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Chapter 3
KINEMATICS OF FLUID MOTION
by
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Learning Outcomes

Upon completing this chapter, the students are expected to be able to:
1. *List and define the fluid flow classification.*
2. *Apply the principles of conservation of mass (the continuity equation).*
3. *Apply the principles of conservation of energy (Bernoulli's equation) to analyze simple pipe flow problems.*
4. *Derive the equation for velocity in pipe for pitot tubes by applying Bernoulli's equation.*
5. *Derive the equation for velocity and flow rate in pipe for Venturi Meter, Orifice meter and Nozzle meter by applying Bernoulli's equation.*

3.1) Flow Classification

(a) Turbulent Flow
- fast flowing fluid
- ordinary flow

(b) Laminar Flow
- slow moving fluid
- very viscous fluid

(c) Steady Flow
- The flow velocity at a point remains the same at all time.

(d) Unsteady Flow
- The flow velocity at a point varies with time.

(e) Uniform Flow
- The vector of flow velocity remains the same along the flow path.

(f) Non-Uniform Flow
- The vector of flow velocity varies along the flow path.

3.2) Terms Used

(b) Stream flow
- The flow path of each of the fluid particle.
- No particle will cross flow other particle during the flow.

(c) Flow rate/Discharge

$$Flowrate = \frac{Volume}{Time} \qquad \boxed{Q = \frac{V}{t}} \qquad \text{unit} = \text{m}^3/\text{s}$$

$Q = Av$

v = flow velocity (m/s)
A = cross-section area (m^2)

Example: The time to fill full a 1500 liter water tank is 38.5 minutes. Then the flow rate in the pipe that fills the tank is: $Q = \dfrac{V}{t} = \dfrac{1500 liter}{38.5\,\mathbf{min}} = 6.49 \times 10^{-4}\, m^3/s$

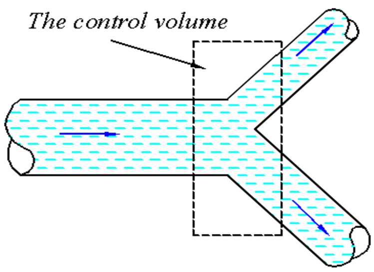
(d) Mass flow rate

$$Mass\ flow\ rate = \frac{Mass}{Time} \qquad \boxed{\dot{m} = \frac{m}{t}} \qquad \text{unit = kg/s}$$

$$\boxed{\dot{m} = \rho Q}$$

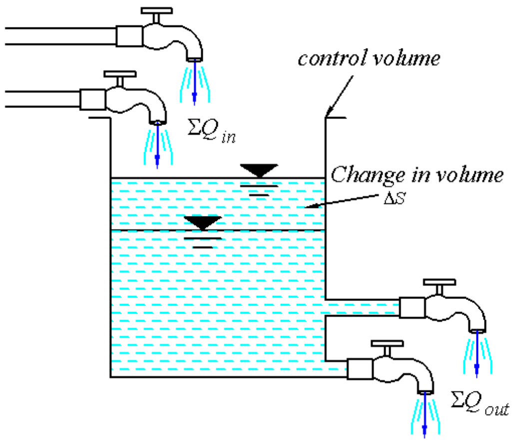
(e) Control Volume

- The volume of part of the system in consideration.



3.3) Conservation of Mass

- No mass can be created or destroyed.
- The flow rate entering a control volume minus the flow rate leaving the control volume must be equal to the rate of change of storage.



$$\boxed{\sum Q_{in} - \sum Q_{out} = \frac{\Delta S}{\Delta t}} \quad \text{Continuity equation}$$

where S = volume of storage (m^3)
t = time (second)

If the control volume is rigid and fully occupied by the fluid, then there is no change in storage – Steady flow.

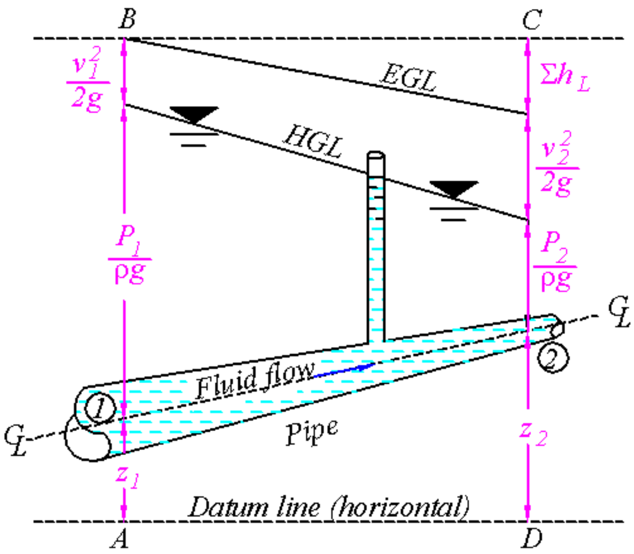
Example – Pipe: $\Delta S = 0$ then $\sum Q_{in} = \sum Q_{out}$

3.4) Conservation of Energy

- In fluid mechanics, the term energy is known as **head**, and is measured in unit length (m).
- There are 3 energy (head) used to analyze flow of fluid in pipes:

1. Pressure Head: $\dfrac{P}{\rho g}$ meter (Vertical height from pipe center to fluid surface in a piezometric tube or HGL)
2. Velocity Head: $\dfrac{v^2}{2g}$ meter (Vertical height from HGL to EGL)
3. Static Head: z meter (Vertical height from a datum to the pipe center)

Conservation of energy: The total head for a flow of fluid in pipe between two points remains the same (as in the following figure).



EGL = Energy Grade Line
HGL = Hydraulic Grade Line
Σh_L = Total head loss due to friction and minor losses (will be discussed in detail in Chapter 5 and 6)

From the above figure, vertical height AB = CD, that is:

$$\boxed{\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2+\sum h_L}$$

which is called the **Bernoulli's Equation.**

Note:
1. EGL = HGL when the fluid in the pipe is not flowing.
2. EGL and HGL is parallel if the $v_1 = v_2$.
3. $v_1 = v_2$ if the pipe diameter $d_1 = d_2$

Simple Data for flow in pipe:

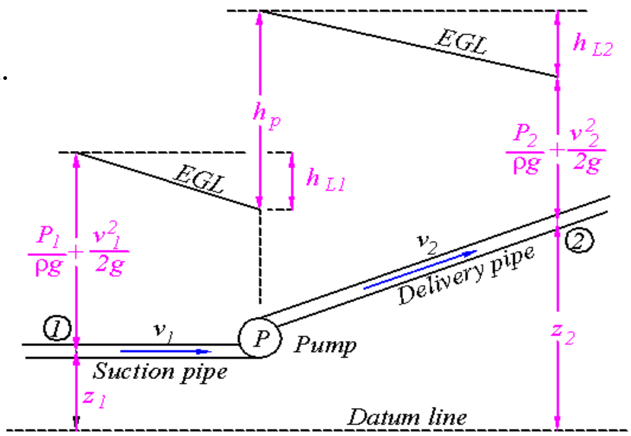
Conditions of Point in pipe	Pressure head: $\dfrac{P}{\rho g}$	Velocity head: $\dfrac{v^2}{2g}$
Fluid surface (i.e. fluid surface in tank, pond, reservoir, etc.)	0	0
Pipe nozzle or pipe ends at atmosphere	0	Has a value (Fluid is flowing)

3.5) Bernoulli's Application – Pumps and Turbines

3.5.1) <u>Pump:</u> Pump adds energy to the fluid.

$$\boxed{\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1+h_p=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2+\sum h_L}$$

Pump head



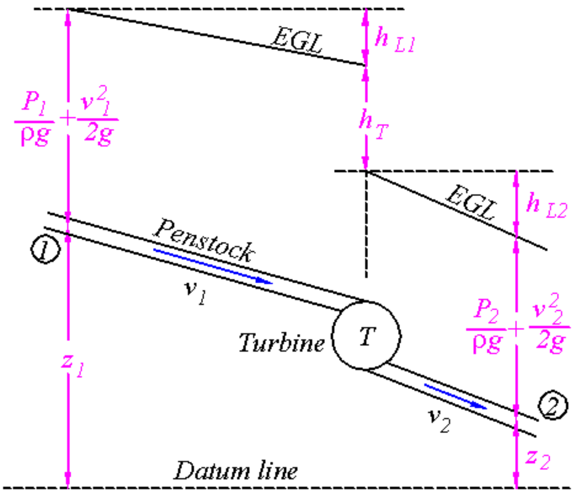
Power required by the pump, $\boxed{P_w=\rho g h_p Q}$ Watt

3.5.2) <u>Turbine:</u> Turbine uses flowing water to induce energy.

Turbine blade.

$$\boxed{\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1-h_T=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2+\sum h_L}$$

Turbine head

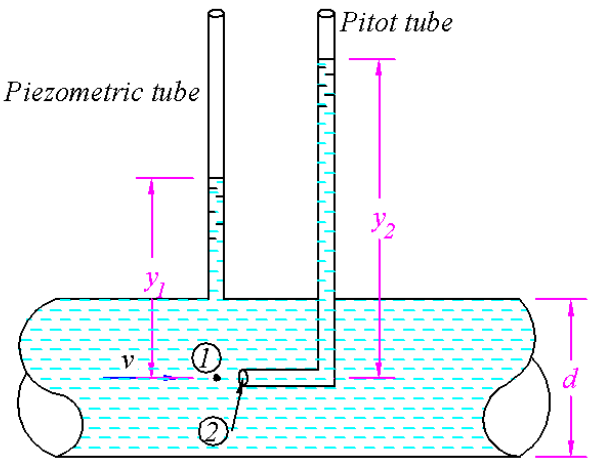


Power produced by a turbine, $\boxed{P_w=\rho g h_T Q}$ Watt

3.6) Bernoulli's Applications – Flow Measuring Devices

Flow parameters to be measured in pipes is the flow velocity, v. By knowing v, the flow rate, Q can be determined knowing the pipe diameter.
There are several devices applicable to measure the flow velocity in pipes.

3.6.1) Single Fluid Pitot Tube



The objective is to determine the flow velocity in the pipe, $\boldsymbol{v_1 = v}$.
Apply Bernoulli's equation from point 1 and 2:

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+\cancel{z_1}=\frac{P_2}{\rho g}+\cancel{\frac{v_2^2}{2g}}+\cancel{z_2}+\cancel{\Sigma h_L} \quad \rightarrow \quad v_2 = 0 \text{ (is called the } \underline{\textbf{stagnation point}}\text{)}$$

$z_1 = z_2$ and $\Sigma h_L = 0$, then:

$$y_1+\frac{v^2}{2g}=y_2$$

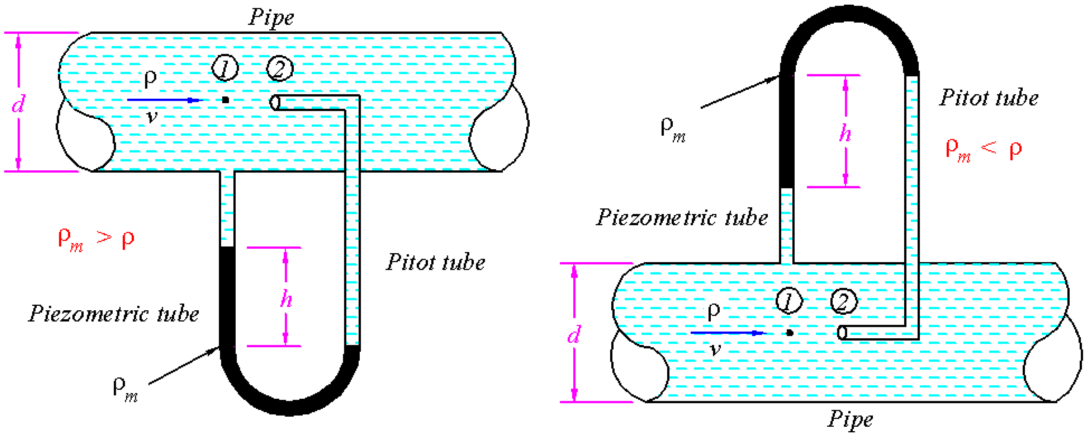
Therefore, $\dfrac{v^2}{2g}=y_2-y_1$

Conclusion:

1. The difference in fluid level in pitot tube and in piezometer tube is the velocity head ($v^2/2g$).
2. The fluid surface in pitot tube is the Energy Grade Line (EGL) while the fluid surface in the piezometer tube is the Hydraulic Grade Line (HGL).
3. The vertical distance from the fluid surface in the pitot tube to the pipe center is:

   $\dfrac{P}{\rho g}+\dfrac{v^2}{2g}$ (pressure head + velocity head).

3.6.2) Multiple Fluid Pitot Tube



(a) U - Tube (b) Inverted U - Tube

The objective is to determine the flow velocity in the pipe, $\boldsymbol{v_1 = v}$.
Apply Bernoulli's equation from point 1 and 2:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + \cancel{z_2} + \cancel{\sum h_L} \qquad \rightarrow \qquad v_2 = 0 \text{ (stagnation point)}$$

$z_1 = z_2$ and $\Sigma h_L = 0$, then:

$$\frac{v^2}{2g} = \frac{P_2 - P_1}{\rho g} \qquad \text{.... (i)}$$

What is $P_2 - P_1$?
Use the manometer concept:

(a) For U-tube:

$$P_2 - P_1 = \rho_m gh - \rho gh$$

$$\frac{P_2 - P_1}{\rho g} = h\left(\frac{\rho_m}{\rho} - 1\right)$$

(b) For Inverted U-tube:

$$P_2 - P_1 = \rho gh - \rho_m gh$$

$$\frac{P_2 - P_1}{\rho g} = h\left(1 - \frac{\rho_m}{\rho}\right)$$

For both U-tube and inverted U-tube:

$$\frac{P_2 - P_1}{\rho g} = h\left|\frac{\rho_m}{\rho} - 1\right| \qquad \text{....(ii)}$$

Put (ii) into (i), therefore Equation (i) becomes:

$$\frac{v^2}{2g} = \frac{P_2 - P_1}{\rho g} = h\left|\frac{\rho_m}{\rho} - 1\right|$$

Therefore,

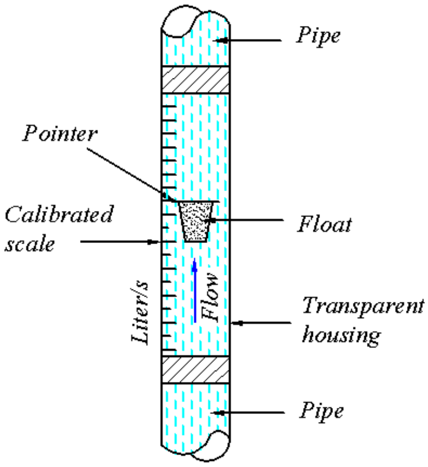
$$\boxed{v = \sqrt{2gh\left|\frac{\rho_m}{\rho} - 1\right|}}$$

3.6.3) Venturi Meter
3.6.4) Orifice Meter
3.6.5) Nozzle Meter
3.6.6) Elbow Meter

To determine the equations for flow velocity and flow rates in Venturi meter, orifice meter and nozzle meter, solve the corresponding problems in Exercise 3.

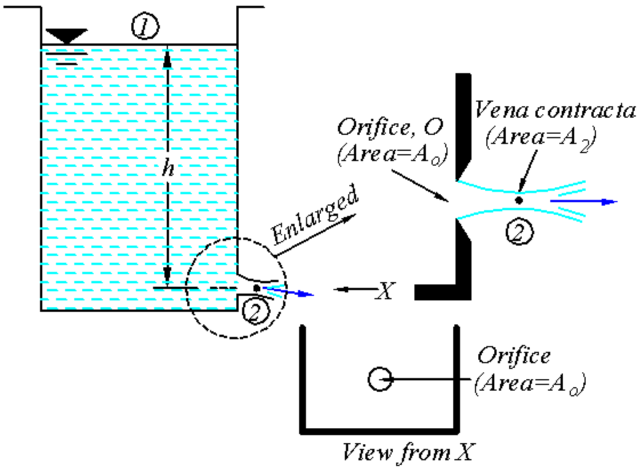
3.6.7) Rotameter

Rotameter directly reads the flow rate in a pipe.



3.7) Bernoulli's Applications – Others Flow Rate Measurements

3.7.1) Orifice in Tanks



Apply Bernoulli's equation from point 1 and 2. Point 2 is at the vena contracta.

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{v_1^2}{2g}} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \cancel{\sum h_L}$$

$$\frac{v_2^2}{2g} = z_1 - z_2 = h$$

$$v_2 = \sqrt{2gh}$$ this is theoretical velocity at 2.

The actual velocity at 2 is, $v_2 = c_v\sqrt{2gh}$ where c_v is the velocity coefficient

But $A_2 = c_c A_o$ where c_c is the coefficient of contraction and A_o is the area of the orifice.

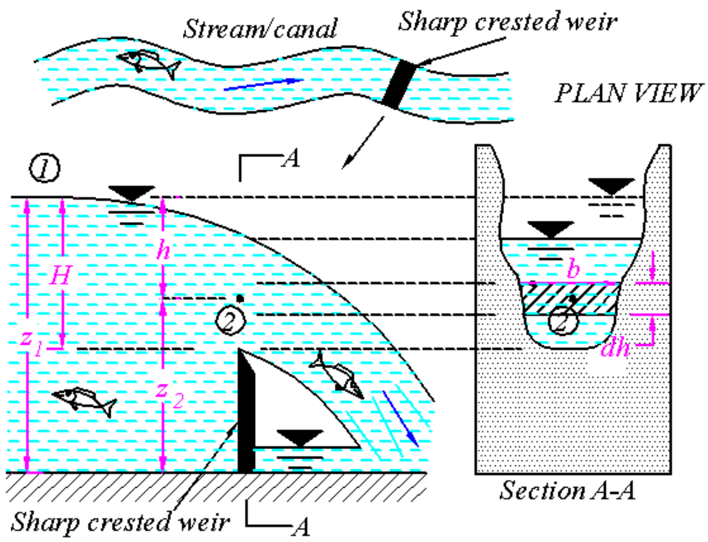
The actual flow rate is

$$Q_a = A_2 v_2$$

$$\boxed{Q_a = c_c A_o c_v \sqrt{2gh}}$$

3.7.2) Sharp Crested Weir (Open Channel)

The objective is to determine the flow rate Q in an open channel (stream, gully, etc)



Apply Bernoulli's equation from point 1 and 2:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \sum h_L$$

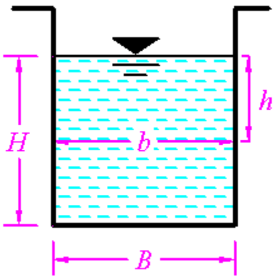
$$\frac{v_2^2}{2g} = z_1 - z_2 = h \longrightarrow v_2 = \sqrt{2gh} \longrightarrow Q = A_2 v_2$$

$$Q = b(dh)\sqrt{2gh} \qquad \text{since } A_2 = b(dh) \longrightarrow \qquad Q = \left(bh^{1/2}\sqrt{2g}\right)dh$$

$$\boxed{Q_{theory} = \sqrt{2g}\int_0^H \left(bh^{1/2}\right)dh} \qquad \text{(i)}$$

This is for general weir shape. The actual shape may be rectangular or triangular as follows:-

3.7.2.1) Rectangular Weir



From (i)

$$Q_{theory} = \sqrt{2g}\int_0^H \left(bh^{1/2}\right)dh$$

But $b = B$ that is constant, then

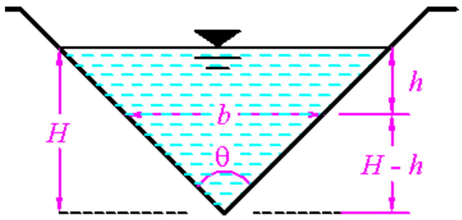
$$Q_{theory} = B\sqrt{2g}\int_0^H \left(h^{1/2}\right)dh$$

$$\boxed{Q_{theory} = \frac{2}{3}B\sqrt{2g}H^{3/2}}$$

3.7.2.2) Triangular Weir

$$\mathbf{tan}\left(\frac{\theta}{2}\right) = \frac{b}{2(H-h)}$$

$$\therefore b = 2(H-h)\mathbf{tan}\left(\frac{\theta}{2}\right)$$



Equation (i) becomes: $\qquad Q_{theory} = \sqrt{2g}\int_0^H \left(bh^{1/2}\right)dh$

$$Q_{thoery} = \sqrt{2g}\int_0^H \left(2(H-h)\mathbf{tan}\left(\frac{\theta}{2}\right)\left(h^{1/2}\right)\right)dh$$

$$\boxed{Q_{theory} = \frac{8}{15}\mathbf{tan}\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}}$$

3.8) Actual vs Theoretical Flow Rate

The actual flow rate is $\boxed{Q_{actual} = c_d Q_{theory}}$

where $\qquad c_d$ = flow rate coefficient (this value should be calibrated in laboratory)