SAB2223 Mechanics of Materials and Structures

TOPIC 5
STATICALLY DETERMINATE PLANE TRUSSES

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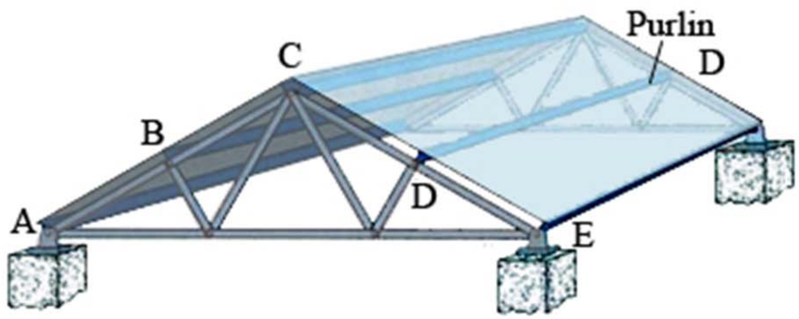
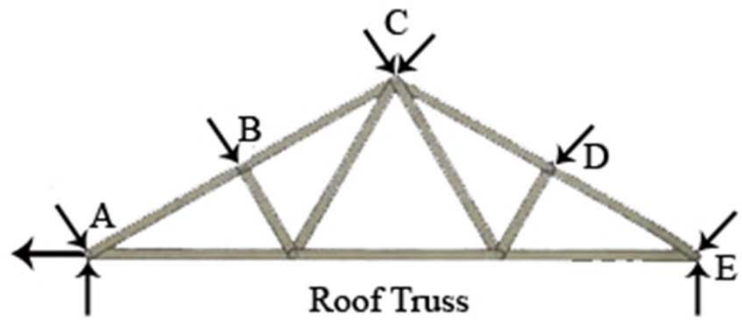
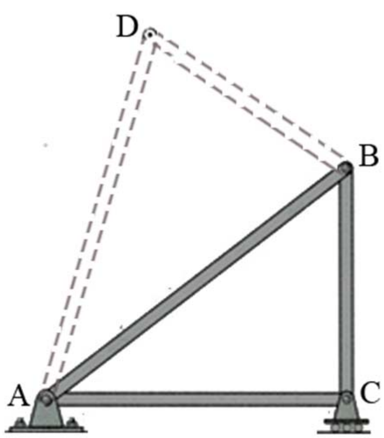
Defining a Simple Truss





- Trusses are structures composed of triangulated members joined together at their end points.
- Pitched trusses are usually used for roofs.



For a statically determinate truss, $M = 2J - 3$.

$M > 2J - 3$: Statically indeterminate

$M = 2J - 3$: Statically determinate

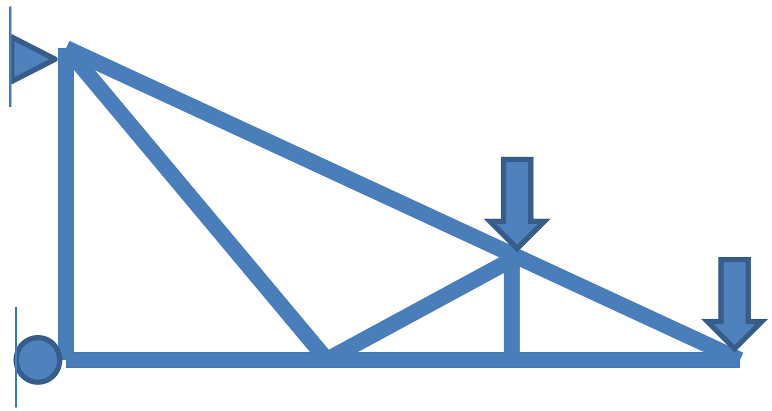
$M < 2J - 3$: Statically not stable

$R > 3$: Statically indeterminate

$R = 3$: Statically determinate

$R < 3$: Statically not stable

Example 1



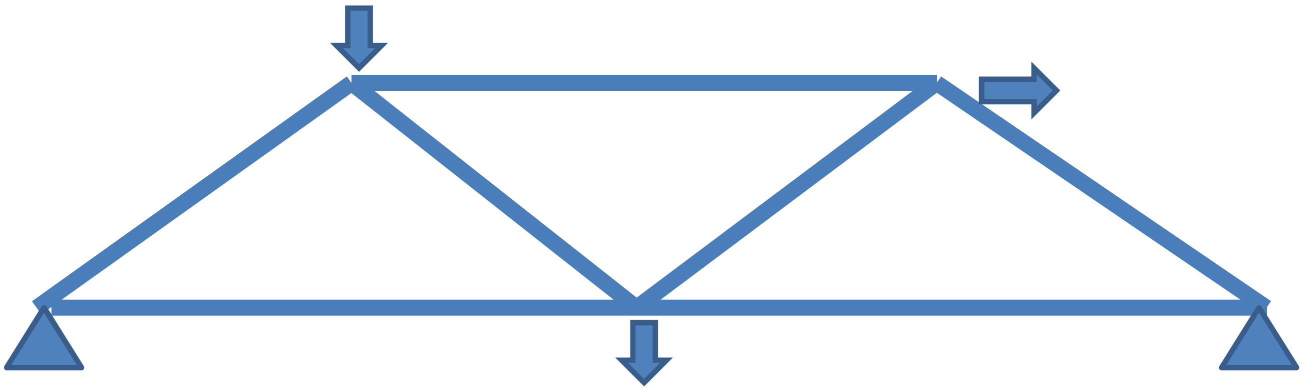
R = 3

R – 3 = 0 ∴ The external forces of the truss is statically determinate

M = 9 2J – 3 = 2(6) – 3 = 9

M = 2J – 3 ∴ The internal forces of the truss is statically determinate

Example 2



R = 4

R – 3 = 1 $\therefore$ The external forces of the truss is statically indeterminate to 1 degree

M = 7 2J – 3 = 2(5) – 3 = 7

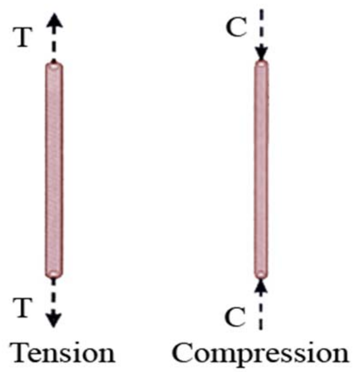
M = 2J – 3 $\therefore$ The internal forces of the truss is statically determinate

Analysis and Design Assumption

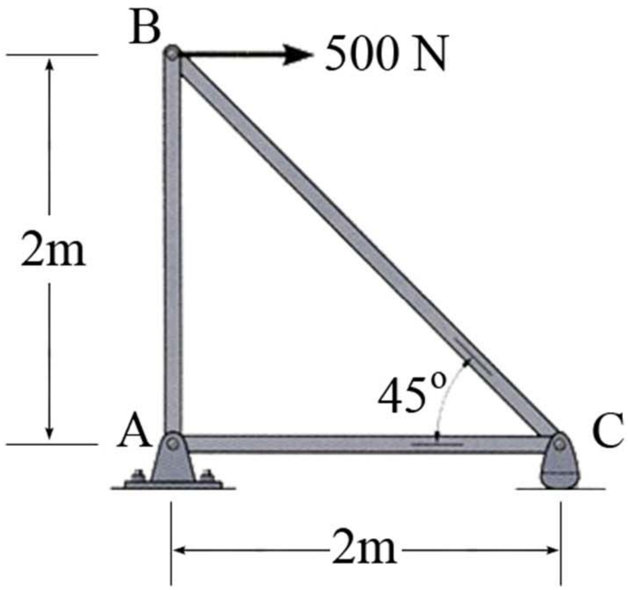
To analyze the forces in truss members, several assumptions are made:

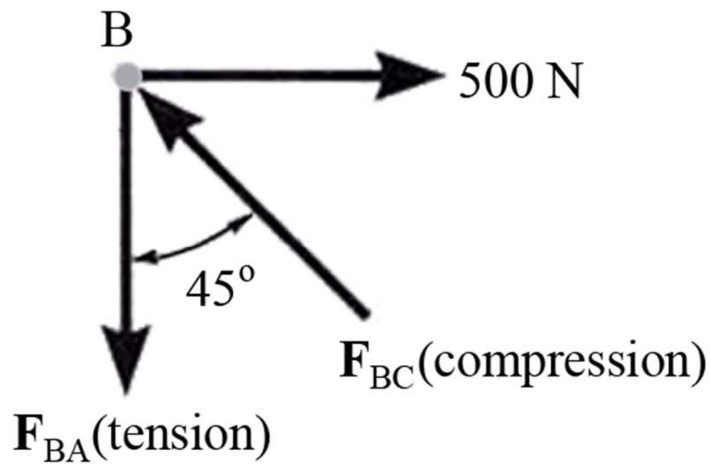
1. All loads are applied at the joints.
2. The members are joined together using simple connection.

With these two assumptions, all members are subject to either tension or compression force only.



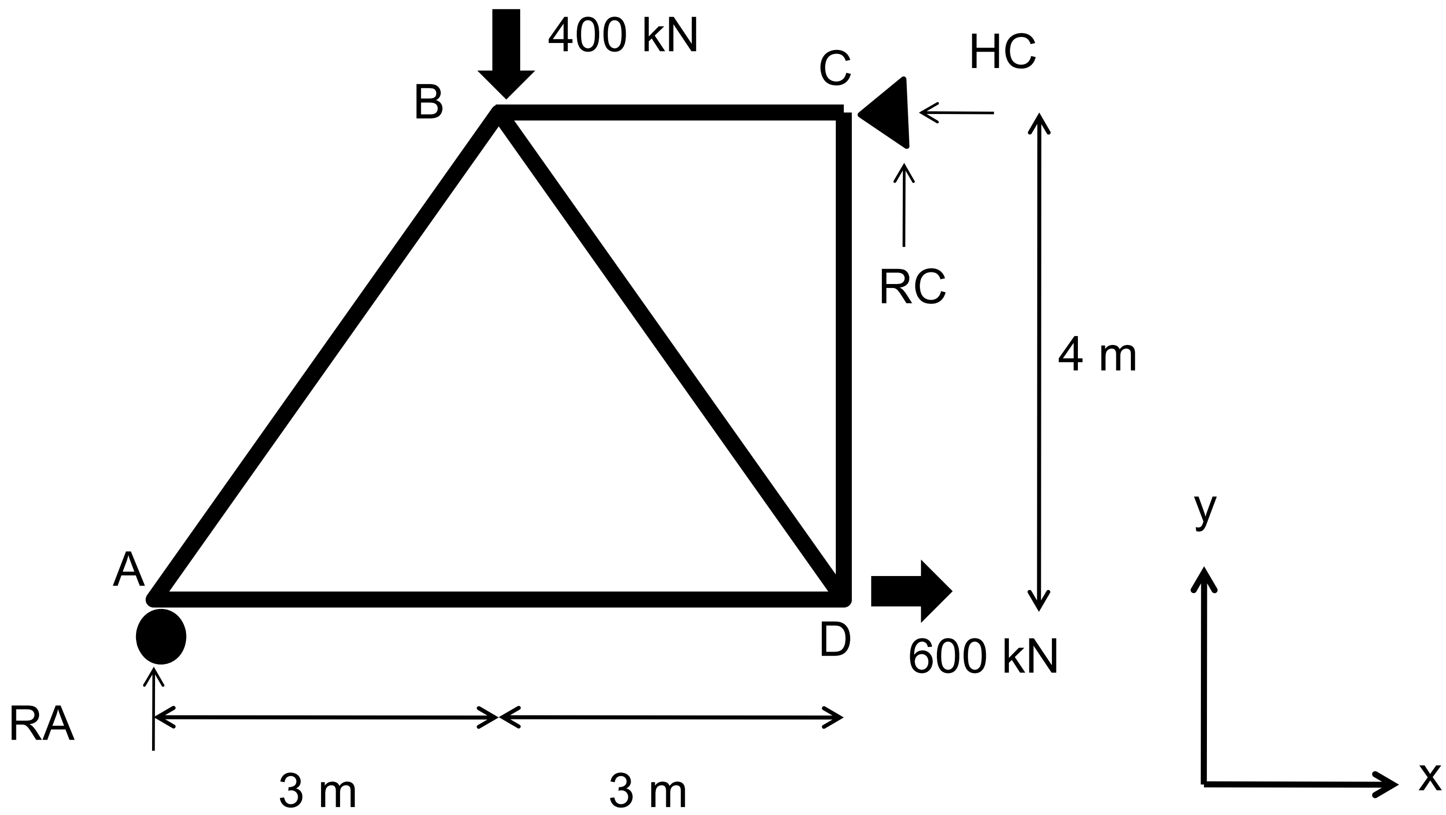
The Method of Joints





- In this method, the equilibrium of a joint is considered.
- All forces acting at the joint are shown in a free body diagram. This includes all external forces (including support reactions) and internal forces.
- Equations of equilibrium ($\sum F_X = 0$ and $\sum F_Y = 0$) are used to solve for the unknown forces acting at the joints.

Example 3



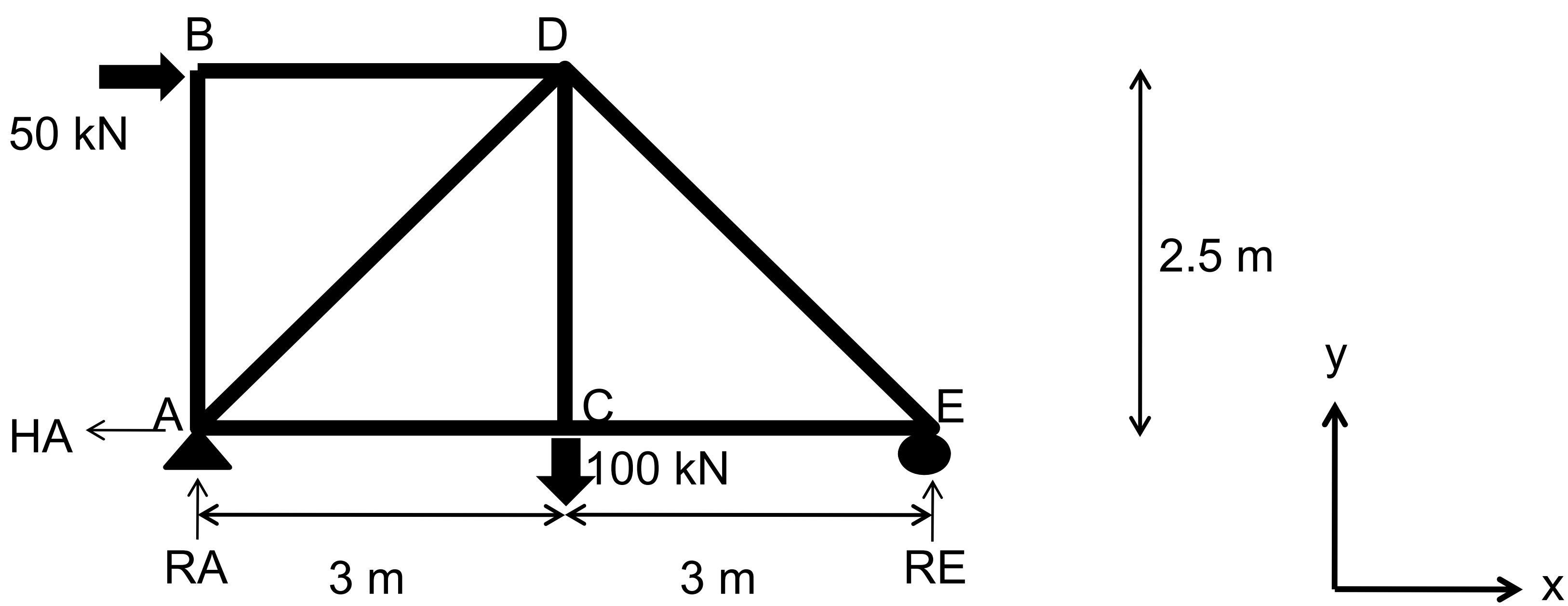
R = 3 M = 2J – 3 M = 0; Fx = 0; Fy = 0

RA = 600; RC = -200; HC = -600

AB = -750 (C); AD = 450 (T); BC = - 600 (C)

BD = 250 (T); CD = -200 (C)

Example 4



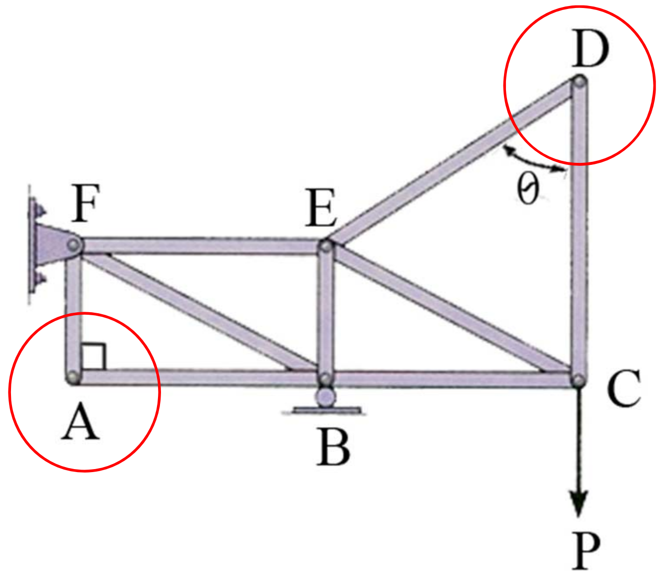
R = 3 M = 2J – 3 M = 0; Fx = 0; Fy = 0

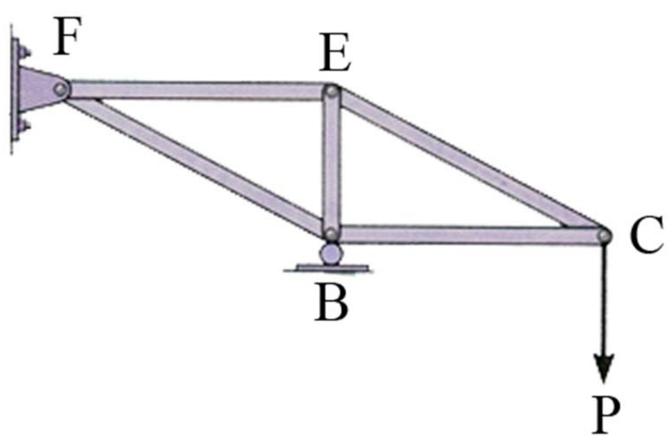
RA =29.17; RE = 70.83; HA = -50

AB = 0 ; AC = 85 (T); AD = -45.56 (C)

BD = -50 (C); CD = 100 (T); CE = 85 (T); DE = -110.64 (C)

Zero-Force Members

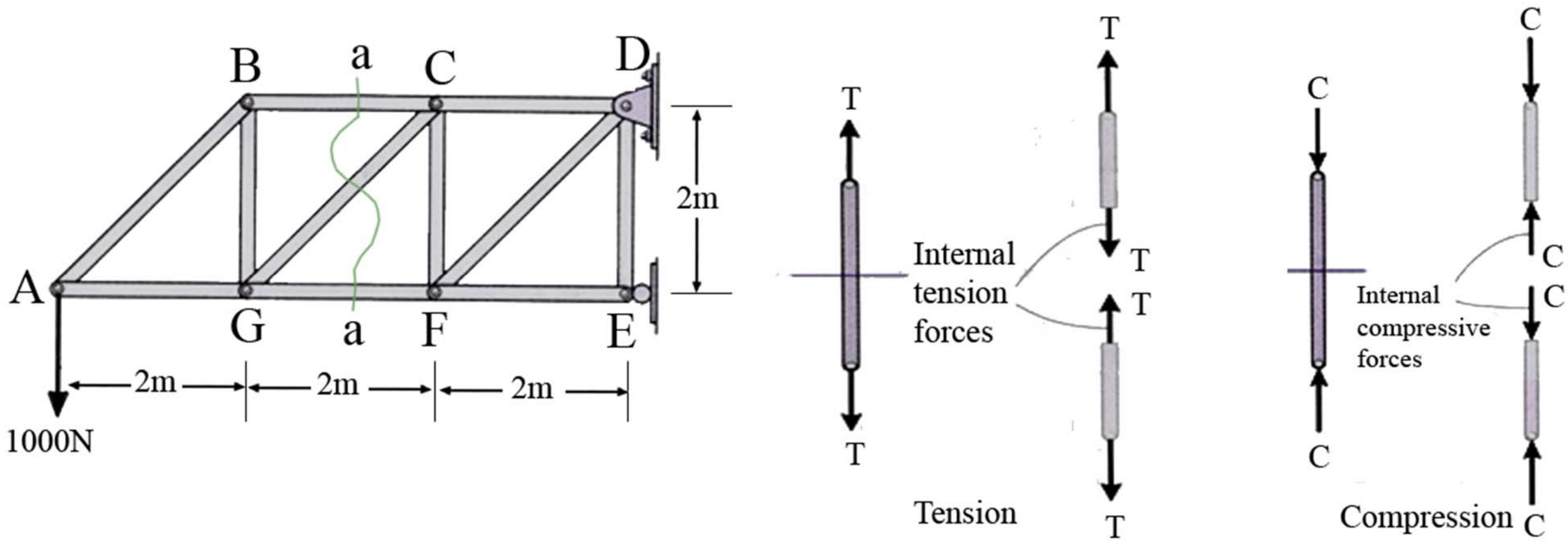




If a joint has only two non-colinear members and there is no external load or support reaction at that joint, then those two members are zero-force members.

In this example members DE, CD, AF, and AB are zero force members.

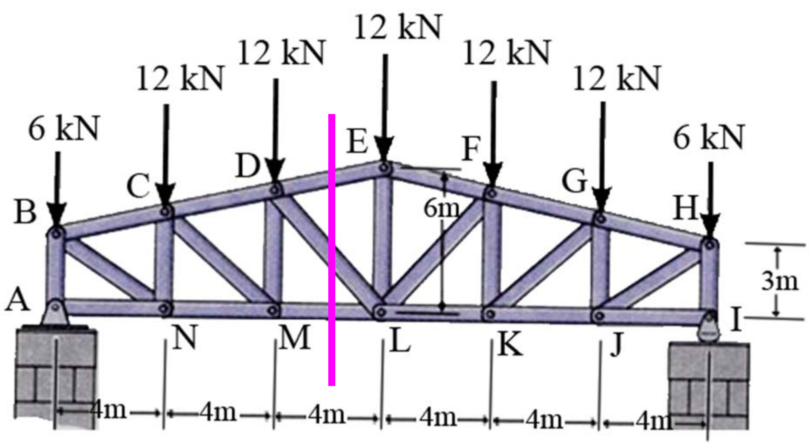
The Method of Sections



In this method, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss.

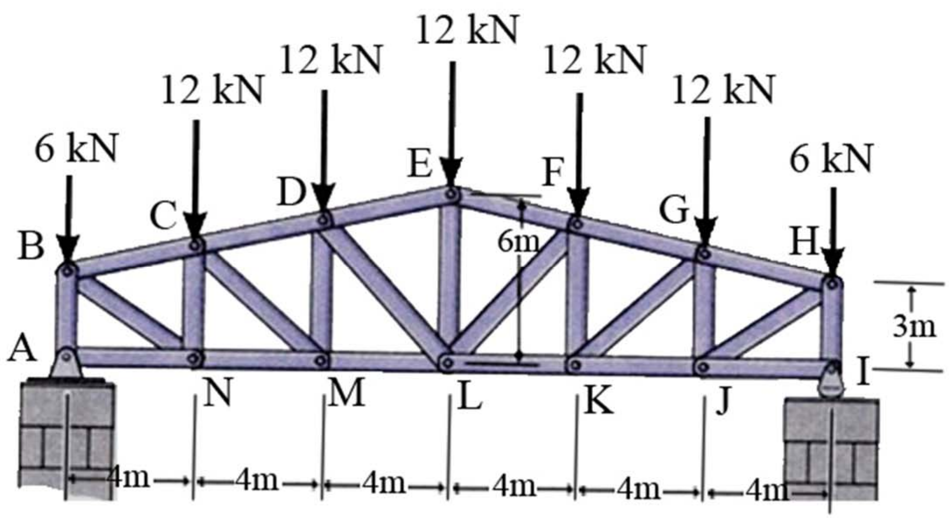
Since truss members are subjected to only tensile or compressive forces along their length, the internal forces at the cut member will also be either tensile or compressive with the same magnitude. This result is based on the equilibrium principle and Newton's third law.

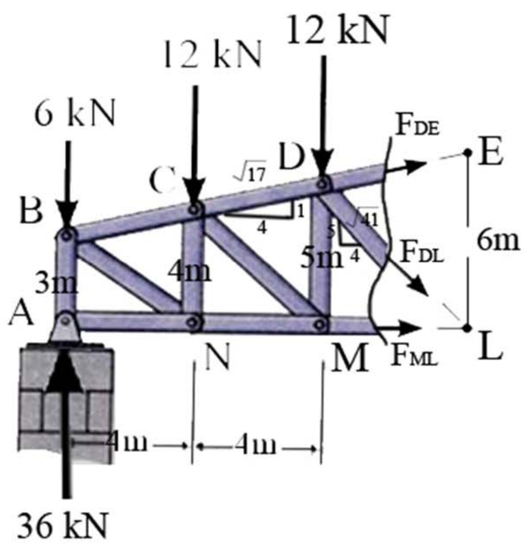
Example 5



Given the loads as shown on the roof truss. Find The force in members DE, DL, and ML.

Example 5 (cont.)





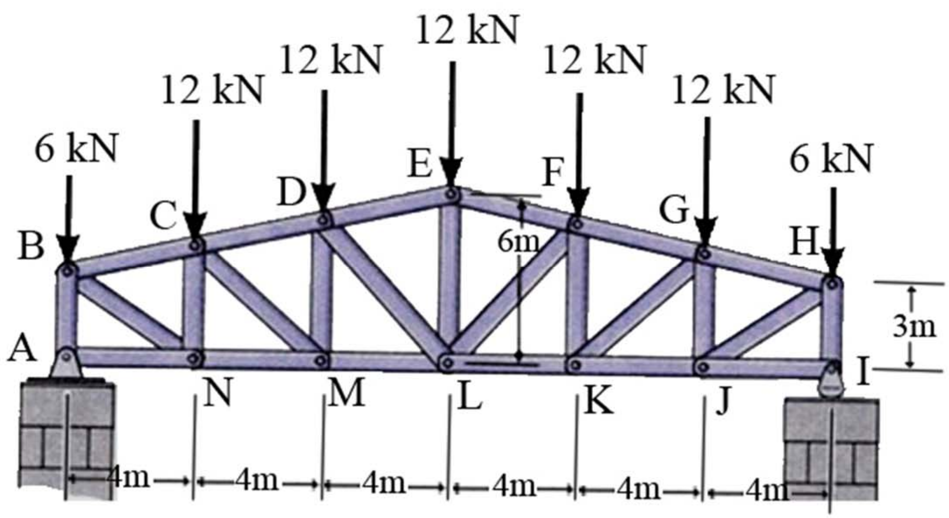
Analyzing the entire truss, we get $\Sigma F_X = A_X = 0$. By symmetry, the vertical support reactions are

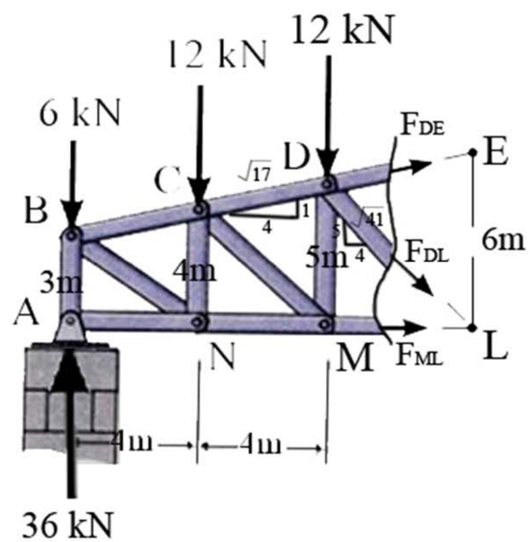
$A_Y = I_Y = 36$ kN

$$\curvearrowright + M_D = -36(8) + 6(8) + 12(4) + F_{ML}(5) = 0$$

$$F_{ML} = 38.4 \text{ kN (T)}$$

Example 5 (Cont.)





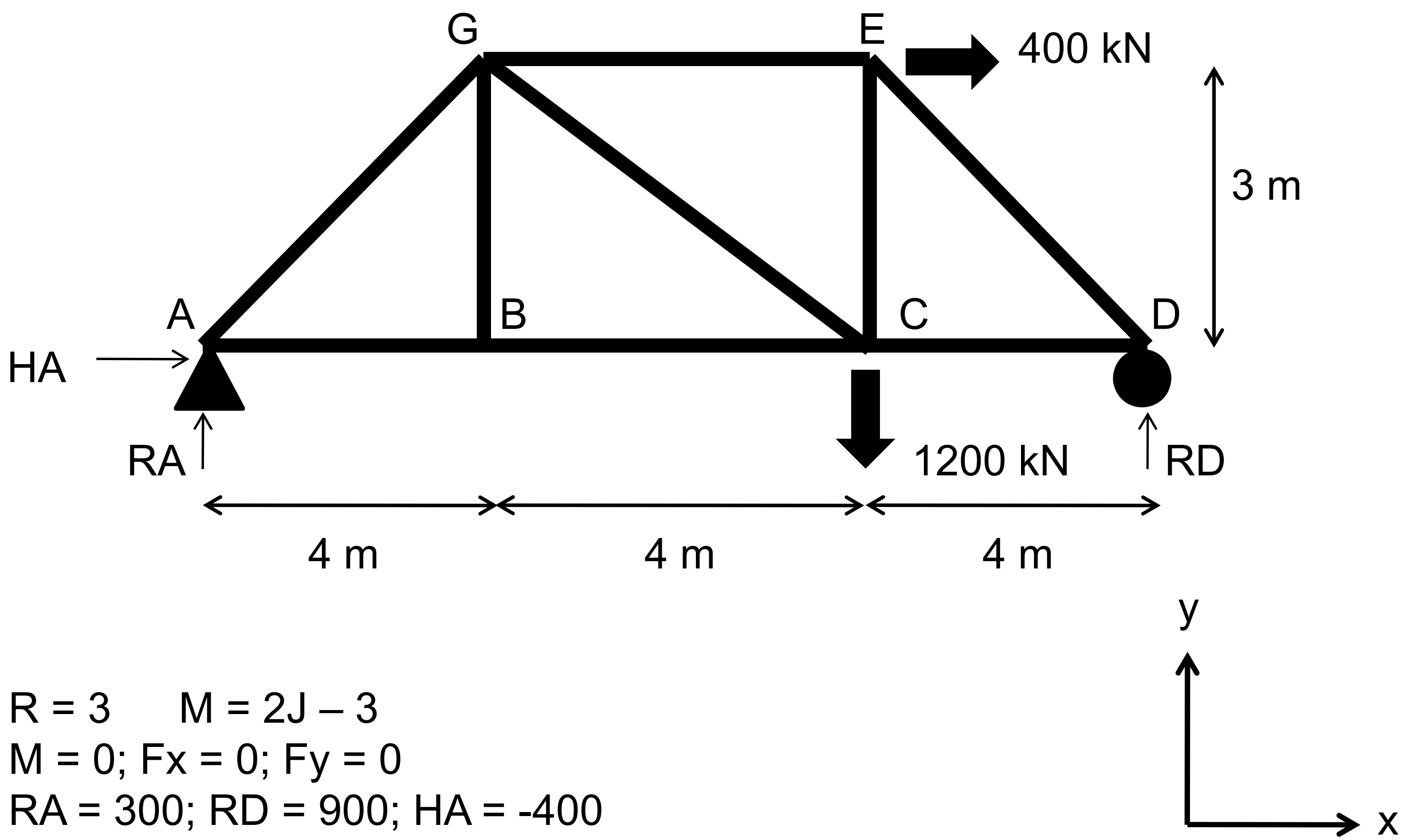
$\circlearrowright + \Sigma M_L = -36\,(12) + 6\,(12) + 12\,(8) + 12\,(4) - F_{DE}\,(4/\sqrt{17})(6) = 0$

$F_{DE} = -37.11$ kN or 37.1 kN (C)

$\rightarrow + \Sigma F_X = 38.4 + (4/\sqrt{17})\,(-37.11) + (4/\sqrt{41})\,F_{DL} = 0$

$F_{DL} = -3.84$ kN or 3.84 kN (C)

Example 6



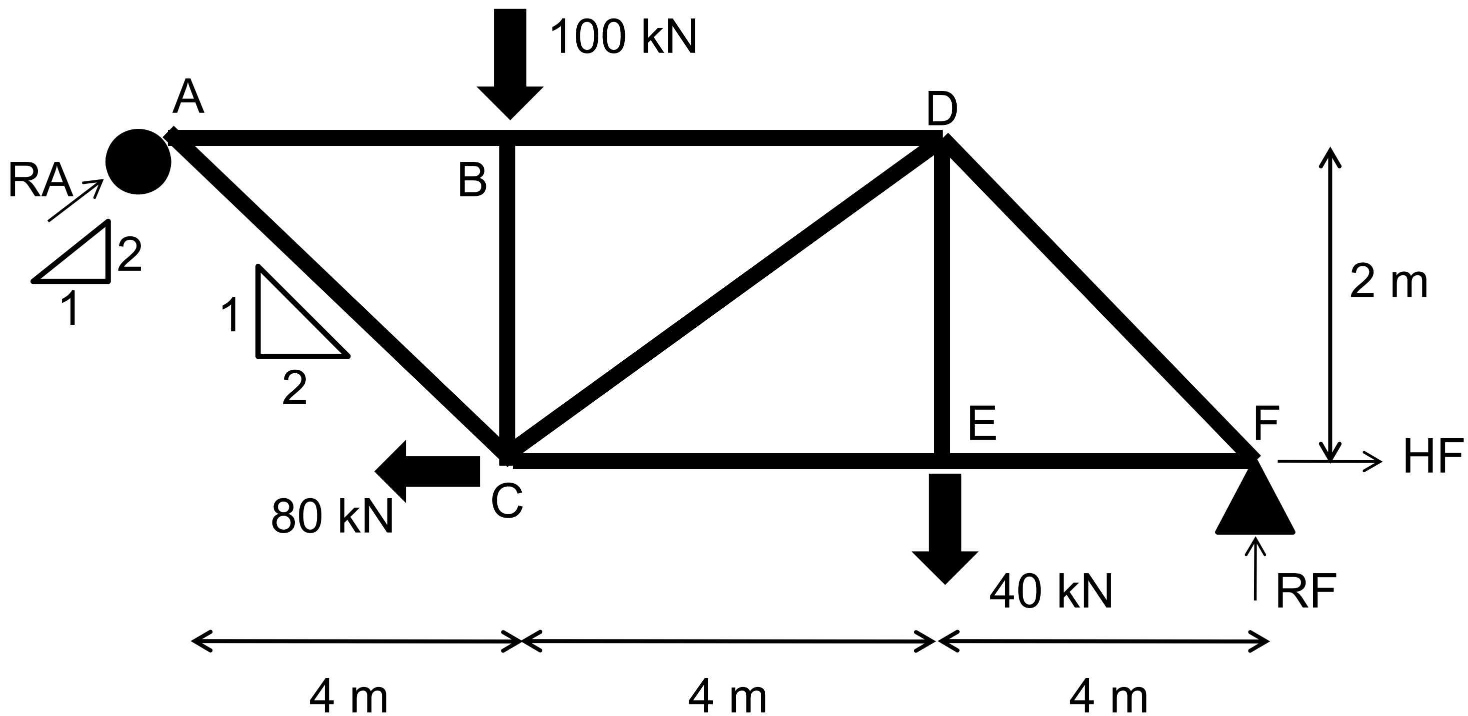
R = 3   M = 2J – 3

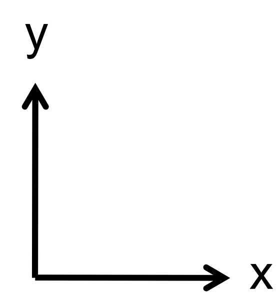
M = 0; Fx = 0; Fy = 0

RA = 300; RD = 900; HA = -400

GE = -800 (C); GC = 800 (T); BC = 500 (T)

Example 7





R = 3   M = 2J – 3

M = 0; Fx = 0; Fy = 0

RA = 82.6; RF = 66.1; HF = 43.1

EC = 175.3 (T); DC = 58.36 (T); BD = 184.4 (C)

Virtual Work Method

- When a structure is loaded, its stressed elements deform. As these deformations occur, the structure changes shape and points on the structure displace.
- Work is the product of a force times a displacement in the direction of the force
- **External Work** is when a force *F* undergoes a displacement *dx* in the same direction as the force.
- **Internal Work** is when internal displacements *δ* occur at each point of internal load u.

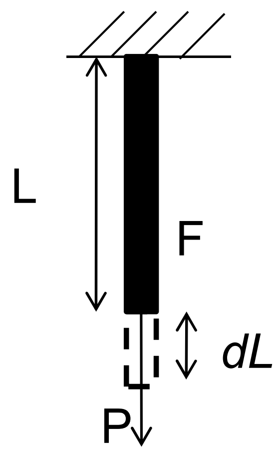
$$\sum P\Delta = \sum u\,\delta$$

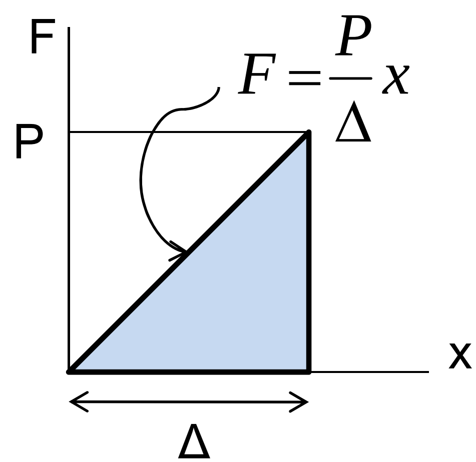
Work of External Load — Work of Internal Load

Virtual Work Method

- When a bar is loaded axially, it will deform and store strain energy *u*.
- A bar (as shown in the figure) subjected to the externally applied load P induces an axial force *F* of equal magnitude (F = P). If the bar behaves **elastically** (Hooke's Law), the magnitude of the strain energy *u* stored in a bar by a force that increases linearly from zero to a final value *F* as the bar undergoes a change in length *dL.*

From **Hooke's Law**, $dL = \dfrac{PL}{AE}$





Displacement of Trusses

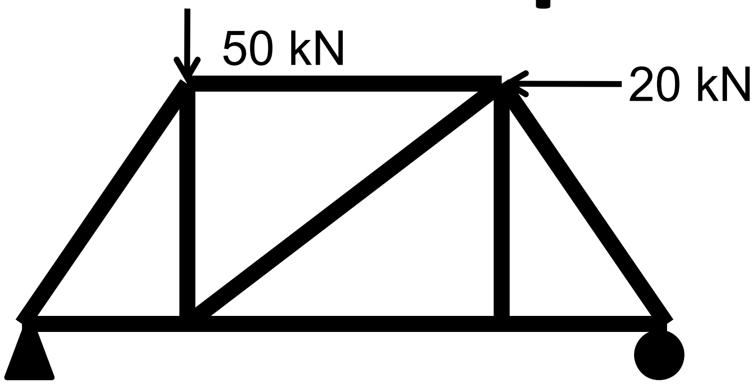
- We can use the **method of virtual work** to determine the displacement of a truss joint when the truss is subjected to an external loading, temperature change, or fabrication errors.
- When a **unit force** acting on a truss joint, and resulted a displacement of Δ, the external work is equals to 1 × Δ.
- Due to the unit force, each truss member will carry an internal forces of *u*, which cause the deformation of the member in length *dL*. Therefore, the **displacement** of a truss joint can be calculated by using the equation of:

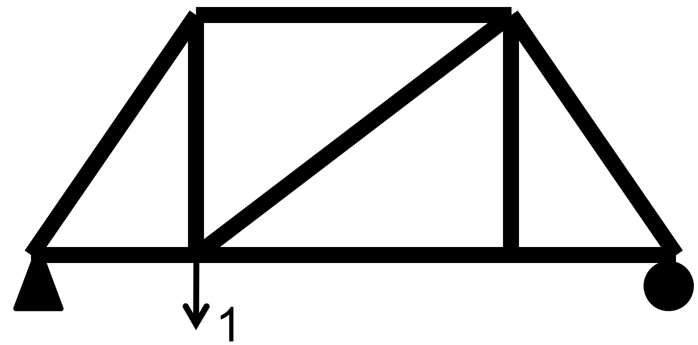
$$1 \cdot \Delta = \sum u \cdot dL$$

Virtual loadings (1 and u)

Real displacements (Δ and dL)

Steps for Analysis





1. Place the unit load on the truss at the joint where the desired displacement is to be determined. The load should be in the same direction as the specified displacement, e.g., horizontal or vertical.
2. With the unit load so placed, and all the real loads removed from the truss, use the method of joints or the method of sections and calculate the internal force in each truss member. Assume that tensile forces are positive and compressive forces are negative.
3. Use the method of joints or the method of sections to determine the internal forces in each member. These forces are caused only by the real loads acting on the truss. Again, assume tensile forces are positive and compressive forces are negative.

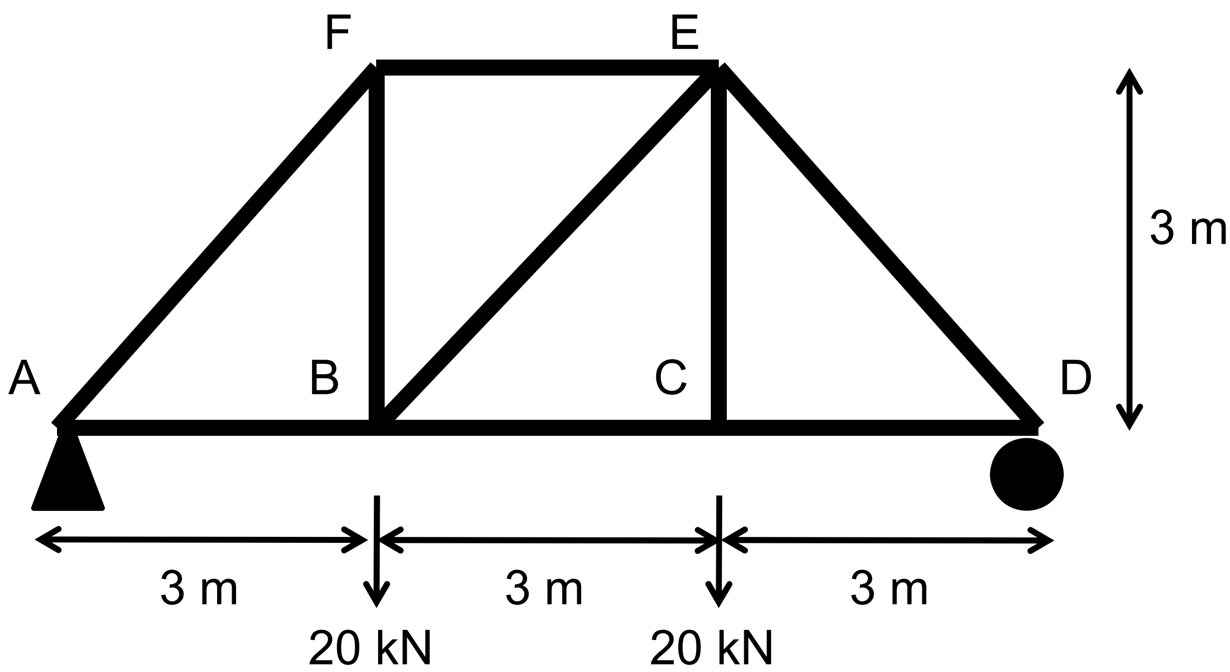
Steps for Analysis (cont.)

Member	Virtual force, u	Real Force, N (kN)	L (m)	uNL (kN.m)
AB	0.601	50	1.8	2
CB	-0.507	-30	1.8	-3
DB	0.515	20	1.8	5
			Total	4

- Apply the equation of virtual work, to determine the desired displacement. It is important to retain the algebraic sign for ach of the corresponding internal forces when substituting these terms into the equation.
- If the resultant sum of displacement is positive, the direction is same as the unit load or vice-versa.
- When applying any formula, attention should be paid to the units of each numerical quantity. In particular, the virtual unit load can be assigned any arbitrary unit (N, kN, etc.).

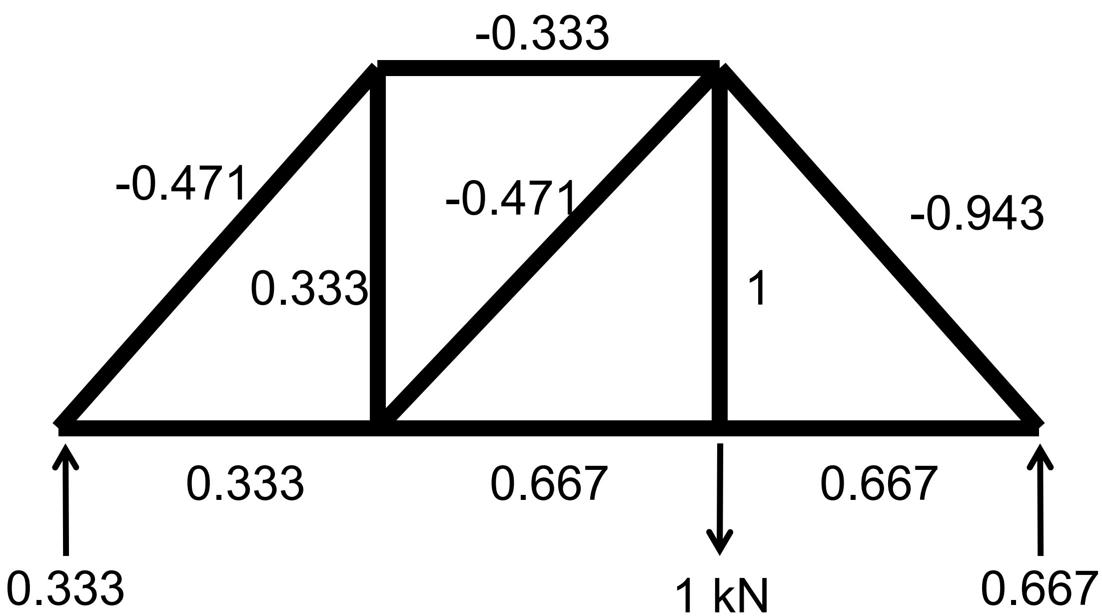
Example 8

Determine the vertical displacement of joint C of the steel truss shown in Figure. The cross-sectional area of each member is A = 300 mm^2 and E = 200 GPa.



Example 8 (cont.)

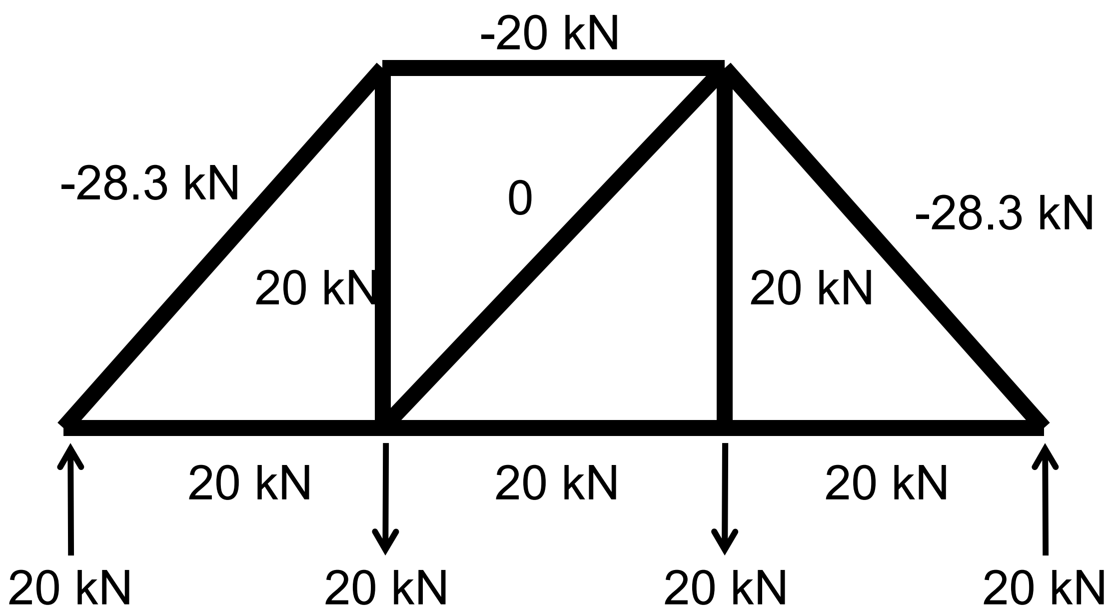
Solution:



Virtual force, u

Example 8 (cont.)

Solution:



Real force, N

Example 8 (cont.)

Solution:

Member	Virtual force, u	Real force, N (kN)	L (m)	uNL (kN.m)
AB	0.333	20	3	20
BC	0.667	20	3	40
CD	0.667	20	3	40
DE	-0.943	-28.3	4.24	113
FE	-0.333	-20	3	20
EB	-0.471	0	4.24	0
BF	0.333	20	3	20
AF	-0.471	-28.3	4.24	56.6
CE	1	20	3	60
			Σ	369.6

Example 8 (cont.)

Solution:

$$1.\Delta = \sum u.dL$$

$$1kN.\Delta_{cv} = \sum \frac{uNL}{AE} = \frac{369.6}{AE}$$

$$\Delta_{cv} = \frac{369.6}{[300 \times 10^{-6}][200 \times 10^{6}]}$$

$$\Delta_{cv} = 6.16\ mm$$

Displacement of Trusses

(due to temperature changes and fabrication error)

In some cases, truss members may change their length due to temperature. If α is the coefficient of thermal expansion for a member and ΔT is the change in its temperature, the change in length of a member is

$$1 . \Delta = \sum u . \alpha . \Delta T . L$$

1 = external virtual unit load acting on the truss joint in the stated direction of Δ

u = internal virtual normal force in a truss member caused by the external virtual unit load

Δ = external joint displacement caused by the temperature change

α = coefficient of thermal expansion of member

ΔT = change in temperature of member

L = length of member

Displacement of Trusses

(due to temperature changes and fabrication error)

Occasionally, errors in fabricating the lengths of the members of a truss may occur. Also, in some cases truss member must be made slightly longer or shorter in order to give the truss a camber. If a truss member is shorter or longer than intended, the displacement of a truss joint from its expected position can be determined from direct application

$$1.\Delta = \sum u.\Delta L$$

1 = external virtual unit load acting on the truss joint in the stated direction of Δ

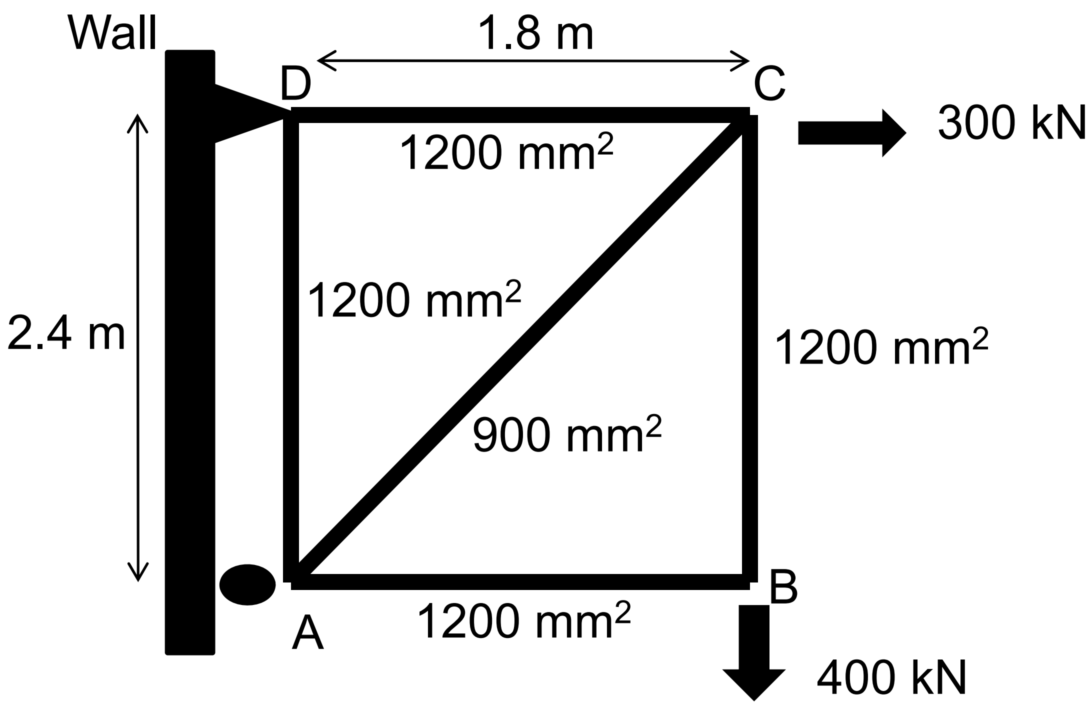
u = internal virtual normal force in a truss member caused by the external virtual unit load

Δ = external joint displacement caused by the fabrication errors

ΔL = difference in length of the member from its intended size as caused by a fabrication error

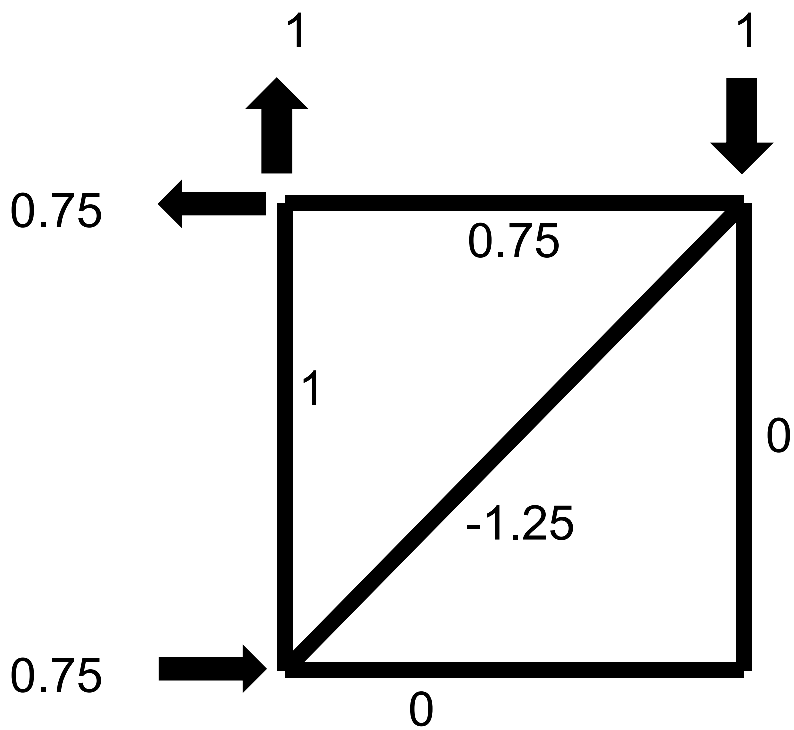
Example 9

Determine the vertical displacement of joint C of the steel truss as shown in the Figure. Due to radiant heating from the wall, member AD is subjected to an increase in temperature of $\Delta T = +60°C$. Take $\alpha = 1.08 \times 10^{-5}/°C$ and $E = 200$ GPa. The cross-sectional area of each member is indicated in the figure.



Example 9 (cont.)

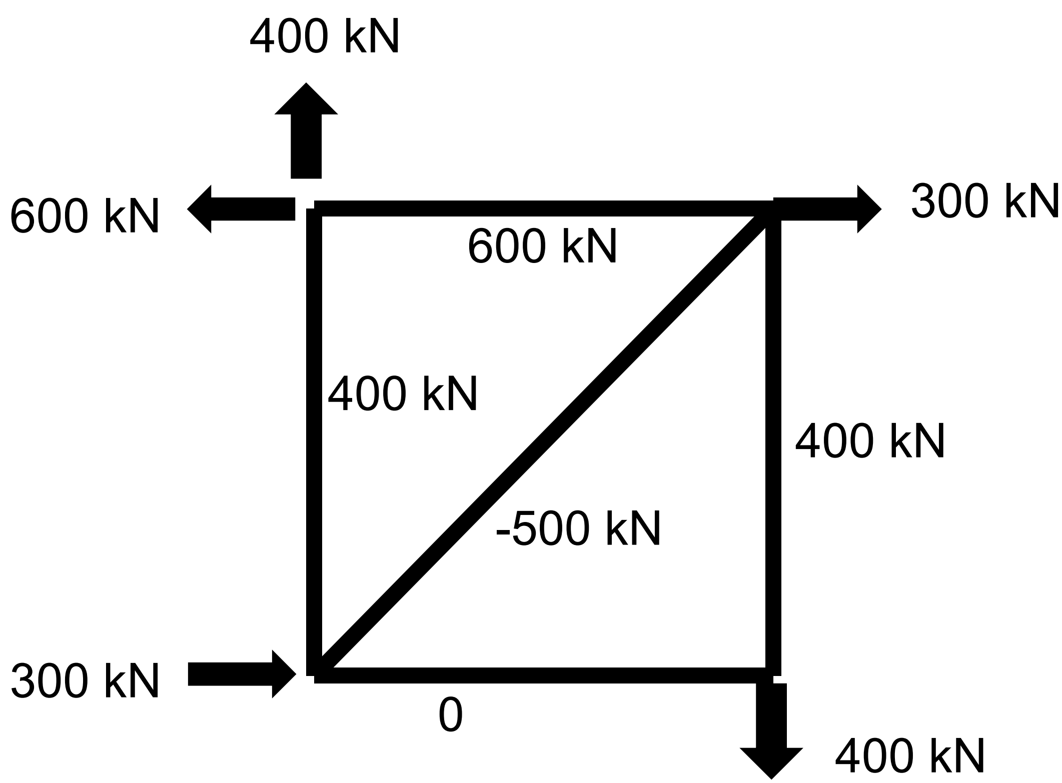
Solution



Virtual force, u

Example 9 (cont.)

Solution



Real force, N

Example 9 (cont.)

Solution. Both loads and temperature affect the deformation; therefore

$$1.\Delta = \sum u.dL + \sum u.\alpha.\Delta T.L$$

$$1kN.\Delta_{cv} = \frac{(0.75)(600)(1.8)}{1200(10^{-6})[200(10^{6})]} + \frac{(1)(400)(2.4)}{1200(10^{-6})[200(10^{6})]}$$

$$+ \frac{(-1.25)(-500)(3)}{900(10^{-6})[200(10^{6})]} + (1)[1.08(10^{-5})](60)(2.4)$$

$$\Delta_{cv} = 0.0193\ m = 19.3\ mm$$

References

1. Hibbeler, R.C., Mechanics Of Materials, 8th Edition in SI units, Prentice Hall, 2011.
2. Gere dan Timoshenko, Mechanics of Materials, 3rd Edition, Chapman & Hall.
3. Yusof Ahmad, 'Mekanik Bahan dan Struktur' Penerbit UTM 2001