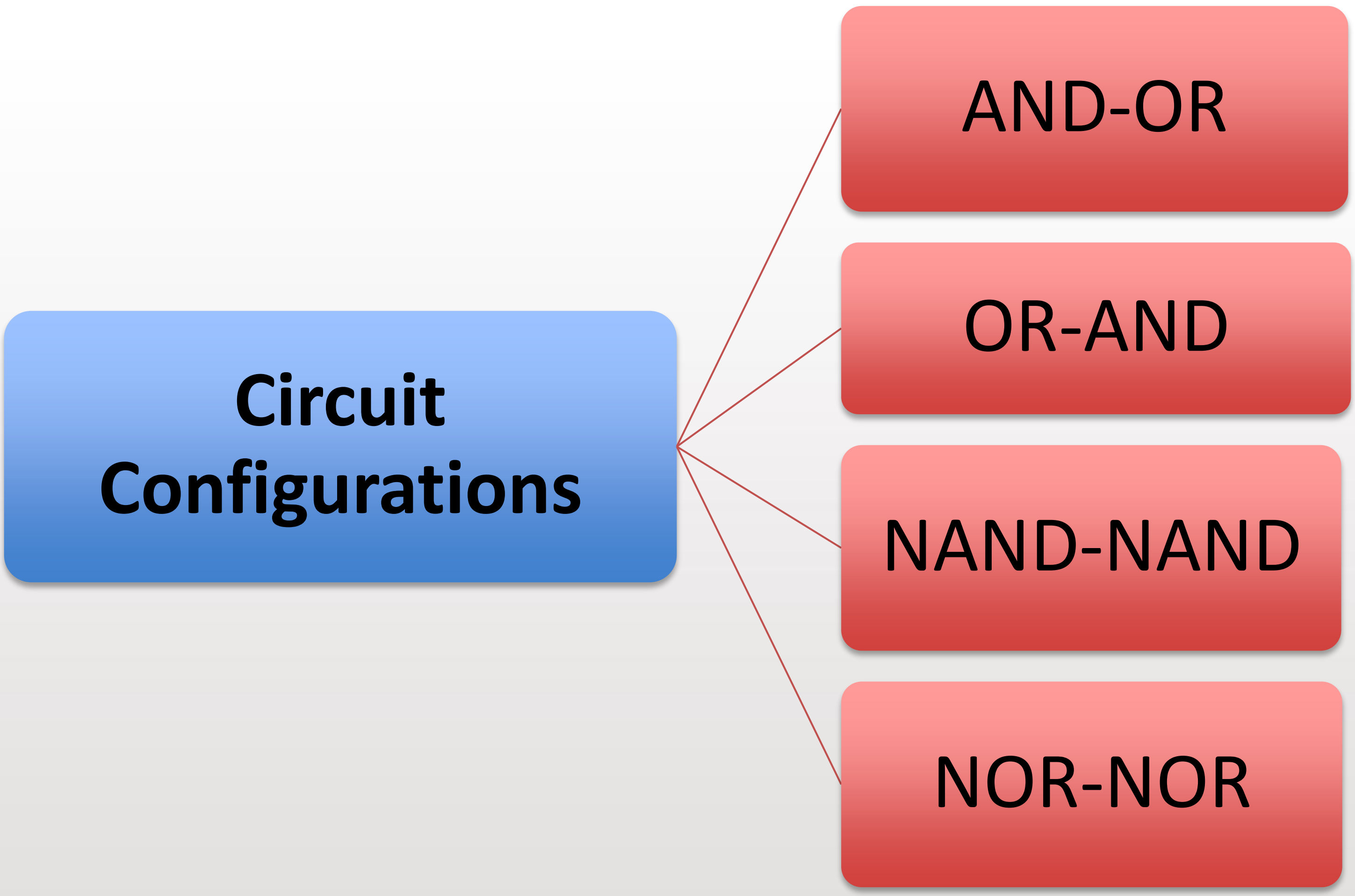
$$\overline{A.B} = \bar{A} + \bar{B}$$

Interpreting the Alternative Symbol

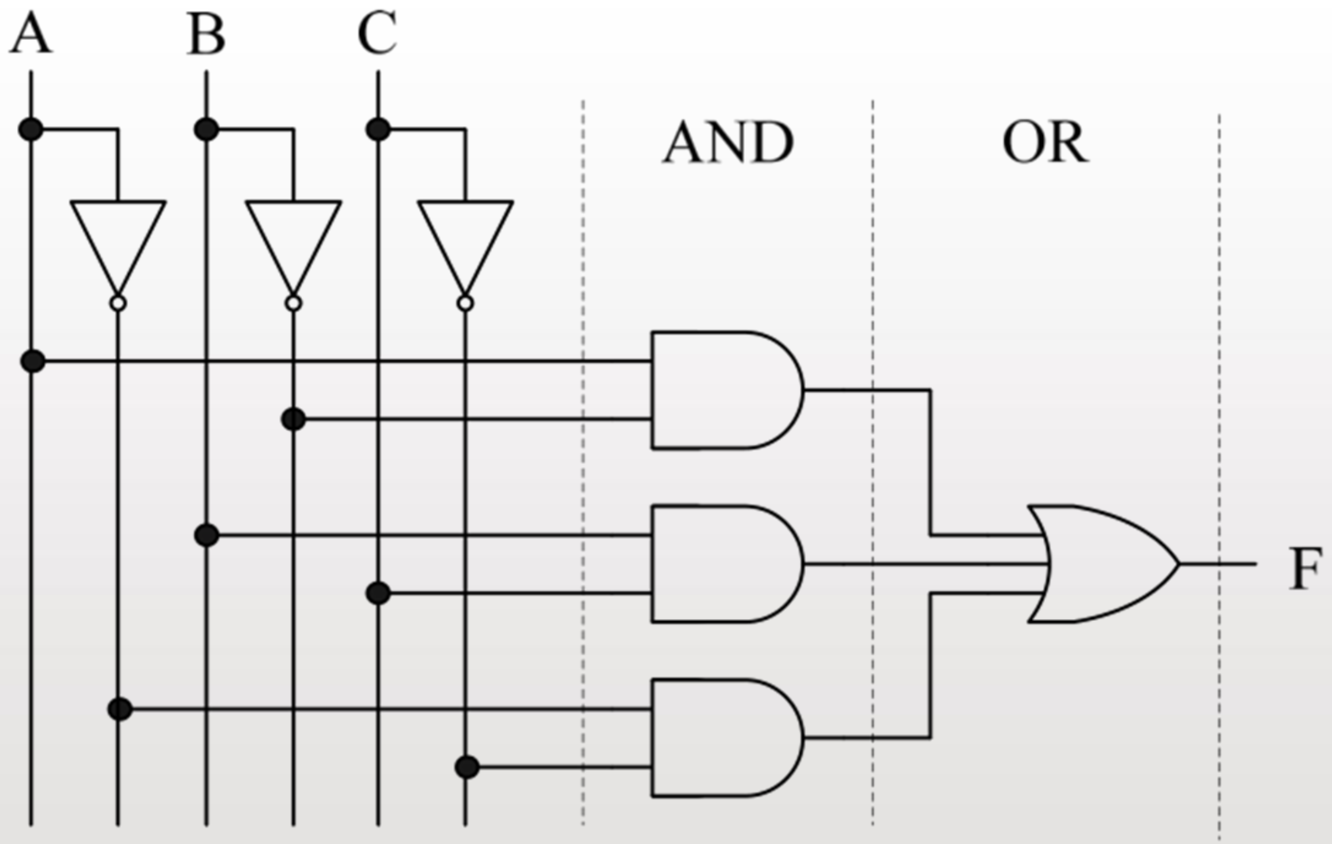




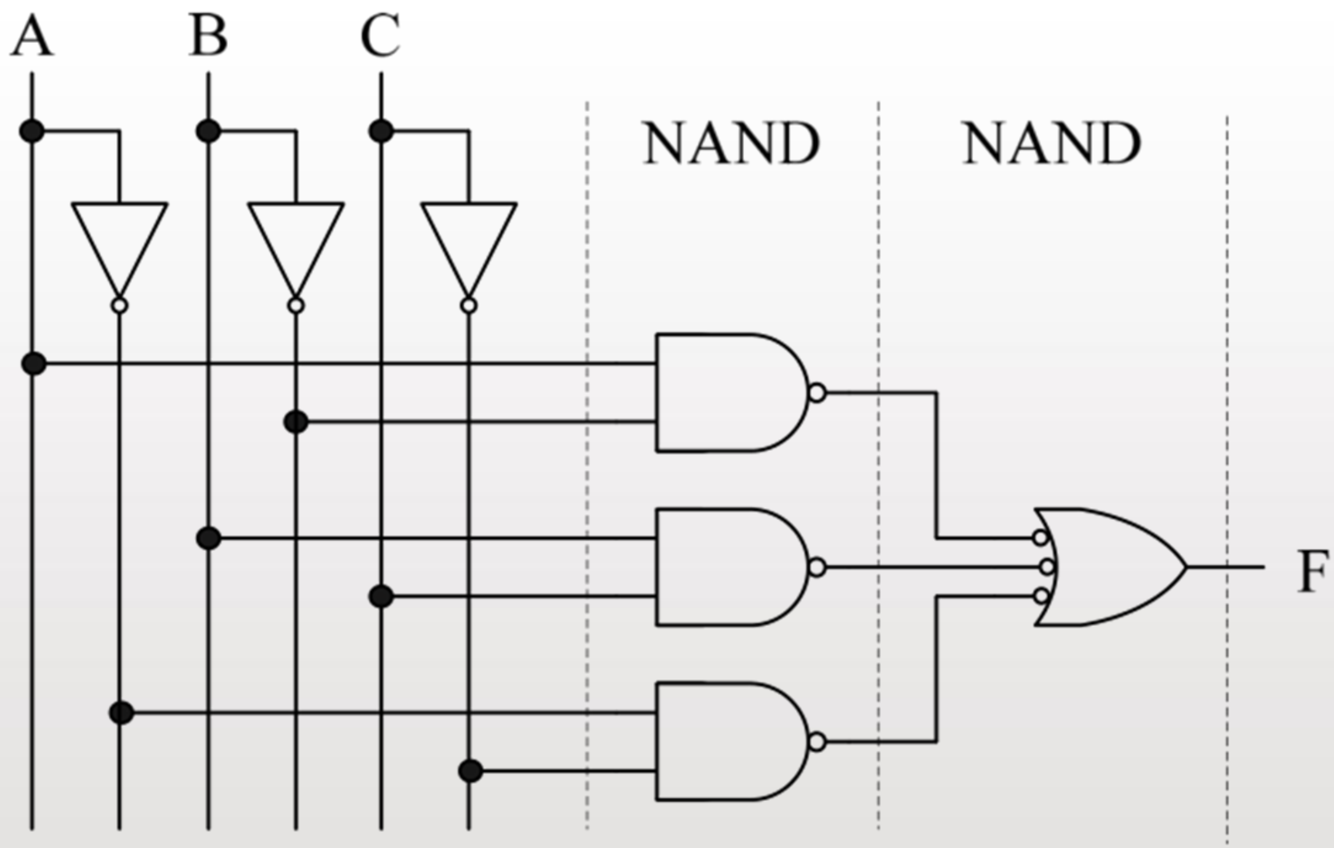
$$\overline{A.B} = \bar{A} + \bar{B}$$


Circuit Configurations
AND-OR
OR-AND
NAND-NAND
NOR-NOR

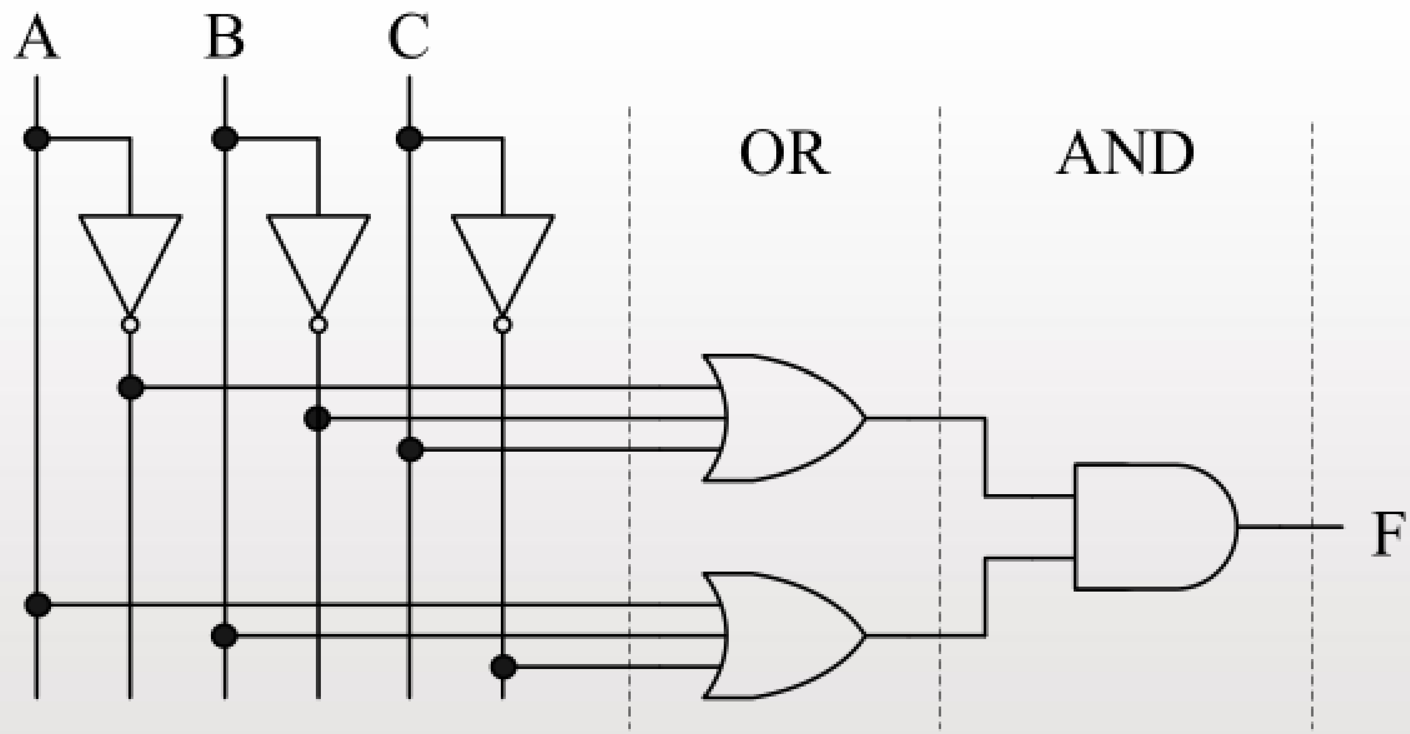
AND-OR Configuration



NAND-NAND Configuration



OR-AND Configuration



NOR-NOR Configuration