SAB2223 Mechanics of Materials and Structures

TOPIC 3
STRESSES IN BEAMS

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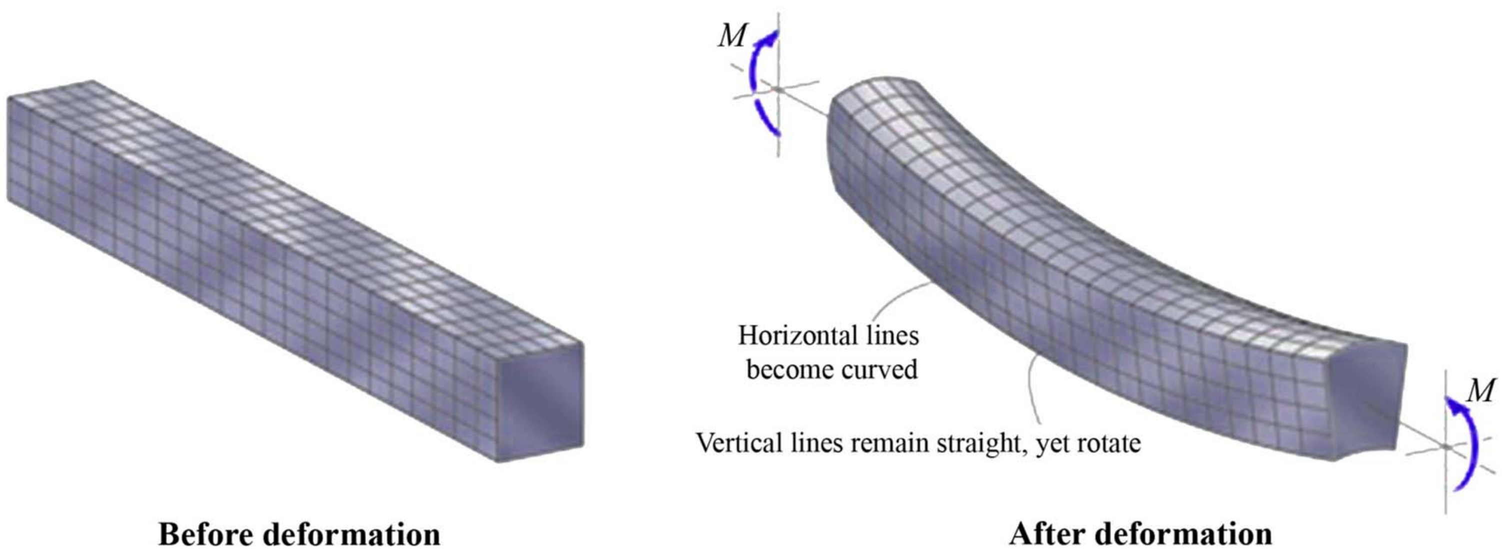
TOPIC 3
STRESSES IN BEAMS

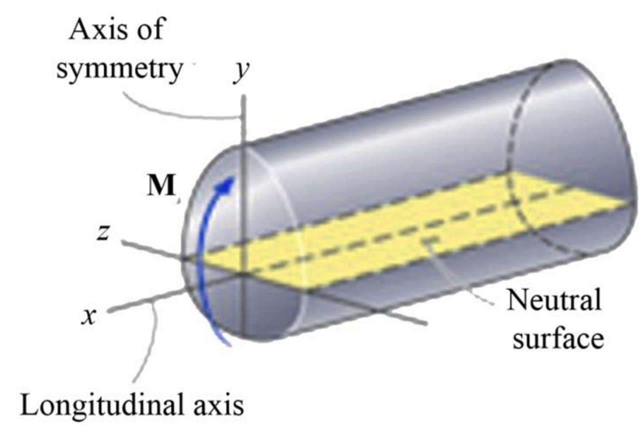
Introduction

- Any deformable material (ie. Rubber) tend to distort/deform when subjected to a bending moment.
- All bending moment induce bending stress and all shear forces induce shear stress.



Introduction

- Any deformable bar subjected to a bending moment causes the material within the **bottom portion** undergo **stretches (tension)** and the **top portion** undergo **compression**.
- The region in between the tension and compression lies the **neutral surface**, which the material do not undergo changes in length.

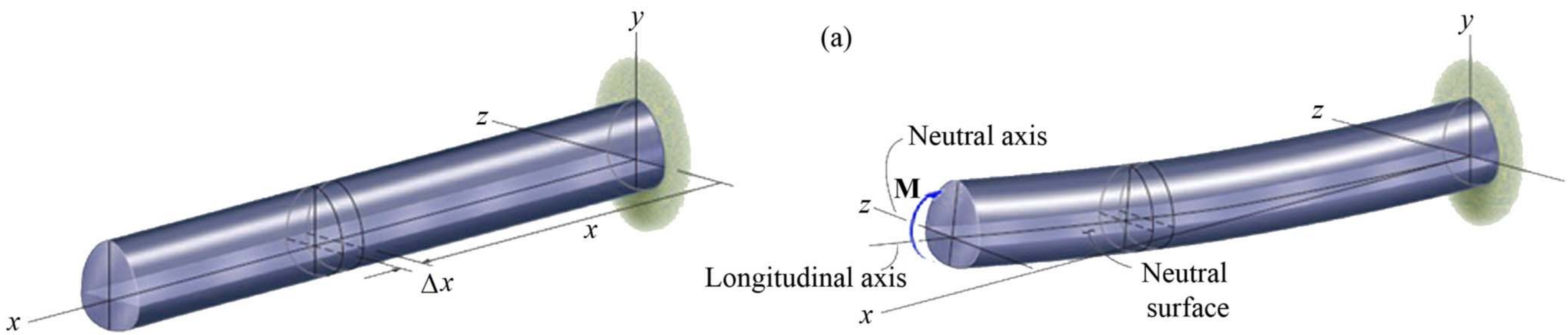


Introduction

- Several assumptions:
  - The longitudinal axis x, which lies within the ***neutral surface*** does not experience any ***change in length***.
  - All ***cross sections*** of the beam remain ***plane*** and perpendicular to the longitudinal axis during the deformation.
  - Any ***deformation*** of the ***cross section*** within its own plane will be ***neglected***.
  - The axis lying in the plane of the cross section and about which the cross section rotates is called the ***neutral axis***.
  - The material is ***homogeneous***, with ***same*** cross sectional area along the length and constant ***Elastic Modulus*** (*E*).

Introduction

- *Longitudinal strain* varies *linearly* from zero at the neutral axis.
- Hooke's law applies when material is **homogeneous.**
- Neutral axis passes through the ***centroid*** of the cross-sectional area for linear-elastic material.



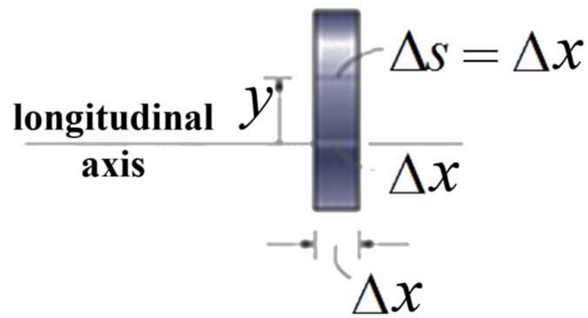
Bending Deformation

Normal Strain:

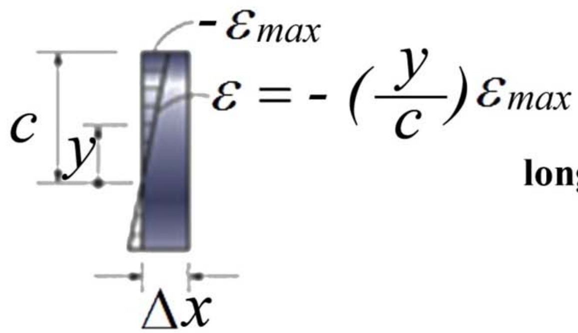
$$\varepsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\varepsilon = \frac{(p-y)\Delta\theta - p\Delta\theta}{p\Delta\theta}$$

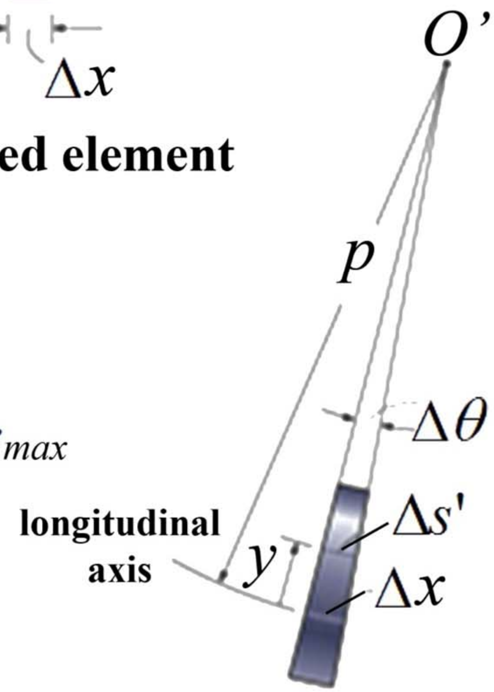
$$\therefore \varepsilon = -\frac{y}{p}$$



Undeformed element



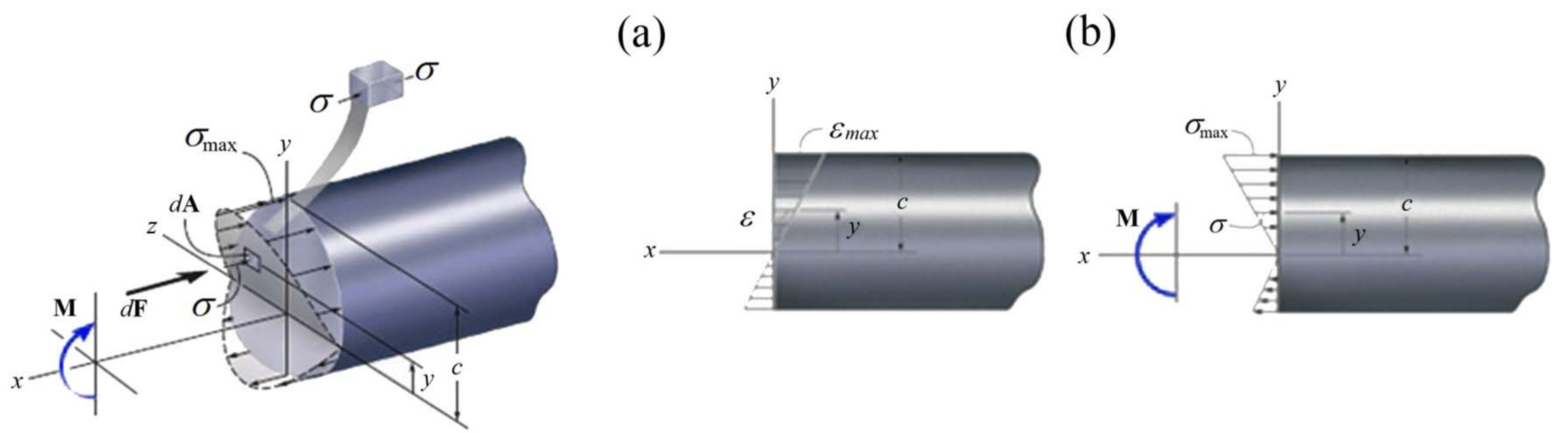
Normal strain distribution



Deformed element

Bending Deformation

- Resultant moment on the cross section is equal to the moment produced by the linear normal stress distribution about the neutral axis.



Bending stress variation

Normal strain variation (profile view)

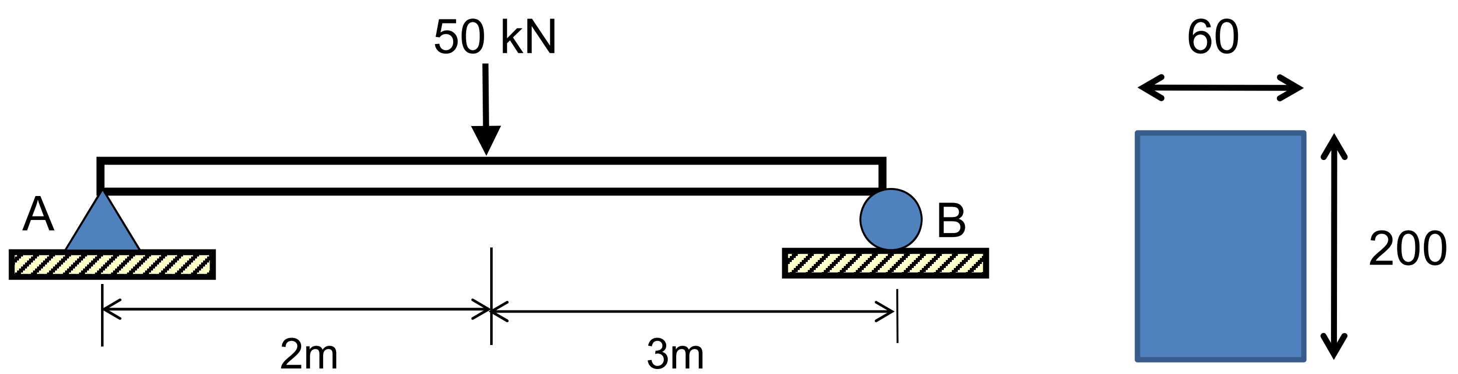
Bending stress variation (profile view)

$$\sigma = -\frac{My}{I}$$

σ = normal stress in the member
M = resultant internal moment
I = moment of inertia
y = perpendicular distance from the neutral axis

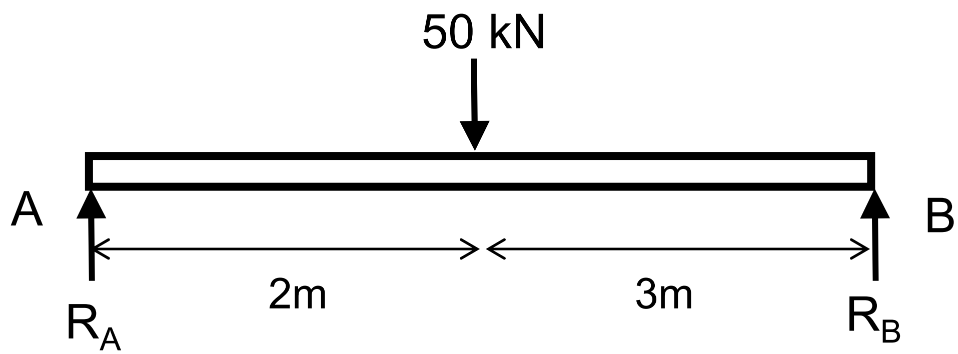
Example 1

Determine the maximum bending stress in beam AB as shown in the figure and draw the stress distribution over the cross section at this location.



Example 1 (cont.)

Solution:



By taking the moment at A,

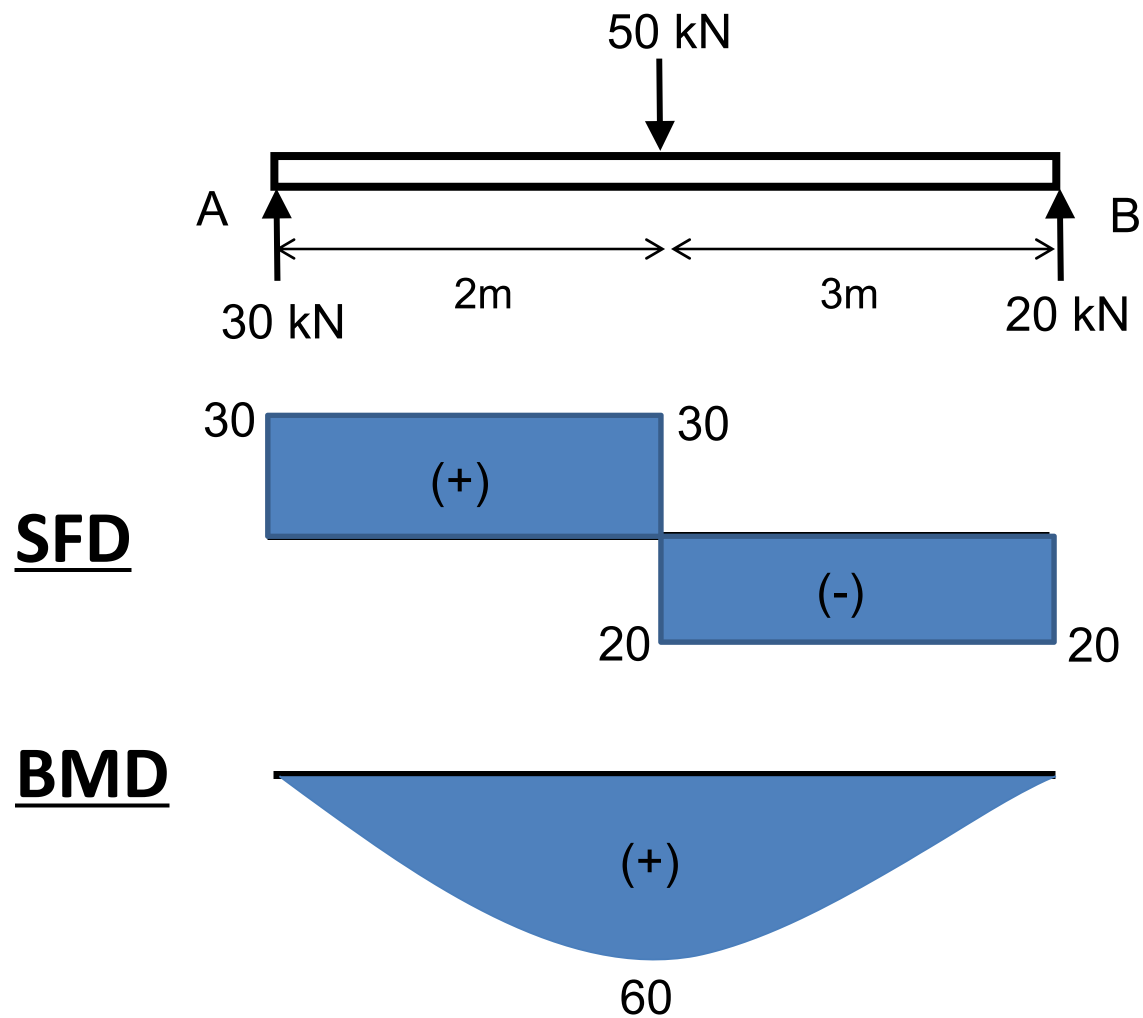
$$\Sigma M_A = 0$$
$$-R_B \times 5 + 50 \times 2 = 0$$
$$R_B = 20 \text{ kN}$$

$$\Sigma F_y = 0$$
$$R_A + R_B = 50$$
$$R_A = 50 - 20$$
$$R_A = 30 \text{ kN}$$

$$\Sigma F_x = 0$$
$$R_{Ax} = 0$$

Example 1 (cont.)

Solution:



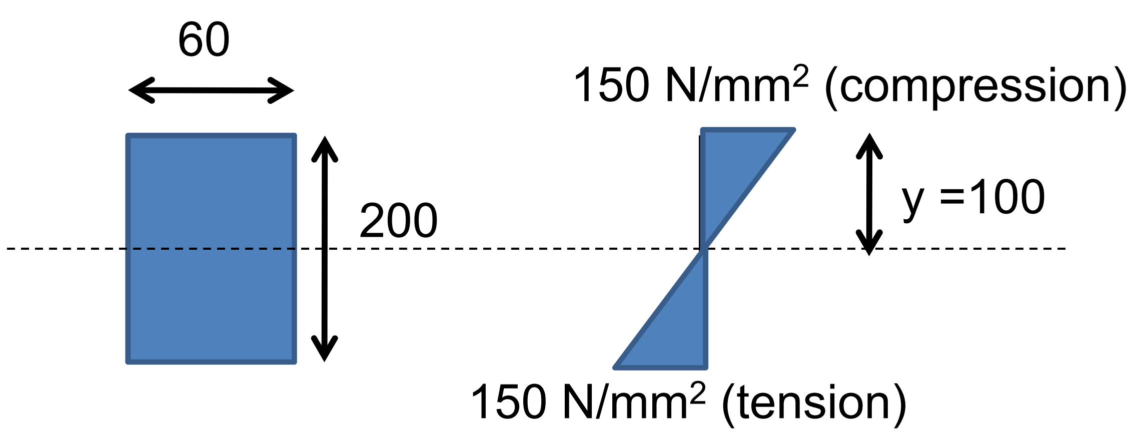
Example 1 (cont.)

Solution:

M_{max} = 60 kNm (occurred at 2m from point A)

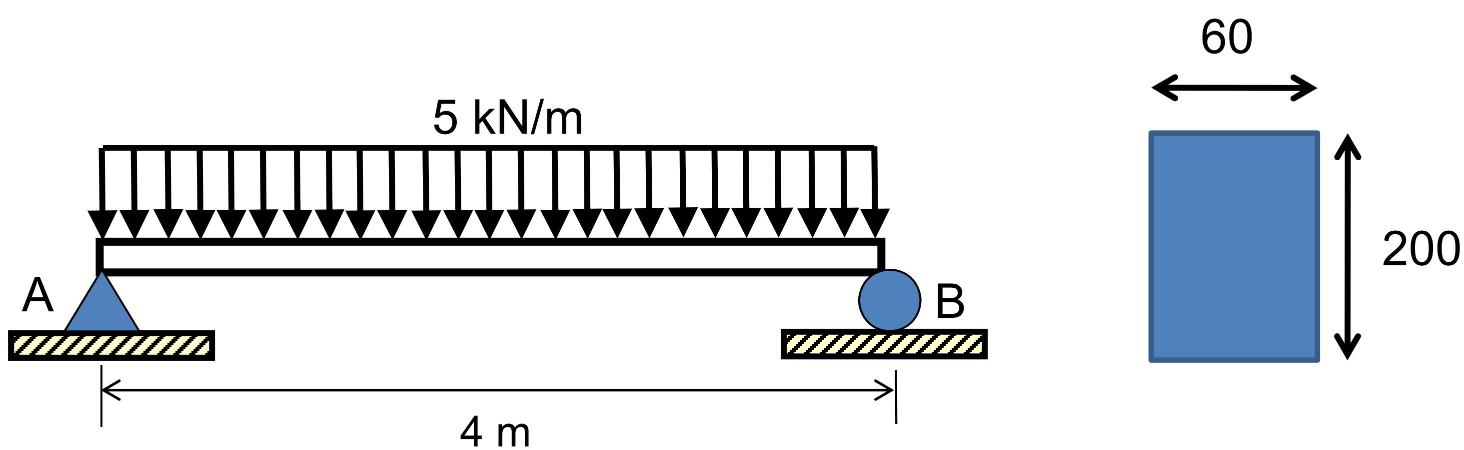
$I = bh^3/12 = 60 \times 200^3 / 12 = 40 \times 10^6$ mm^4

$\sigma_{max} = My / I = (60 \times 10^6 \times 100) / (40 \times 10^6) = 150$ N/mm^2



Example 2

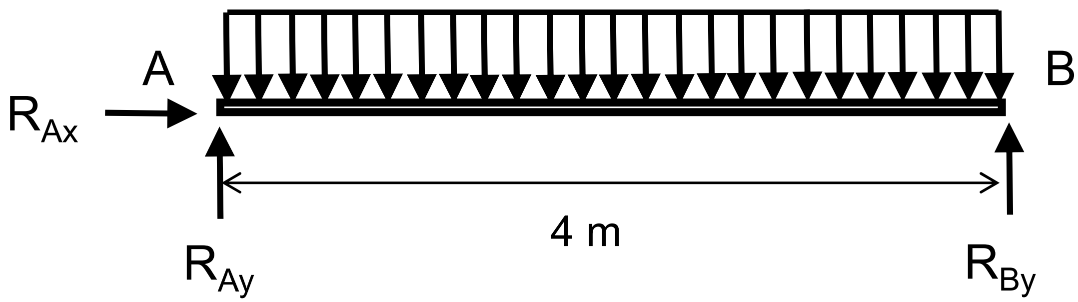
Determine the maximum bending stress in beam AB as shown in the figure and draw the stress distribution over the cross section at this location.



Example 2 (cont.)

Solution

a)



By taking the moment at A,

$\Sigma M_A = 0$

$-R_B \times 4 + 5 \times 4 \times 4/2 = 0$

$R_B = 10$ kN

$\Sigma F_y = 0$

$R_A + R_B = 5 \times 4$

$R_A = 20 - 10$

$R_A = 10$ kN

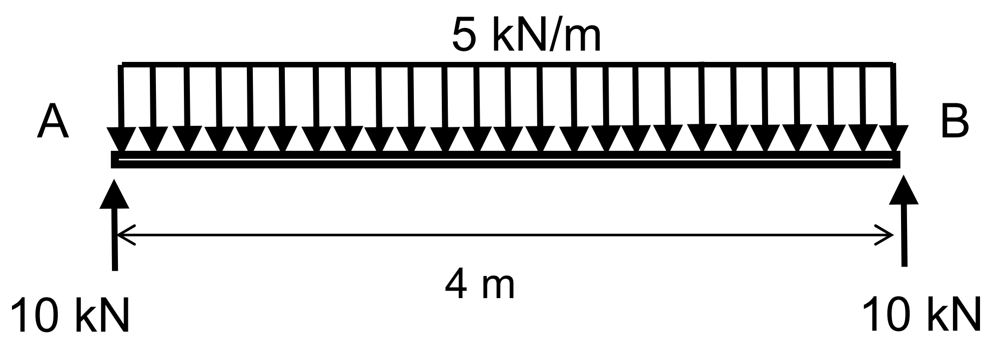
$\Sigma F_x = 0$

$R_{Ax} = 0$

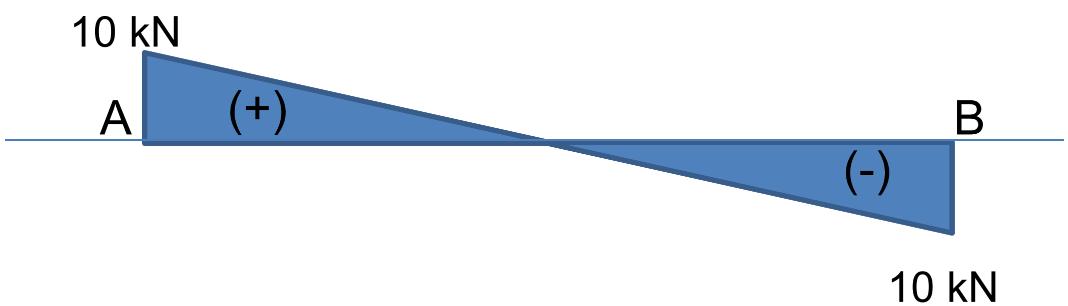
Example 2 (cont.)

Solution (cont.)

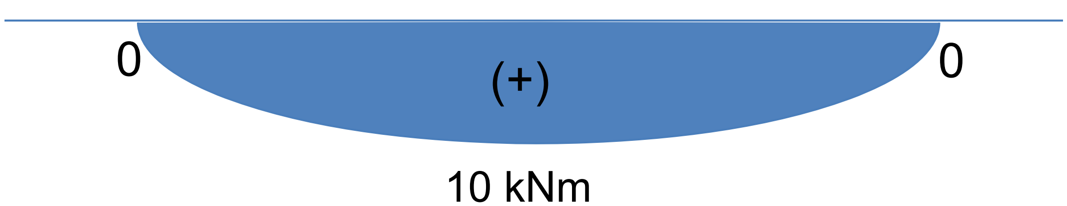
a)



SFD



BMD



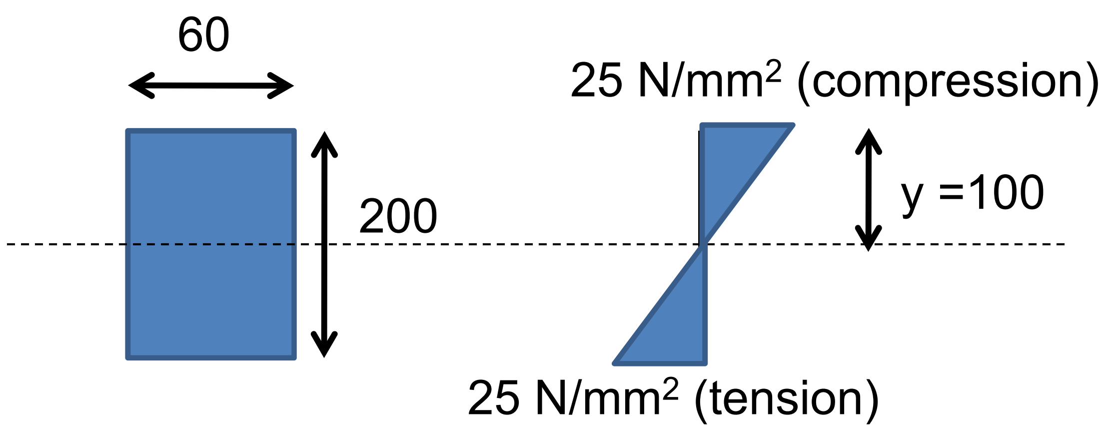
Example 2 (cont.)

Solution:

M_{max} = 10 kNm (occurred at 2m from point A)

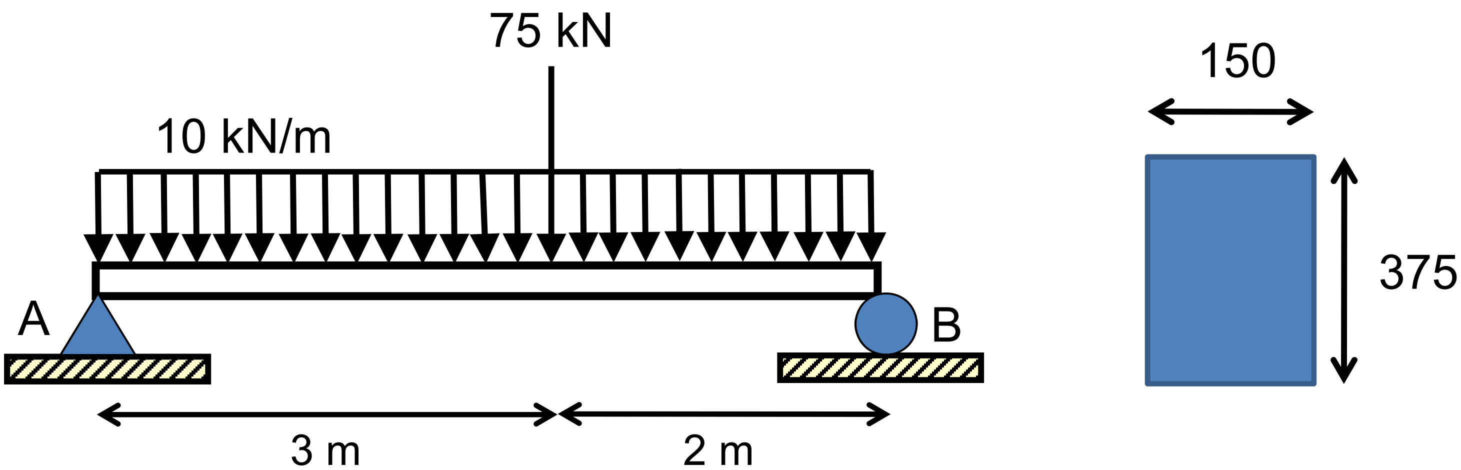
$I = bh^3/12 = 60 \times 200^3 / 12 = 40 \times 10^6$ mm^4

$\sigma_{max} = My / I = (10 \times 10^6 \times 100) / (40 \times 10^6) = 25$ N/mm^2



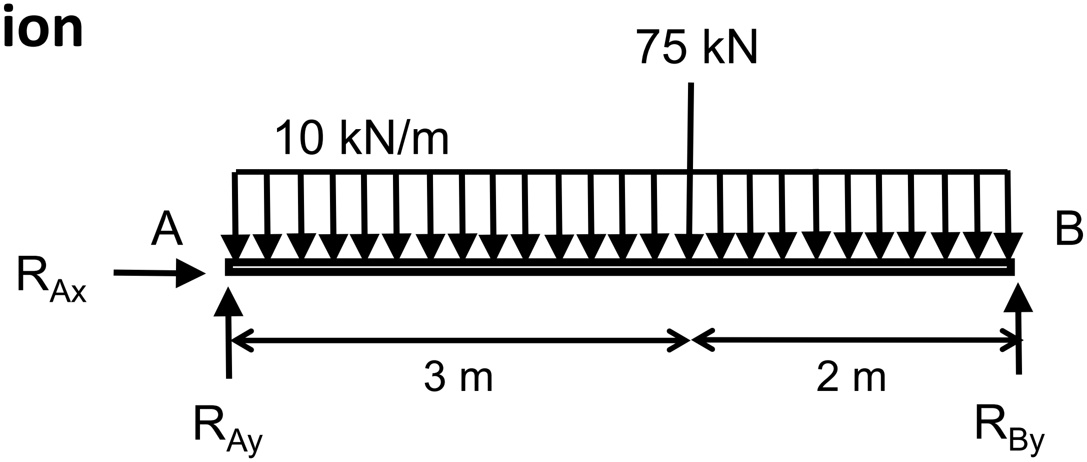
Example 3

Determine the maximum bending stress in beam AB as shown in the figure and draw the stress distribution over the cross section at this location.



Example 3 (cont.)

Solution



$$\Sigma F_x = 0$$
$$R_{Ax} = 0$$

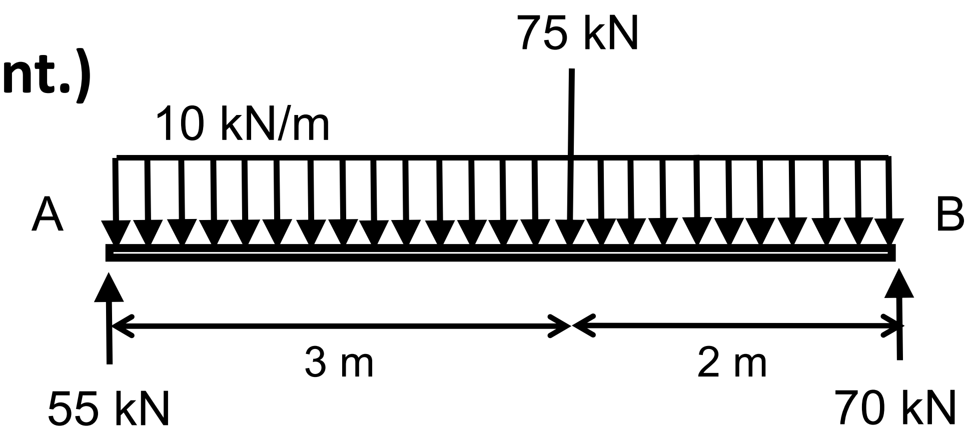
By taking the moment at A,

$$\Sigma M_A = 0$$
$$-R_B \times 5 + 10 \times 5 \times 5/2 + 75 \times 3 = 0$$
$$R_B = 70 \text{ kN}$$

$$\Sigma F_y = 0$$
$$R_A + R_B = 10 \times 5 + 75$$
$$R_A = 125 - 70$$
$$R_A = 55 \text{ kN}$$

Example 3 (cont.)

Solution (cont.)



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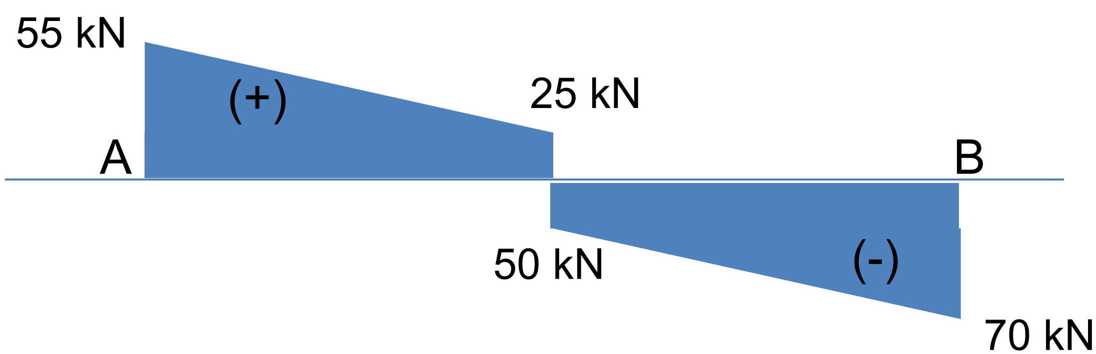

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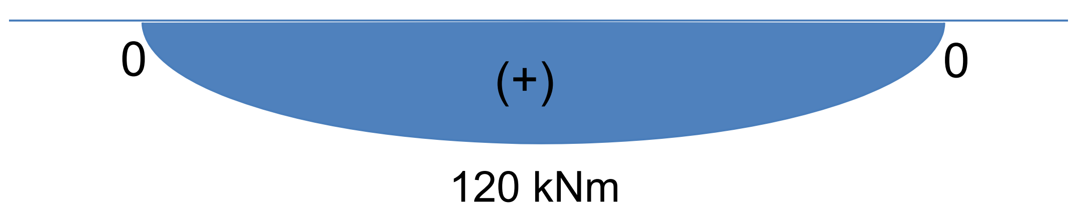


Example 3 (cont.)

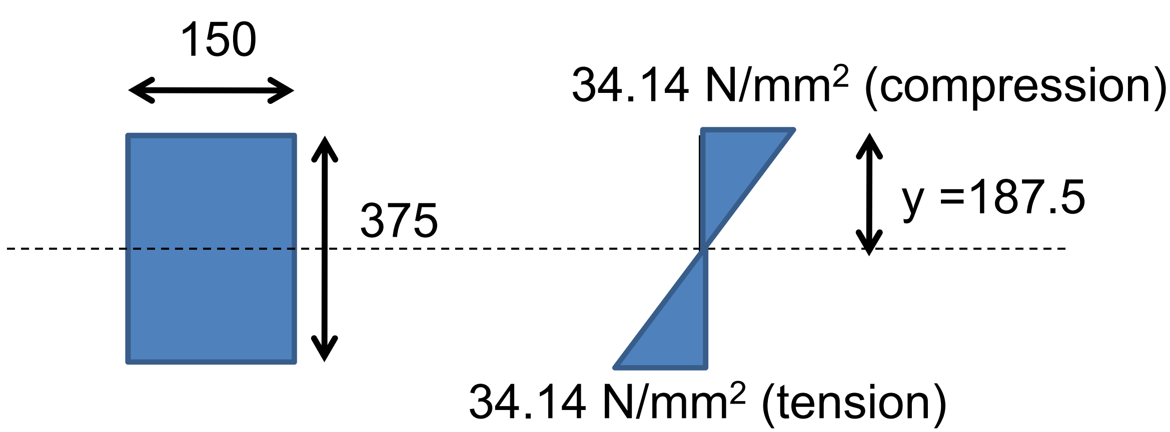
Solution:

M_{max} = 120 kNm (occurred at 3m from point A)

$I = bh^3/12 = 150 \times 375^3 / 12 = 6.59 \times 10^8$ mm^4

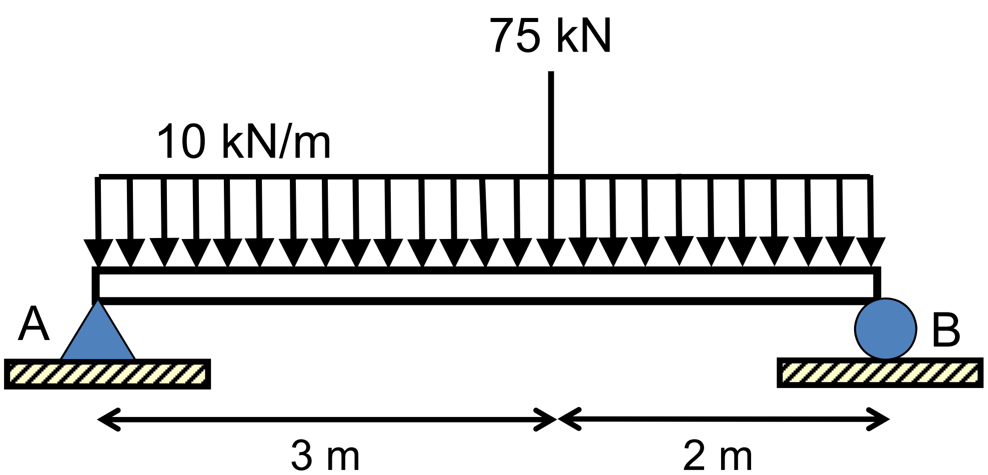
σ_{max} (above) $= - My / I = - (120 \times 10^6 \times 187.5) / (6.59 \times 10^8)$

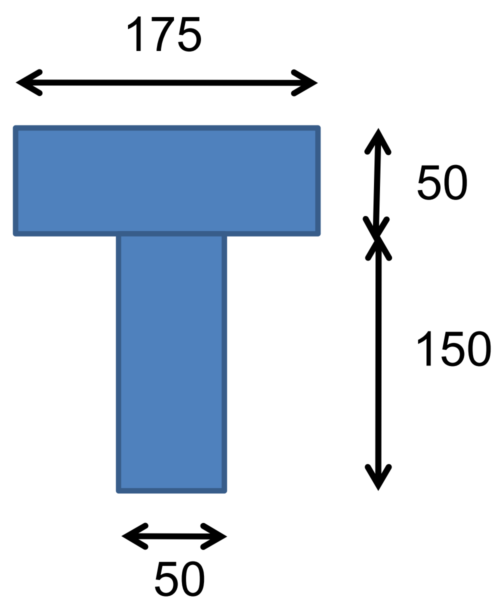
$= - 34.14$ N/mm^2



Example 4

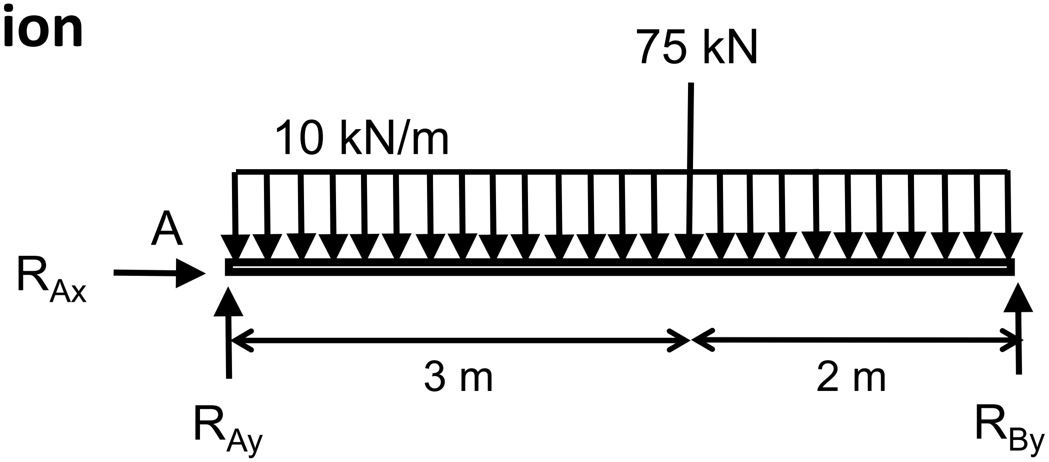
Determine the maximum bending stress in beam AB as shown in the figure and draw the stress distribution over the cross section at this location.





Example 4 (cont.)

Solution



$$\Sigma F_x = 0$$

$$R_{Ax} = 0$$

By taking the moment at A,

$$\Sigma M_A = 0$$

$$-R_B \times 5 + 10 \times 5 \times 5/2 + 75 \times 3 = 0$$

$$R_B = 70 \text{ kN}$$

$$\Sigma F_y = 0$$

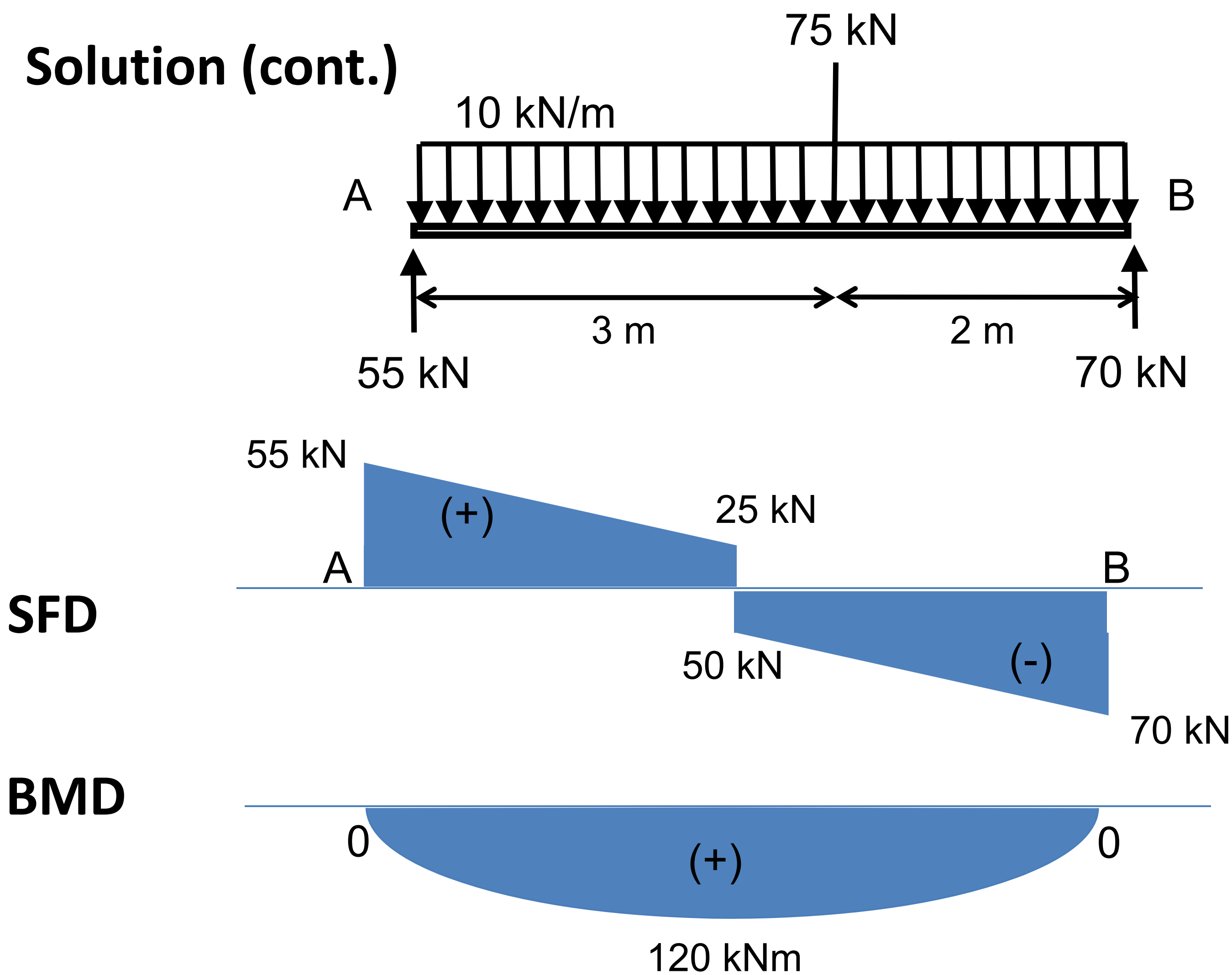
$$R_A + R_B = 10 \times 5 + 75$$

$$R_A = 125 - 70$$

$$R_A = 55 \text{ kN}$$

Example 4 (cont.)

Solution (cont.)



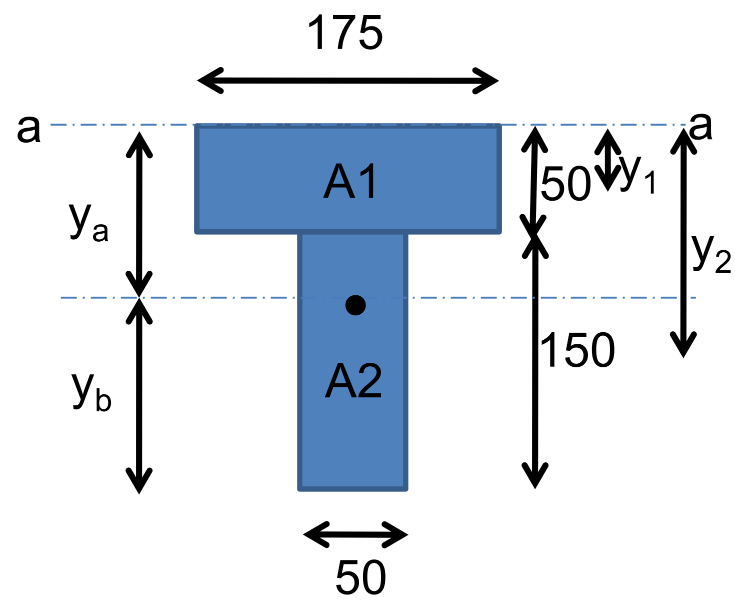
Example 4 (cont.)

Solution:

Determine the centroid location:

Area 1: 175 × 50 = 8750 mm^2

Area 2: 150 x 50 = 7500 mm^2



Moment at a – a:

$\Sigma A.y_a = A_1.y_1 + A_2.y_2$

$\Sigma A.y_a = 8750 \times 25 + 7500 \times 125$

$y_a = 1156250 / (8750 + 7500) = 71.15$ mm

$y_b = (150 + 50) - 71.15 = 128.85$ mm

Example 4 (cont.)

Solution:

M_{max} = 120 kNm (occurred at 3m from point A)

$I = \Sigma bh^3/12 + Ay^2$

$= 175 \times 50^3 / 12 + 8750 \times 46.15^2 + 50 \times 150^3 / 12 + 7500 \times 53.85^2$

$= 20.46 \times 10^6 + 35.81 \times 10^6 \text{ mm}^4$

$= 56.27 \times 10^6 \text{ mm}^4$

σ_{max} (above) = - My / I

$= - (120 \times 10^6 \times 71.15) / (56.27 \times 10^6)$

$= - 151.73 \text{ N/mm}^2$

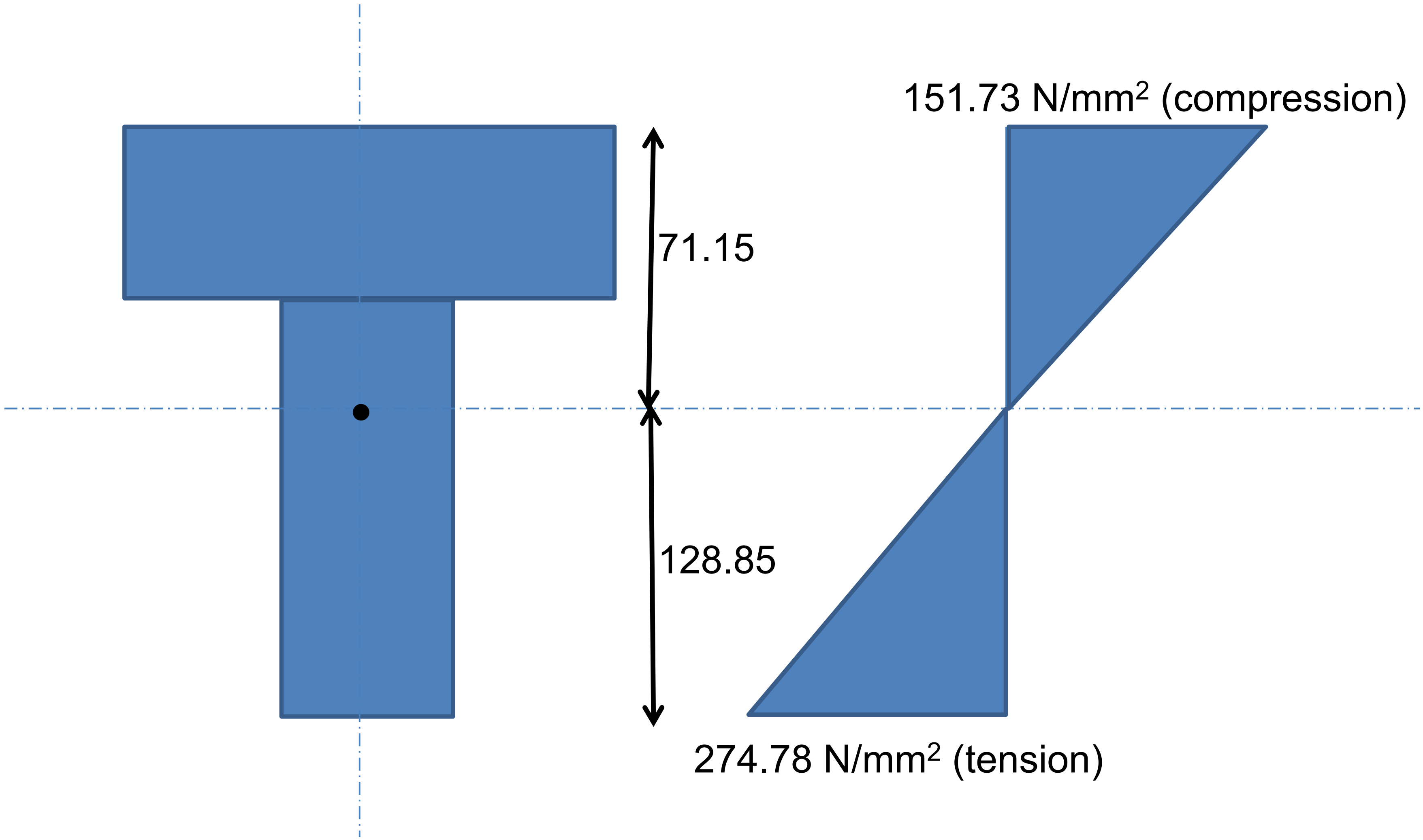
σ_{max} (below) = - My / I

$= - (120 \times 10^6 \times (- 128.85)) / (56.27 \times 10^6)$

$= 274.78 \text{ N/mm}^2$

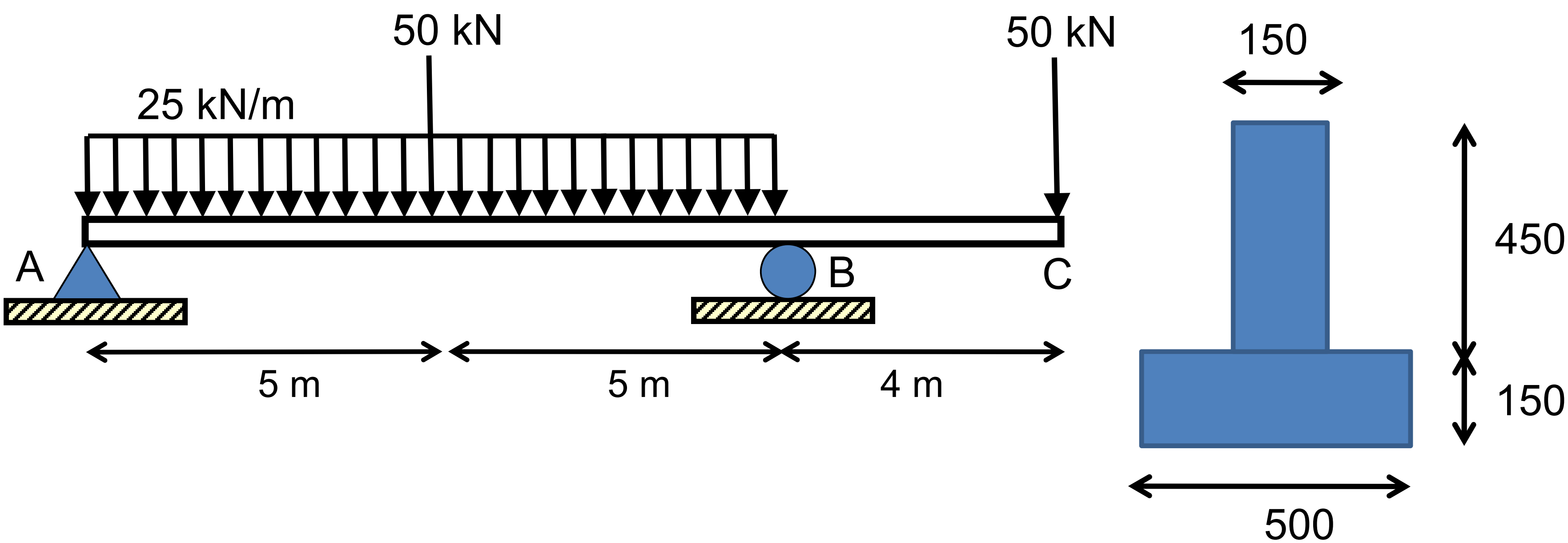
Example 4 (cont.)

Solution:



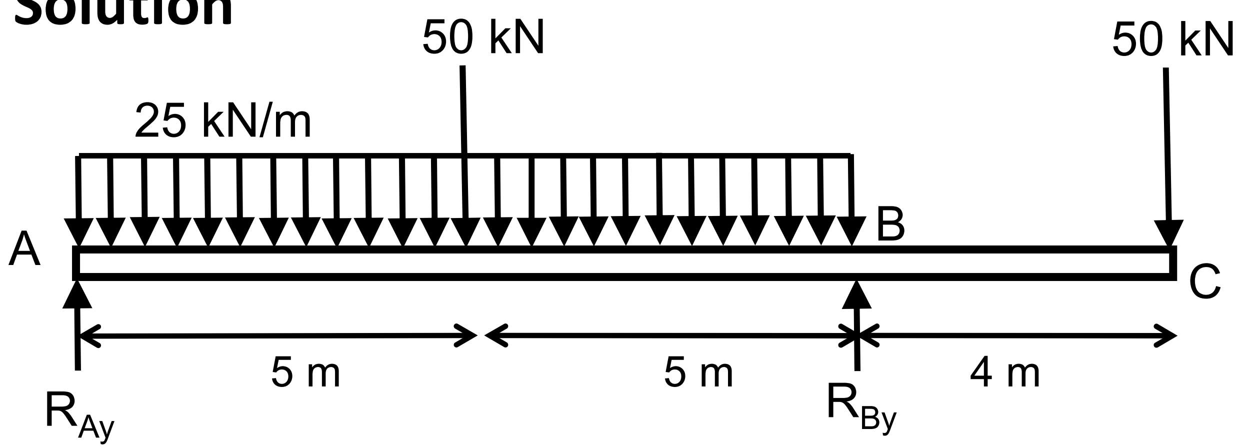
Example 5

Determine the maximum compressive and tensile stress in beam AB as shown in the figure and draw the stress distribution over the cross section at this location.



Example 5 (cont.)

Solution



$\Sigma F_x = 0$

$R_{Ax} = 0$

By taking the moment at A,

$\Sigma M_A = 0$

$-R_B \times 10 + 25 \times 10 \times 10/2 + 50 \times 5 + 50 \times 14 = 0$

$R_B = 220$ kN

$\Sigma F_y = 0$

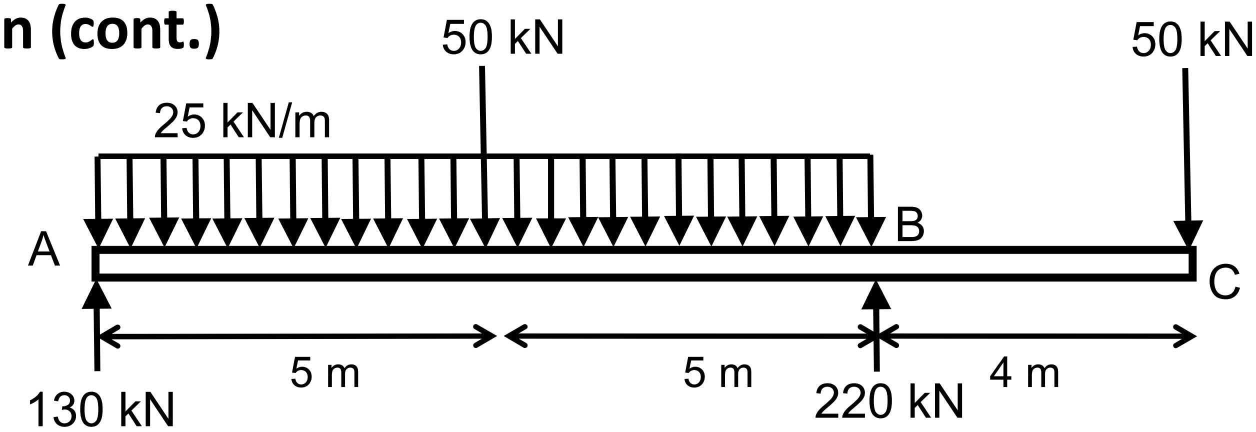
$R_A + R_B = 25 \times 10 + 50 + 50$

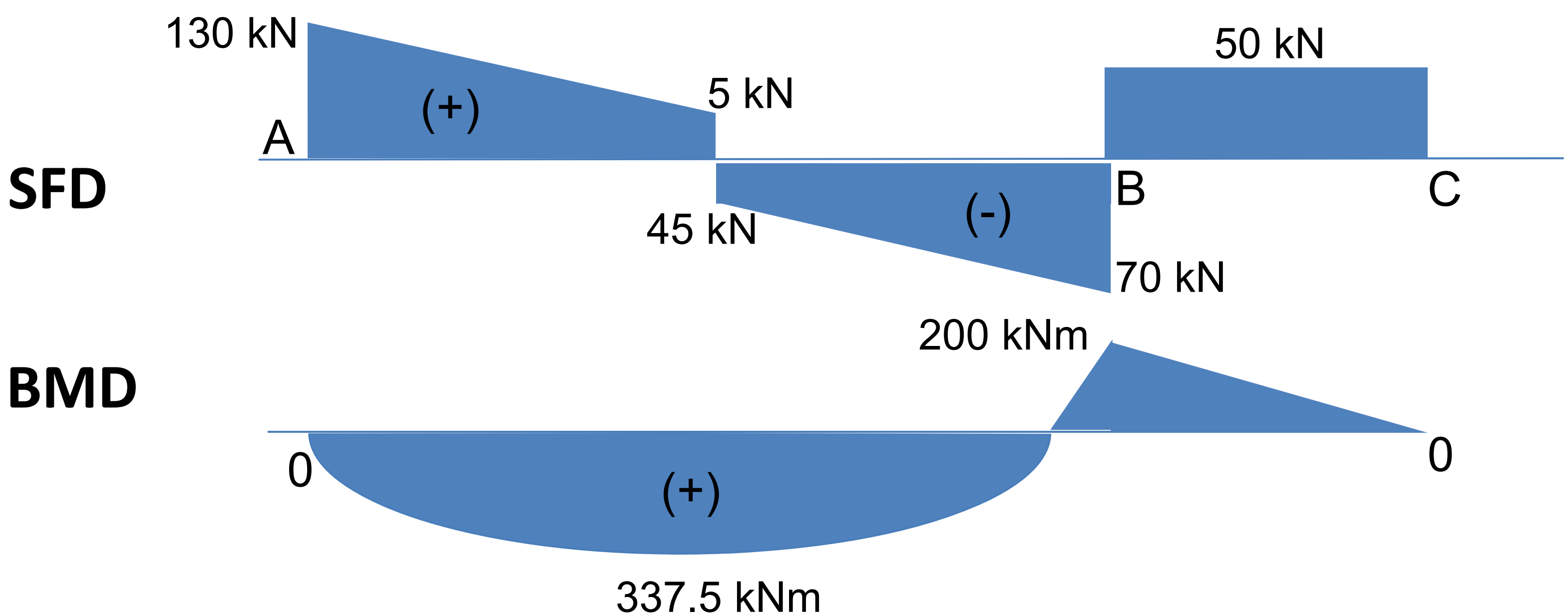
$R_A = 350 - 220$

$R_A = 130$ kN

Example 5 (cont.)

Solution (cont.)





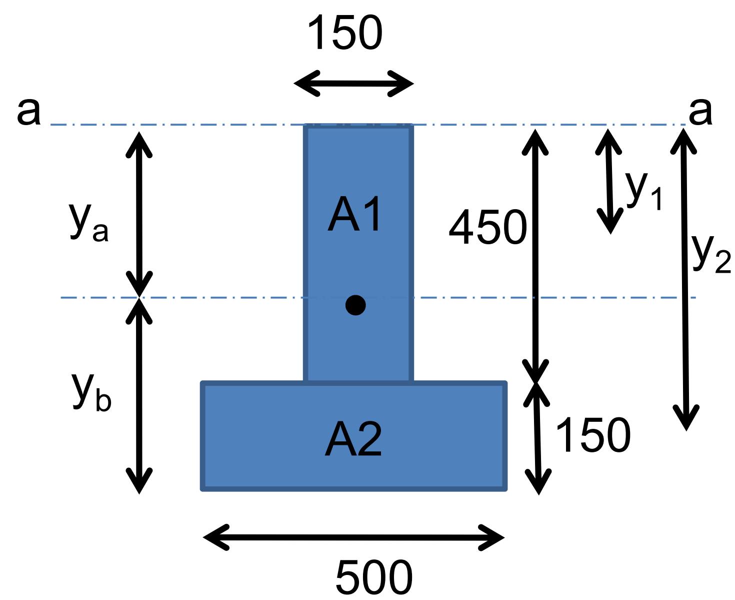
Example 5 (cont.)

Solution:

Determine the centroid location:

Area 1: $450 \times 150 = 67500\ mm^2$

Area 2: $150 \times 500 = 75000\ mm^2$



Moment at a – a:

$\Sigma A.y_a = A_1.y_1 + A_2.y_2$

$\Sigma A.y_a = 67500 \times 225 + 75000 \times 525$

$y_a = 54562500 / (67500 + 75000) = 382.89\ mm$

$y_b = (450 + 150) - 382.89 = 217.11\ mm$

Example (cont.)

Solution:

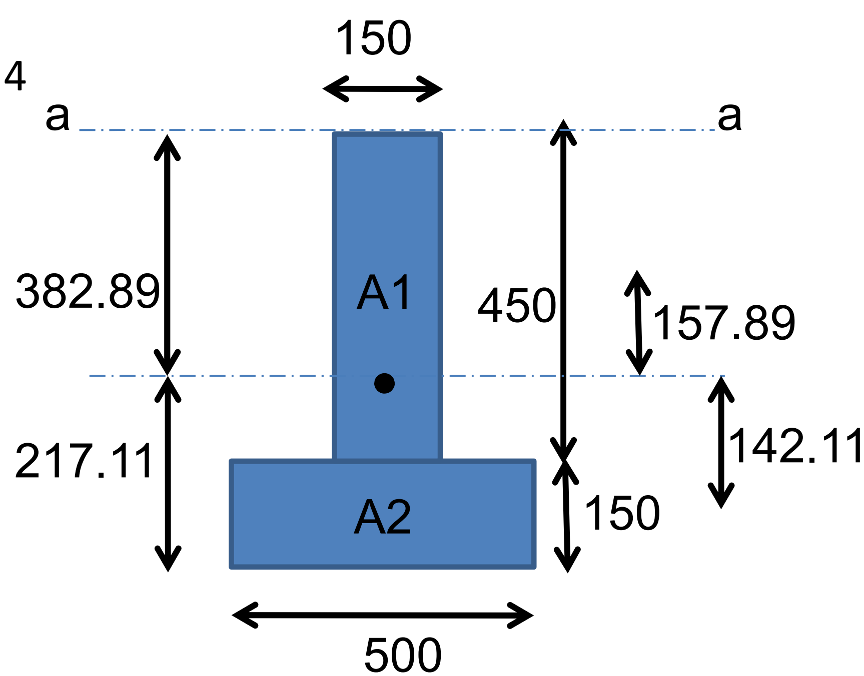
Second Moment of Area:

$I = \Sigma bh^3/12 + Ay^2$

$= 150 \times 450^3 / 12 + 67500 \times 157.89^2 + 500 \times 150^3 / 12 + 75000 \times 142.11^2$

$= 28.22 \times 10^8 + 16.55 \times 10^8$ mm^4

$= 44.77 \times 10^8$ mm^4



Example (cont.)

Solution:

M_{max} = 335 kNm (occurred at 5m from point A)

$\sigma_{max\ (above)}$ = - My / I

= - (335 × 10^6 × 382.89) / (44.77 × 10^8)

= - 28.65 N/mm^2

$\sigma_{max\ (below)}$ = - My / I

= - (335 × 10^6 × (-217.11)) / (44.77 × 10^8)

= 16.24 N/mm^2

Example 5 (cont.)

Solution:

M_{max} = - 200 kNm (occurred at support B)

$\sigma_{max\ (above)} = - My / I$

$= - ((- 200 \times 10^6) \times 382.89) / (44.77 \times 10^8)$

$= 17.10\ N/mm^2$

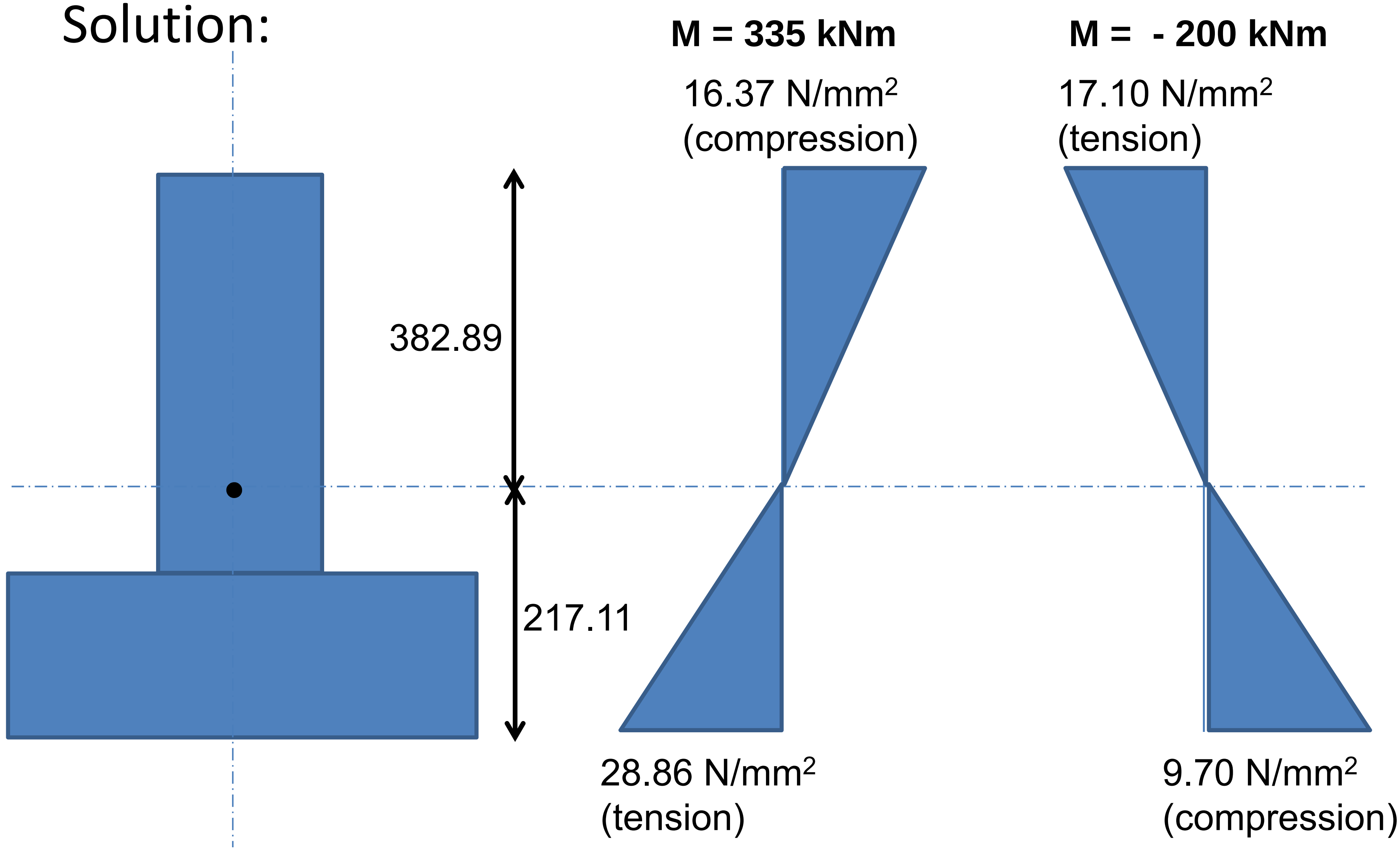
$\sigma_{max\ (below)} = - My / I$

$= - ((-200 \times 10^6) \times (- 217.11)) / (44.77 \times 10^8)$

$= - 9.70\ N/mm^2$

Example 5 (cont.)

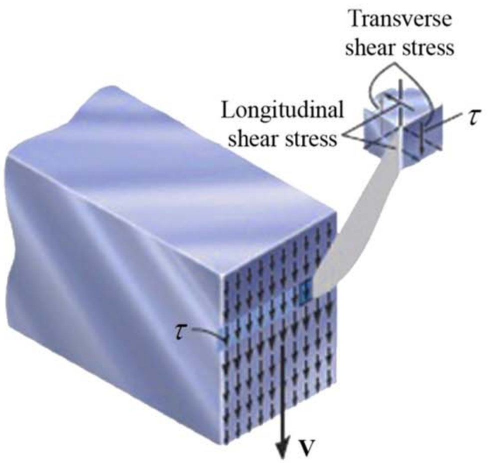
Solution:

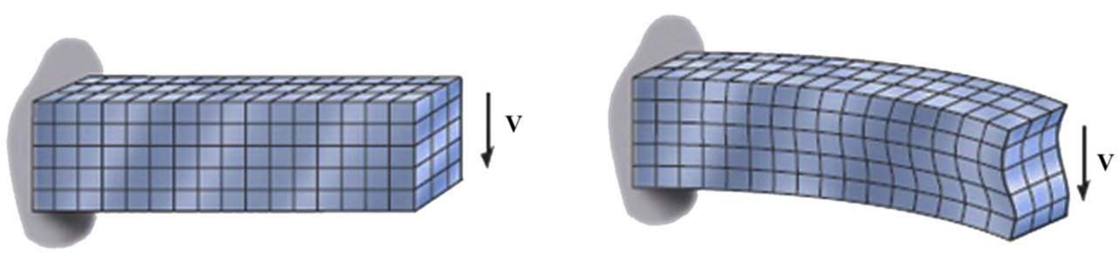


Shear in Straight Members

- A shear, V can induce non-uniform shear-strain distribution over the cross section and the relationship is given by:

$$V = dM/dx$$





(a) Before deformation (b) After deformation

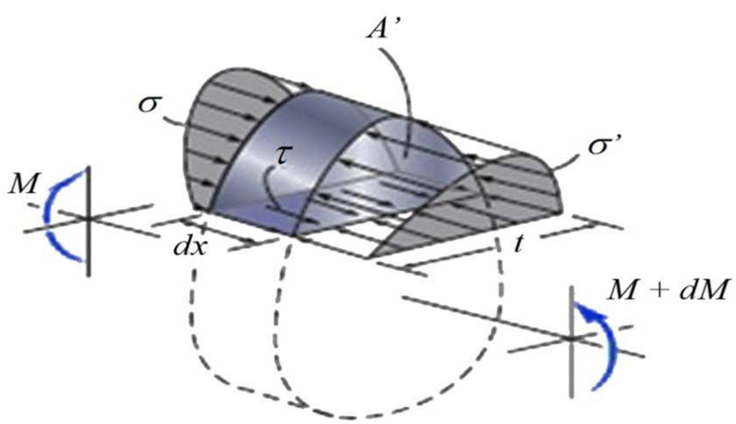
Shear in Straight Members

- The transverse shear stress on the beam's cross-sectional area can be obtained by using:

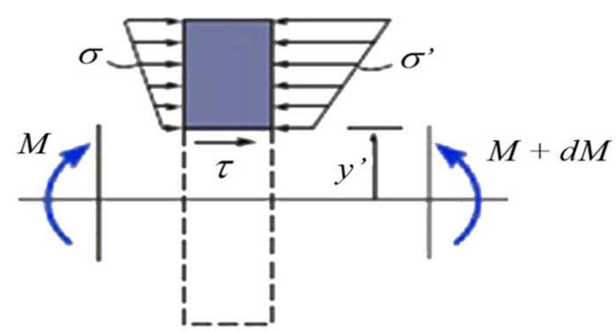
$$\tau = \frac{VQ}{It}$$

$$\text{where } Q = \int_{A'} y\,dA = \bar{y}' A'$$

τ = the shear stress in the member
V = internal resultant shear force
I = moment of inertia of the *entire* cross-sectional area
t = width of the member's cross-sectional area



Three-dimensional view

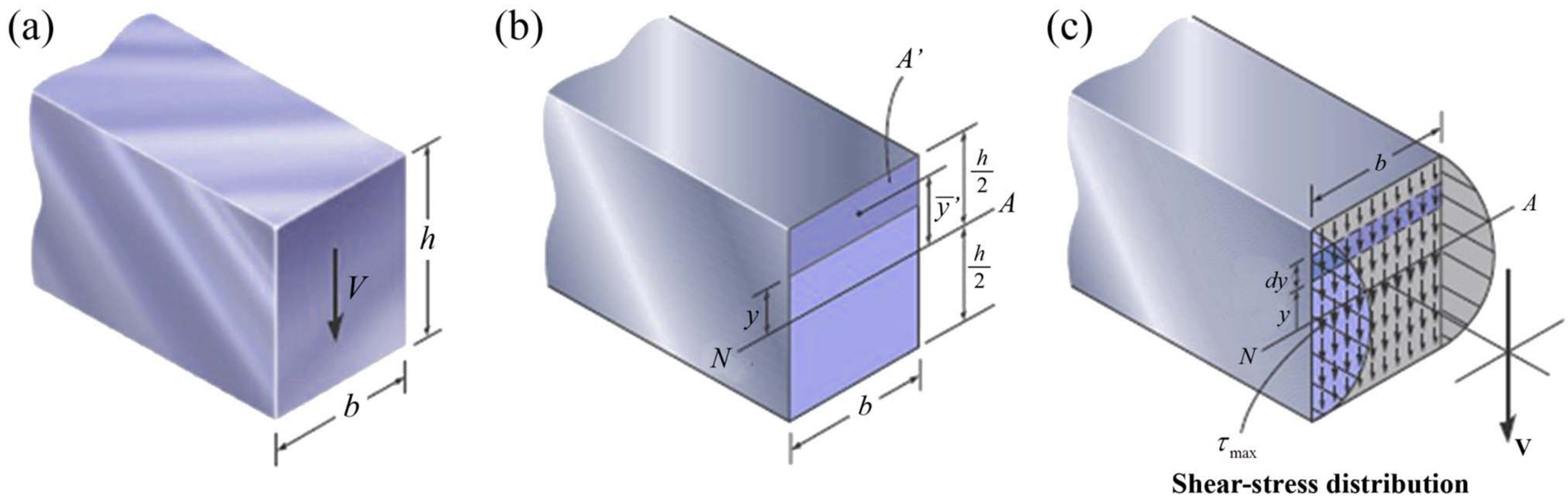


Profile view

Shear Stresses in Beams

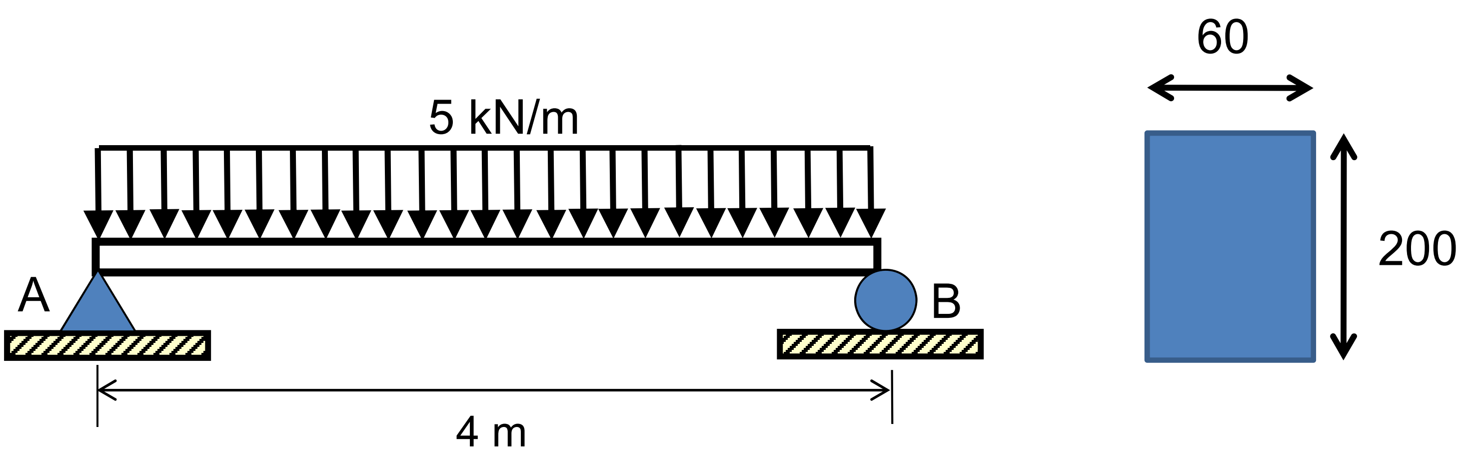
- For rectangular cross section, *shear stress varies parabolically* with depth and maximum shear stress is along the neutral axis, and the formula can be simplified as:

$$\tau_{\max} = 1.5\frac{V}{A}$$



Example 6

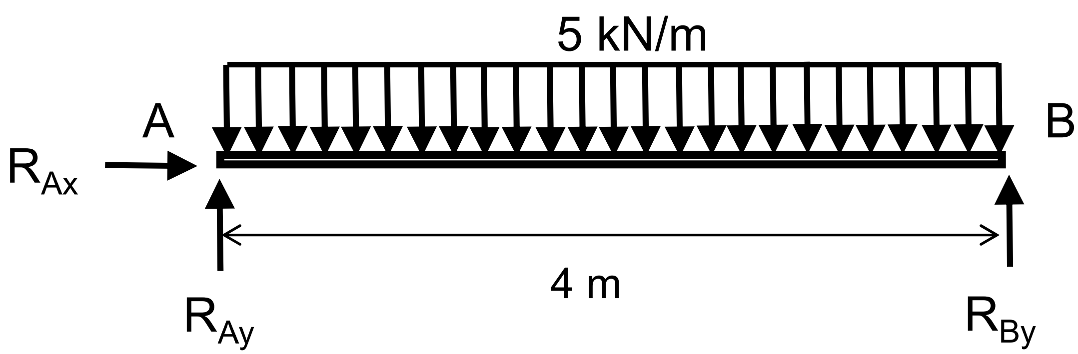
Determine the maximum shear stress in beam AB at 1m from support A and draw the stress distribution over the cross section at every 25mm interval.



Example 6 (cont.)

Solution

a)



By taking the moment at A,

$\Sigma M_A = 0$

$-R_B \times 4 + 5 \times 4 \times 4/2 = 0$

$R_B = 10$ kN

$\Sigma F_y = 0$

$R_A + R_B = 5 \times 4$

$R_A = 20 - 10$

$R_A = 10$ kN

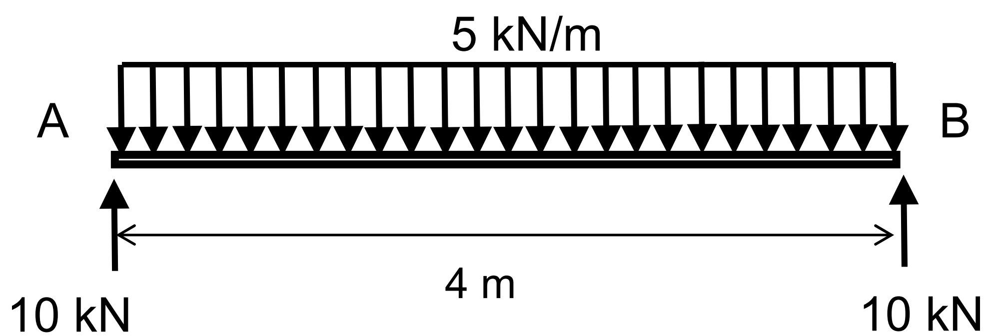
$\Sigma F_x = 0$

$R_{Ax} = 0$

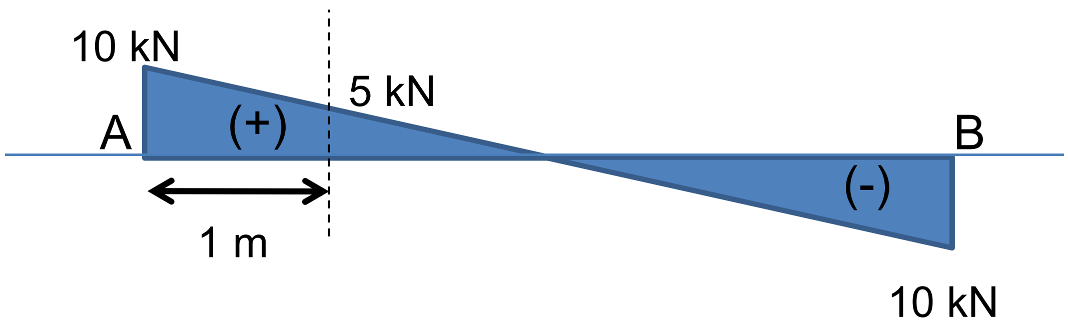
Example 6 (cont.)

Solution (cont.)

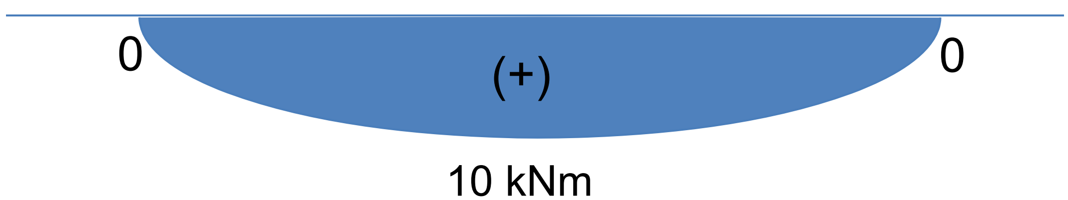
a)



SFD



BMD

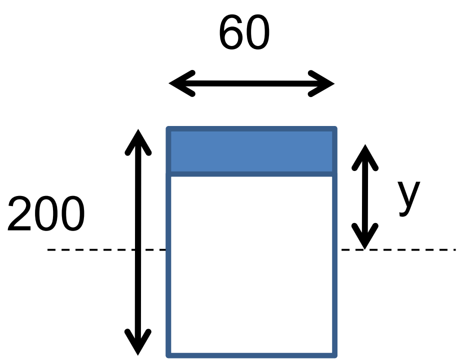


Example 6 (cont.)

Solution:

V_{1m} = 5 kN

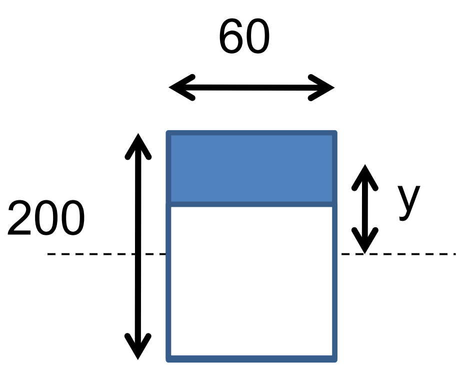
I = bh³/12 = 60 × 200³ / 12 = 40 × 10⁶ mm⁴



A = 60 × 25 = 1500 mm²

y = 100 – (25/2) = 87.5 mm

$$\tau_1 = \frac{VAy}{It} = \frac{5\times10^3\times1500\times87.5}{40\times10^6\times60} = 0.273\mathrm{N/mm^2}$$



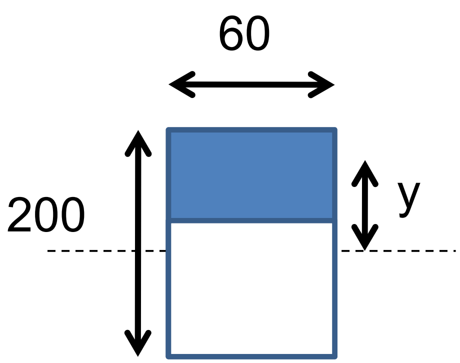
A = 60 × 50 = 3000 mm²

y = 100 – (50/2) = 75 mm

$$\tau_2 = \frac{VAy}{It} = \frac{5\times10^3\times3000\times75}{40\times10^6\times60} = 0.469\mathrm{N/mm^2}$$

Example 6 (cont.)

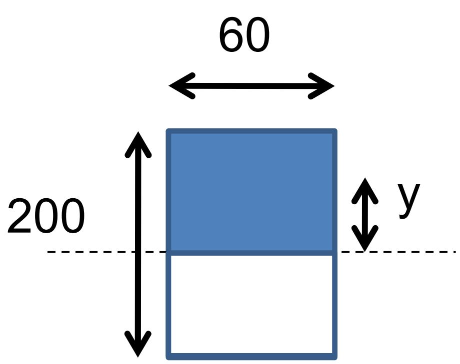
Solution:



A = 60 × 75 = 4500 mm²

y = 100 – (75/2) = 62.5 mm

$$\tau_3 = \frac{VAy}{It} = \frac{5\times10^3 \times 4500 \times 62.5}{40\times10^6 \times 60} = 0.586\text{N/mm}^2$$



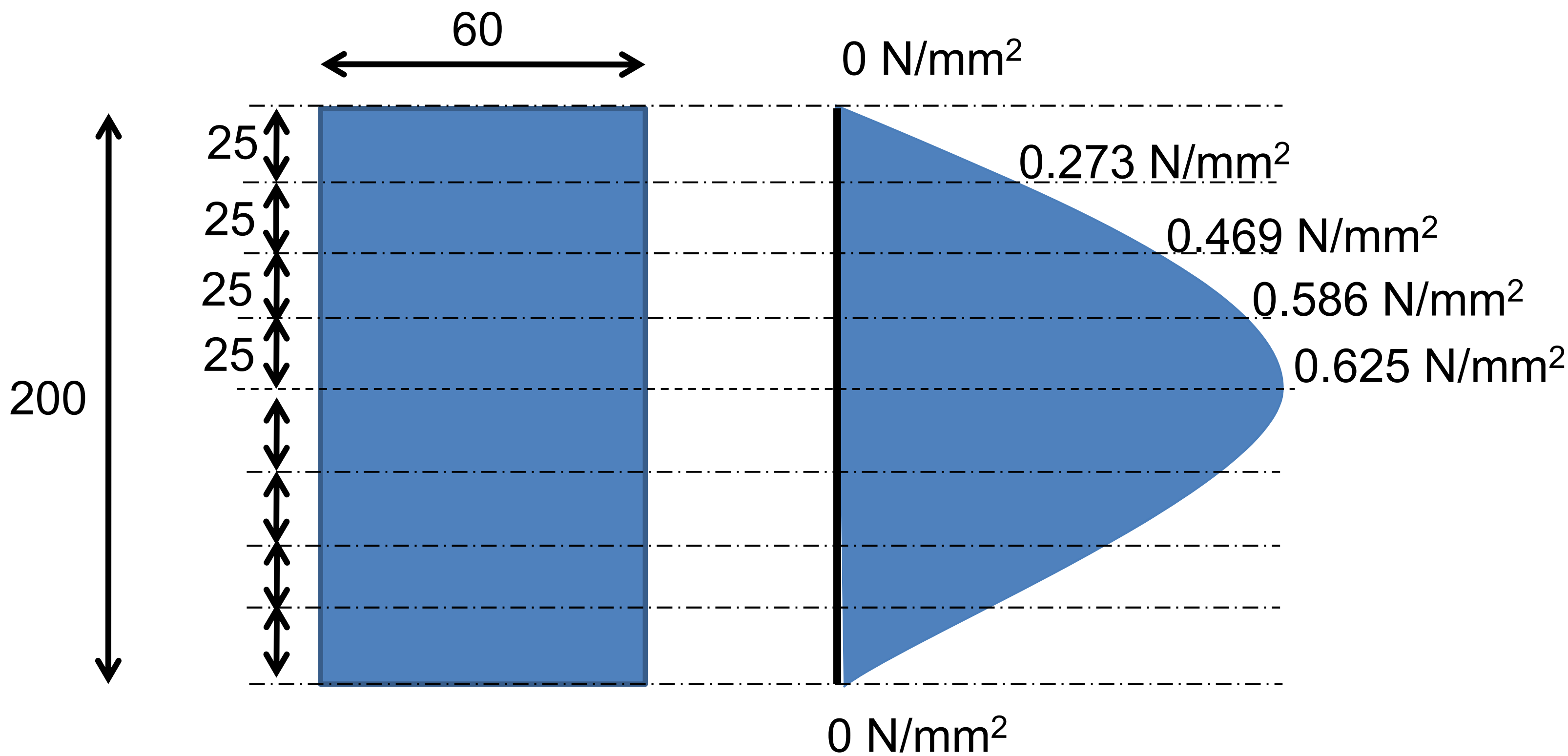
A = 60 × 100 = 6000 mm²

y = 100 – (100/2) = 50 mm

$$\tau_4 = \frac{VAy}{It} = \frac{5\times10^3 \times 6000 \times 50}{40\times10^6 \times 60} = 0.625\text{N/mm}^2$$

Example 6 (cont.)

Solution:

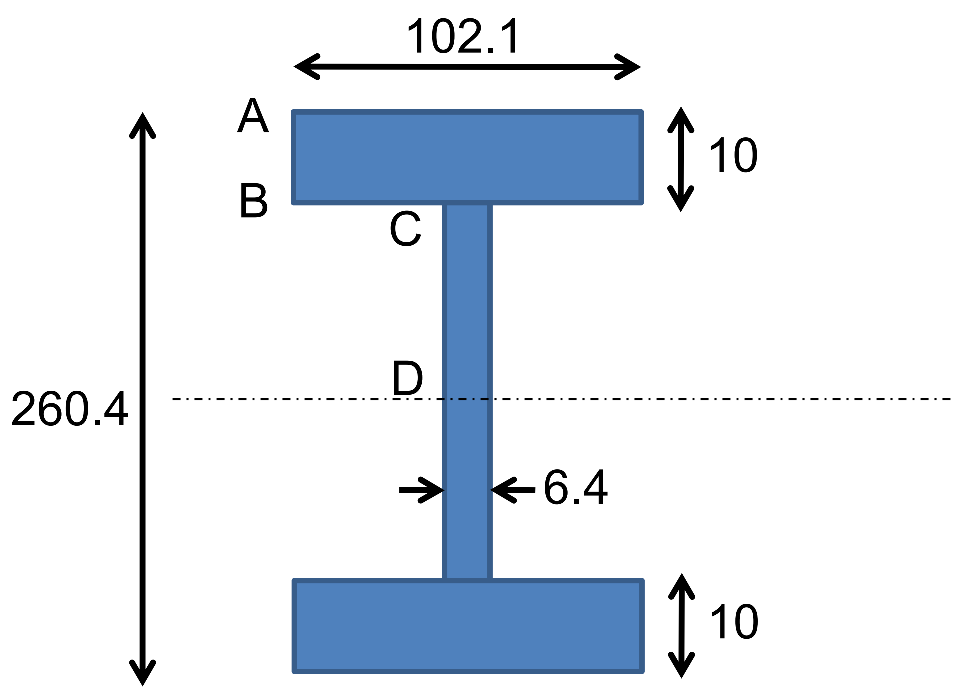


When V = 5 kN, $\tau_{max} = 0.625\ N/mm^2$

When V = 10 kN, $\tau_{max} = 1.5V/A = 1.5 \times 10 \times 10^3 / 12000 = 1.25\ N/mm^2$

Example 7

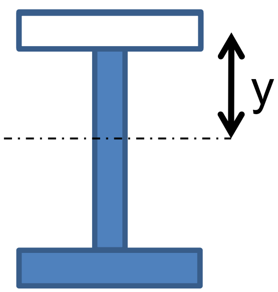
Determine the shear stress distribution over the cross section at A, B, C and D as shown in the figure. Given V = 25 kN and I = 4008 cm^4.



Example 7 (cont.)

Solution:

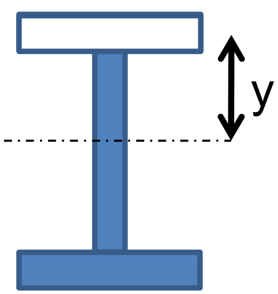
$\tau_A = 0$



A = 102.1 × 10 = 1021 mm^2

y = (260.4/2) – (10/2) = 125.2 mm

$$\tau_B = \frac{VAy}{It} = \frac{25 \times 10^3 \times 1021 \times 125.2}{40.08 \times 10^6 \times 102.1} = 0.781 \text{N/mm}^2$$



A = 102.1 × 10 = 1021 mm^2

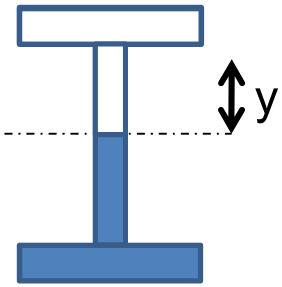
y = (260.4/2) – (10/2) = 125.2 mm

$$\tau_C = \frac{VAy}{It} = \frac{25 \times 10^3 \times 1021 \times 125.2}{40.08 \times 10^6 \times 6.4} = 12.458 \text{N/mm}^2$$

Example 7 (cont.)

Solution:

$$\tau_A = 0$$



A_1 = 102.1 × 10 = 1021 mm^2

y_1 = (260.4/2 – 10/2) = 125.2 mm

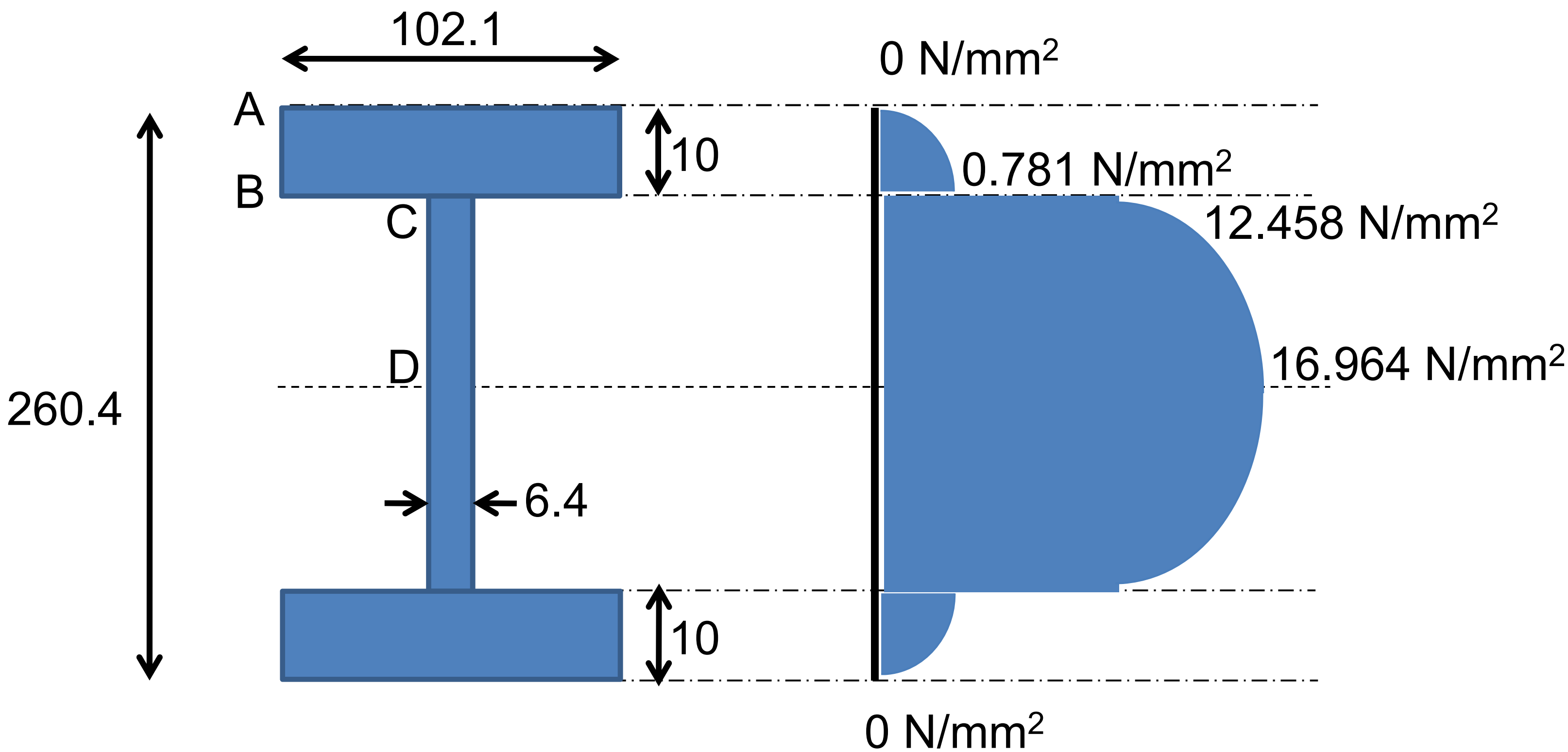
A_2 = (260.4/2 – 10) × 6.4 = 769.28 mm^2

y_2 = (260.4/2 – 10)/2 = 60.1 mm

$$\tau_D = \frac{V \sum A_i y_i}{It} = \frac{25 \times 10^3 \times (1021 \times 125.2 + 769.28 \times 60.1)}{40.08 \times 10^6 \times 6.4} = 16.964 \text{N/mm}^2$$

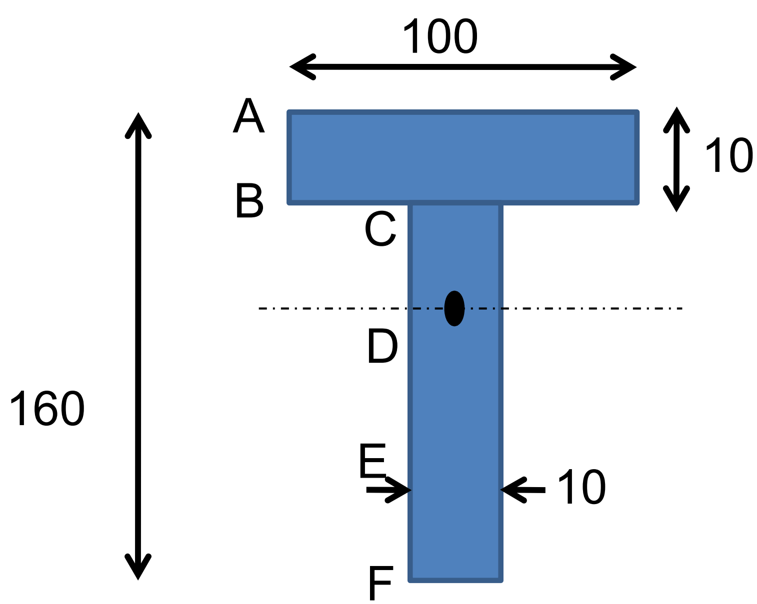
Example 7 (cont.)

Solution:



Example 8

Determine the shear stress distribution over the cross section at A, B, C, D, E and F as shown in the figure. Given V = 25 kN.



Example 8 (cont.)

Solution:

Determine the centroid,

Total Area, A = 100 × 10 + 150 × 10 = 2500 mm^2

$$y_a = \frac{(100\times10)(10/2)+(10\times150)(150/2+10)}{2500}$$

$$y_a = 53mm$$

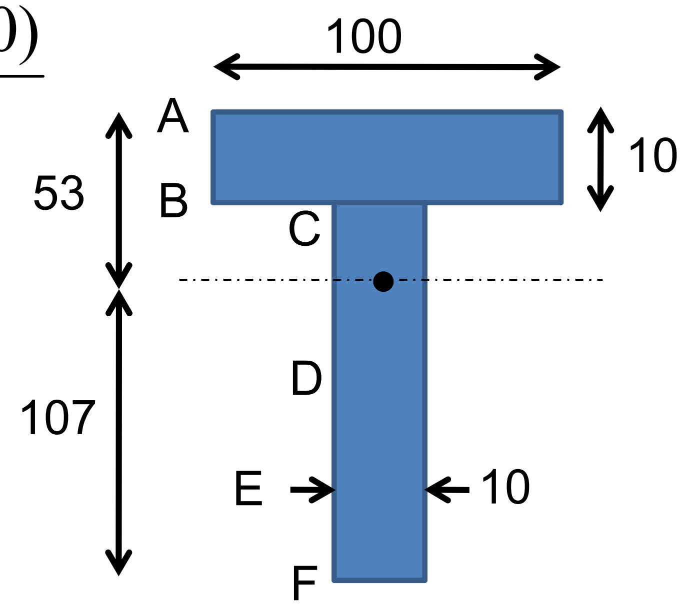
$$y_b = 160-53 = 107mm$$

$$I = bh^3/12 + Ay$$

$$I = 100\times10^3/12 + 100\times10\times(53-5)^2$$

$$+10\times150^3/12 + 10\times150\times(150/2+10-53)^2$$

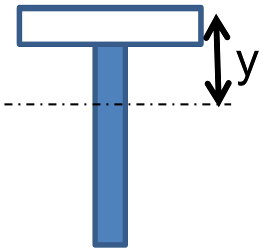
$$I = 2.31\times10^6 + 4.35\times10^6 = 6.66\times10^6\,mm^4$$



Example 8 (cont.)

Solution:

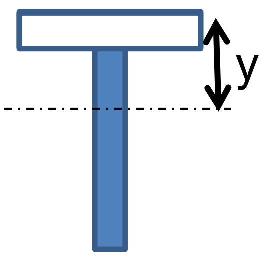
$$\tau_A = \tau_F = 0$$



A = 100 × 10 = 1000 mm²

y = (53 – 10/2) = 48 mm

$$\tau_B = \frac{VAy}{It} = \frac{25\times10^3\times100\times10\times48}{6.66\times10^6\times100} = 1.80\text{N/mm}^2$$



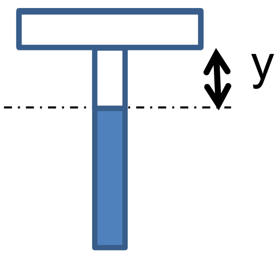
A = 100 × 10 = 1000 mm²

y = (53 – 10/2) = 48 mm

$$\tau_C = \frac{VAy}{It} = \frac{25\times10^3\times100\times10\times48}{6.66\times10^6\times10} = 18.02\text{N/mm}^2$$

Example 8 (cont.)

Solution:



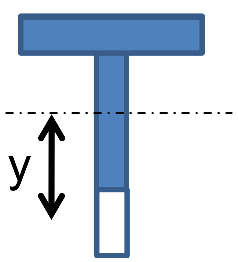
$A_1 = 100 \times 10 = 1000\ mm^2$

$y_1 = (53 – 10/2) = 48\ mm$

$A_2 = (53 – 10) \times 10 = 430\ mm^2$

$y_2 = (53 – 10 – 43/2) = 21.5\ mm$

$$\tau_D = \frac{V \sum A_i y_i}{It} = \frac{25 \times 10^3 \times (1000 \times 48 + 430 \times 21.5)}{6.66 \times 10^6 \times 10} = 21.49 \mathrm{N/mm^2}$$



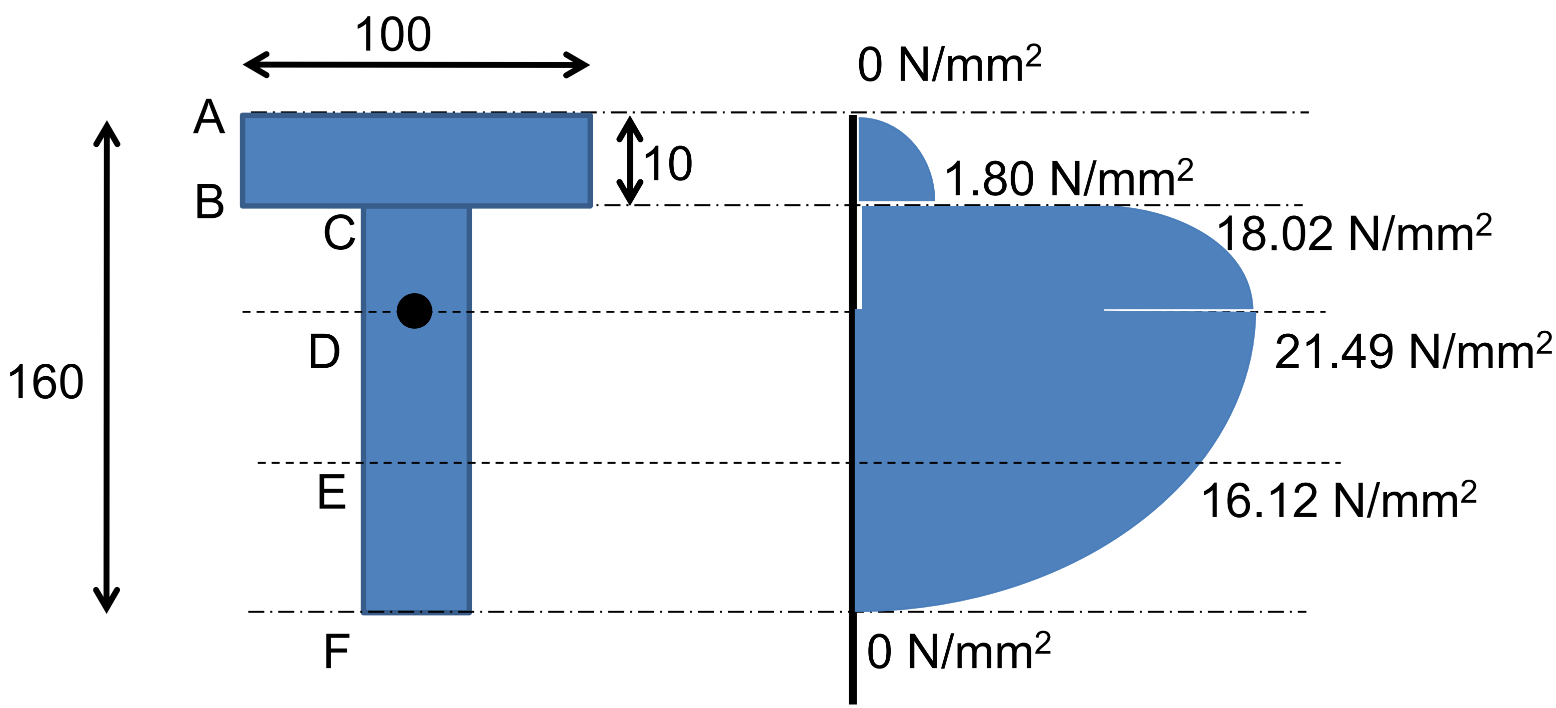
$A = 53.5 \times 10 = 535\ mm^2$

$y = (107/2 + 107/4) = 80.25\ mm$

$$\tau_E = \frac{VAy}{It} = \frac{25 \times 10^3 \times 535 \times 80.25}{6.66 \times 10^6 \times 10} = 16.12 \mathrm{N/mm^2}$$

Example 8 (cont.)

Solution:



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1. Hibbeler, R.C., Mechanics Of Materials, 8th Edition in SI units, Prentice Hall, 2011.
2. Gere dan Timoshenko, Mechanics of Materials, 3rd Edition, Chapman & Hall.
3. Yusof Ahmad, 'Mekanik Bahan dan Struktur' Penerbit UTM 2001