SAB2223 Mechanics of Materials and Structures

TOPIC 2
SHEAR FORCE AND BENDING MOMENT

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SHEAR FORCE AND BENDING MOMENT

Shear Force and Bending Moment

- Introduction
  - Types of beams
  - Effects of loading on beams
  - The force that cause shearing is known as shear force
  - The force that results in bending is known as bending moment
  - Draw the shear force and bending moment diagrams

Shear Force and Bending Moment

- Members with support loadings applied perpendicular to their longitudinal axis are called ***beams***.
- Beams classified according to the way they are supported.

Overhanging beam

Simply supported beam

Cantilevered beam

Shear Force and Bending Moment

- Types of beam
  a) Determinate Beam

  The force and moment of reactions at supports can be determined by using the 3 equilibrium equations of statics i.e

  $\Sigma F_x=0$, $\Sigma F_y=0$, and $\Sigma M=0$

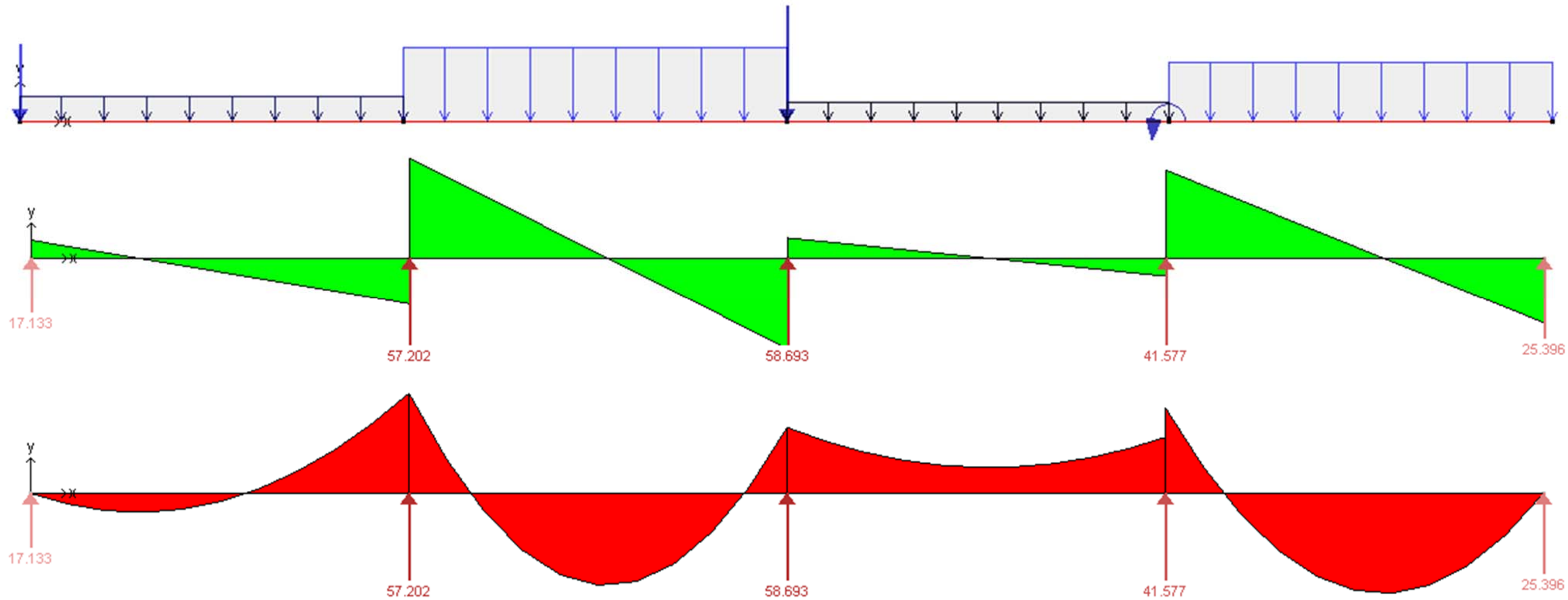
  b) Indeterminate Beam

  The force and moment of reactions at supports are more than the number of equilibrium equations of statics.

  (The extra reactions are called redundant and represent the amount of degrees of indeterminacy).

Shear Force and Bending Moment

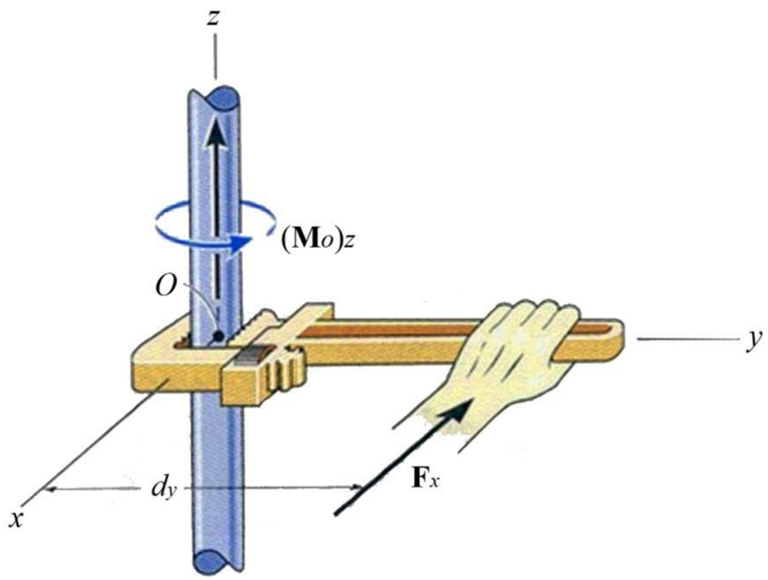
- In order to properly design a beam, it is important to know the ***variation*** of the shear and moment along its axis in order to find the points where these values are a maximum.

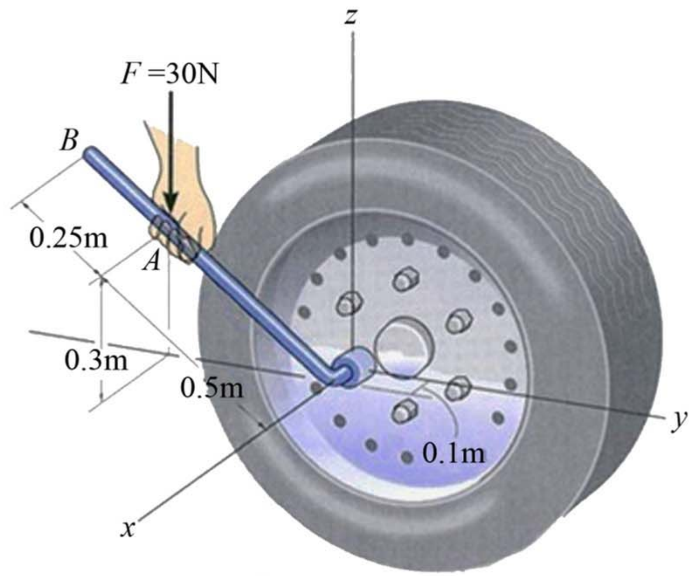


Principle of Moments

- The moment of a force indicates the tendency of a body to turn about an axis passing through a specific point O.
- The principle of moments, which is sometimes referred to as ***Varignon's Theorem*** (Varignon, 1654 – 1722) states that ***the moment of a force about a point is equal to the sum of the moments of the force's components about the point***.

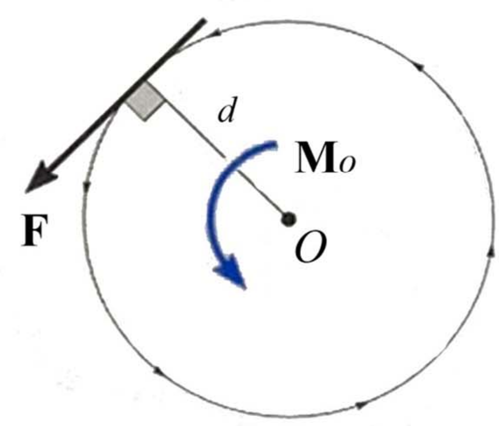
Principle of Moments





In the 2-D case, the magnitude of the moment is

$$M_o = F\ d$$

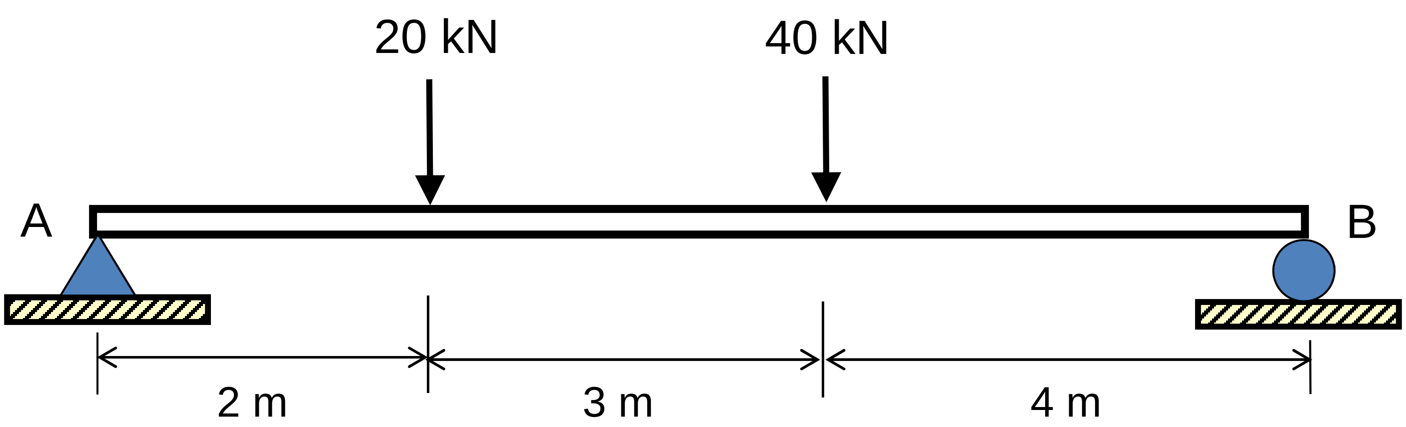


Beam's Reactions

- If a support ***prevents translation*** of a body in a particular direction, then the support exerts a ***force*** on the body in that direction.
- Determined using $\Sigma F_x=0$, $\Sigma F_y=0$, and $\Sigma M=0$

Example 1:

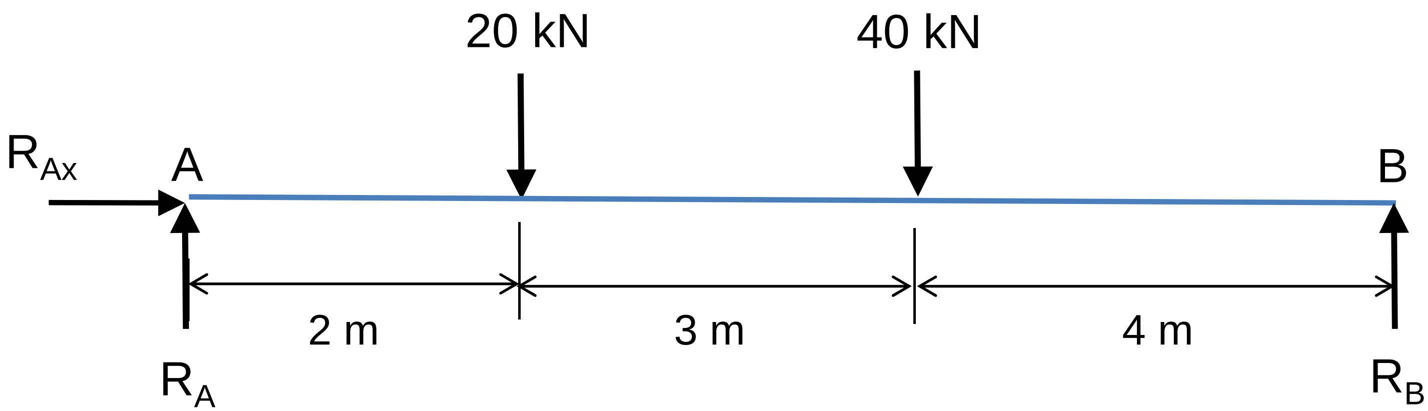
The beam shown below is supported by a pin at A and roller at B. Calculate the reactions at both supports due to the loading.



Example 1

Solution

Draw free body diagram



By taking the moment at B,

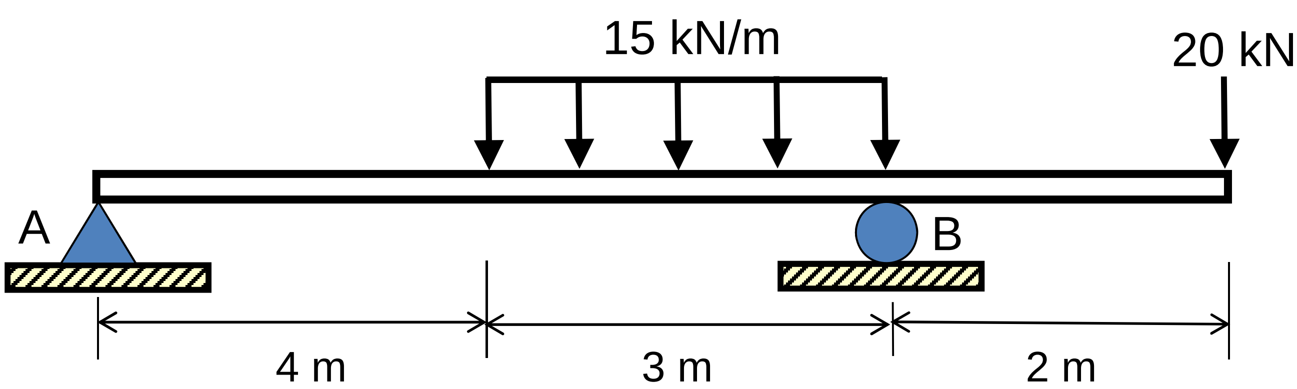
$$\Sigma M_B = 0$$
$$R_A \times 9 - 20 \times 7 - 40 \times 4 = 0$$
$$9R_A = 140 + 160$$
$$R_A = 33.3 \text{ kN}$$

$$\Sigma F_y = 0$$
$$R_A + R_B - 20 - 40 = 0$$
$$R_B = 20 + 40 - 33.3$$
$$R_B = 26.7 \text{ kN}$$

$$\Sigma F_x = 0$$
$$R_{Ax} = 0$$

Example 2

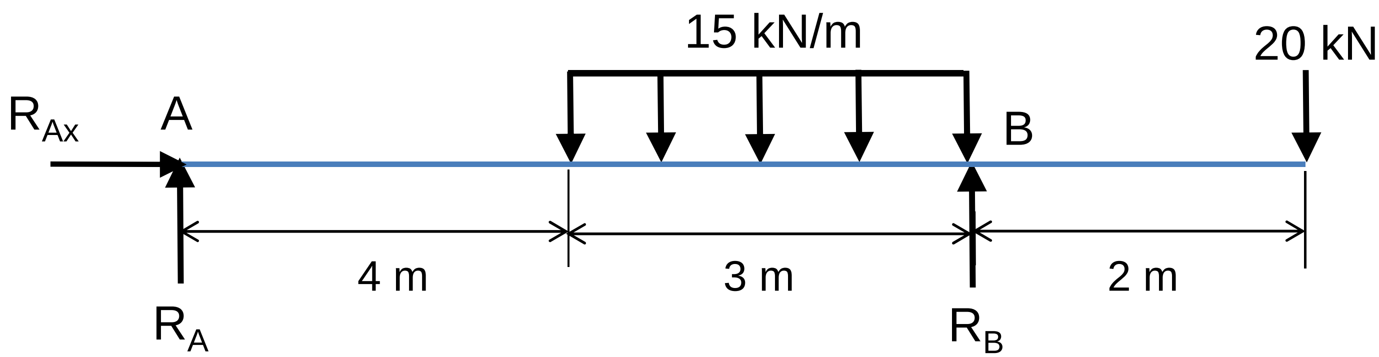
Determine the reactions at support A and B for the overhanging beam subjected to the loading as shown.



Example 2 (cont.)

Solution

Draw free body diagram



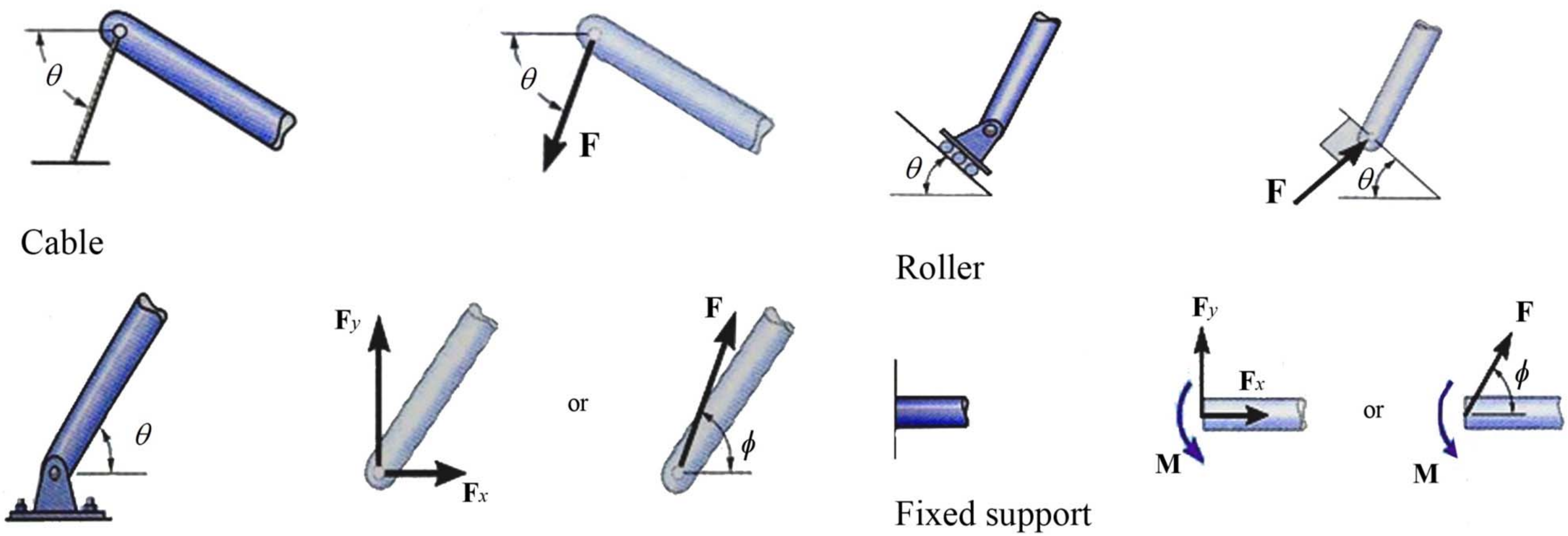
By taking the moment at A,

$$\Sigma M_A = 0$$
$$-R_B \times 7 + 20 \times 9 - (15 \times 3) \times 5.5 = 0$$
$$7R_B = 247.5 + 180$$
$$R_B = 61.07 \text{ kN}$$

$$\Sigma F_y = 0$$
$$R_A + R_B - 20 - 45 = 0$$
$$R_A = 20 + 45 - 61.07$$
$$R_A = 3.93 \text{ kN}$$

$$\Sigma F_x = 0$$
$$R_{Ax} = 0$$

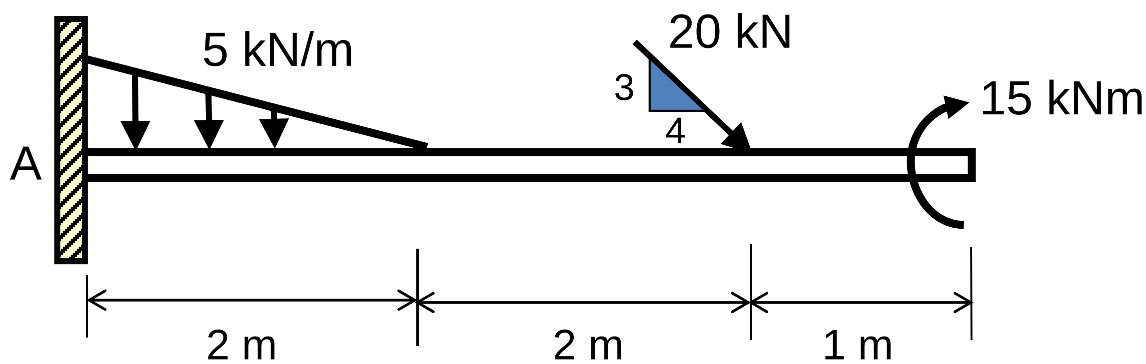
Types of Support



As a general rule, if a ***support prevents translation*** of a body in a given direction, then ***a force is developed*** on the body in the opposite direction. Similarly, if ***rotation is prevented, a couple moment*** is exerted on the body.

Example 3

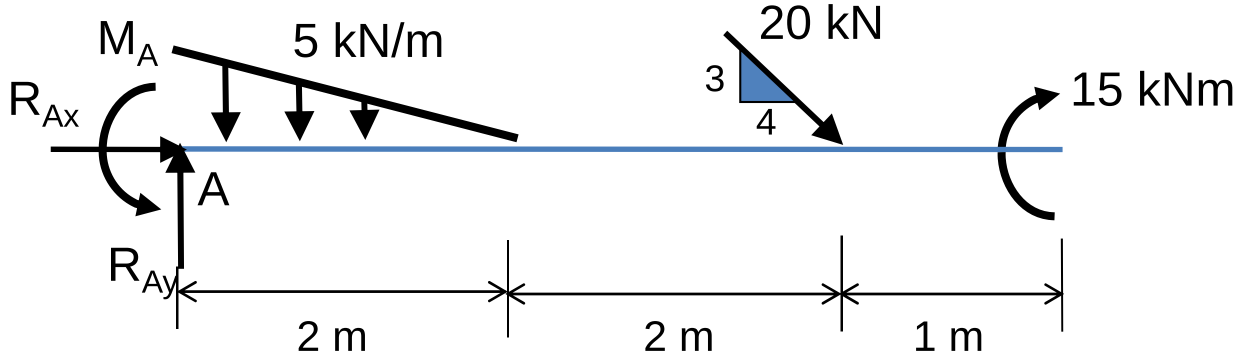
A cantilever beam is loaded as shown. Determine all reactions at support A.



Example 3 (cont.)

Solution

Draw free body diagram



$\Sigma F_x = 0$

$-R_{Ax} + 20\ (4/5) = 0$

$-R_{Ax} = 16\text{ kN}$

$\Sigma F_y = 0$

$R_{Ay} - 0.5\ (5)(2) - 20(3/5) = 0$

$R_{Ay} - 5 - 12 = 0$

$R_{Ay} = 17\text{ kN}$

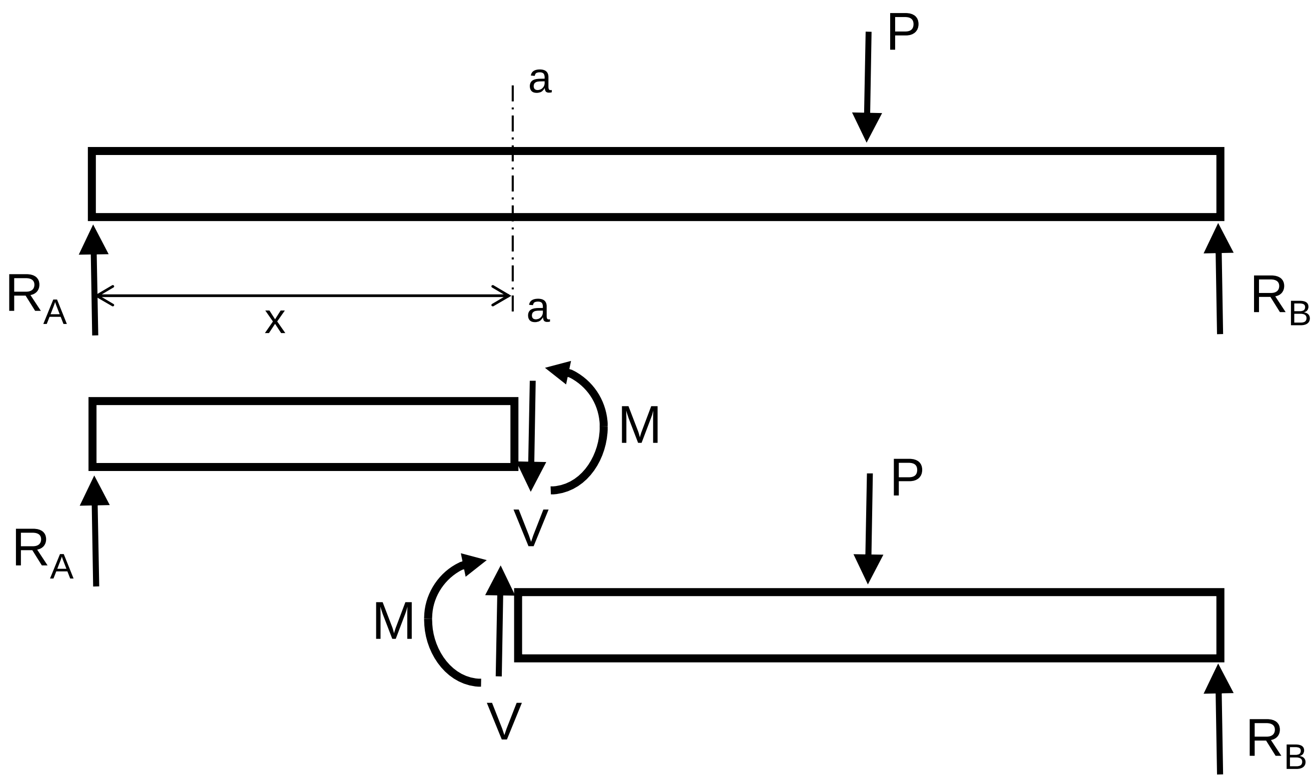
$\Sigma M_A = 0$

$-M_A + 0.5(5)(2)(1/3)(2) + 20(3/5)\ (4) + 15 = 0$

$M_A = 3.3 + 48 + 15$

$M_A = 66.3\text{ kNm}$

Shear Force and Bending Moment



V = shear force

= the force that tends to separate the member

= balances the reaction R_A

Shear Force and Bending Moment

M = bending moment

= the reaction moment at a particular point (section)

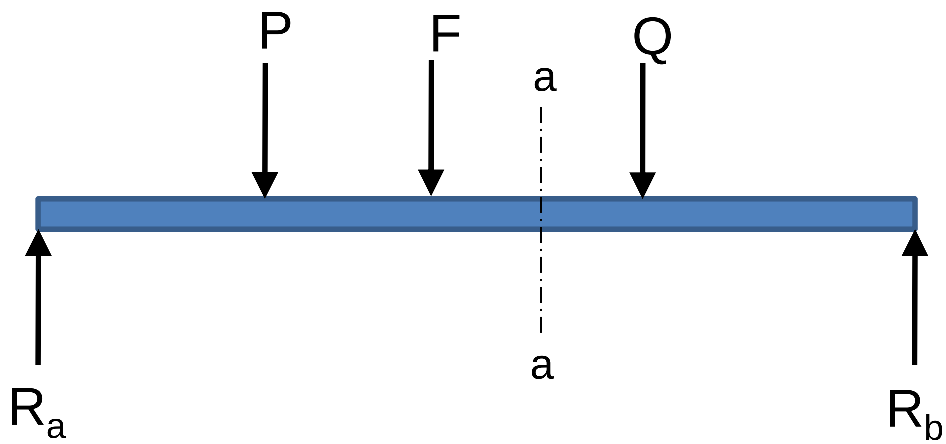
= balances the moment, $R_A.x$

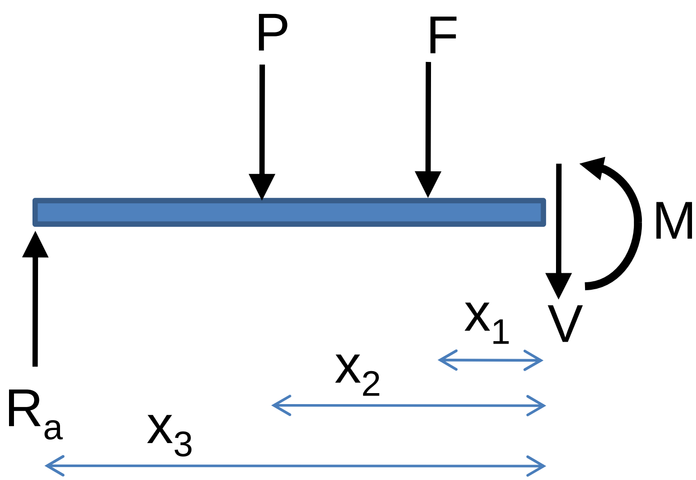
From the equilibrium equations of statics,

$+\uparrow \Sigma F_y = 0;$ $R_A - V = 0$ $\therefore V = R_A$

$+\circlearrowright \Sigma M_a = 0;$ $-M + R_A.x = 0$ $\therefore M = R_A.x$

Shear Force and Bending Moment





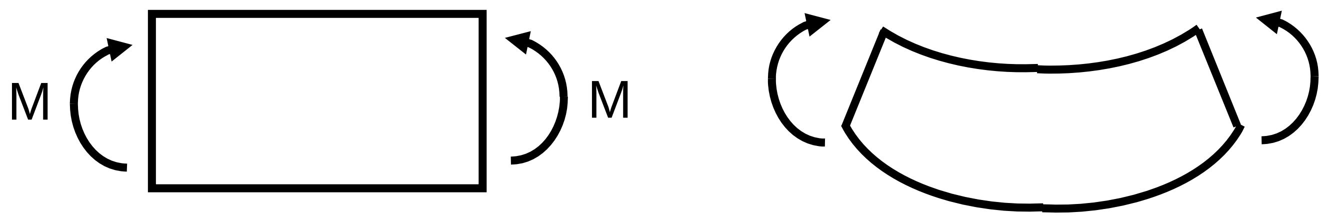
$$\Sigma F_y = 0$$
$$R_a - P - F - V = 0$$
$$V = R_a - P - F$$

$$\Sigma M_a = 0$$
$$-M - F.x_1 - P.x_2 + R_a.x_3 = 0$$
$$M = R_a.x_3 - F.x_1 - P.x_2$$

Shear Force and Bending Moment

Shape deformation due to shear force

Shape deformation due to bending moment

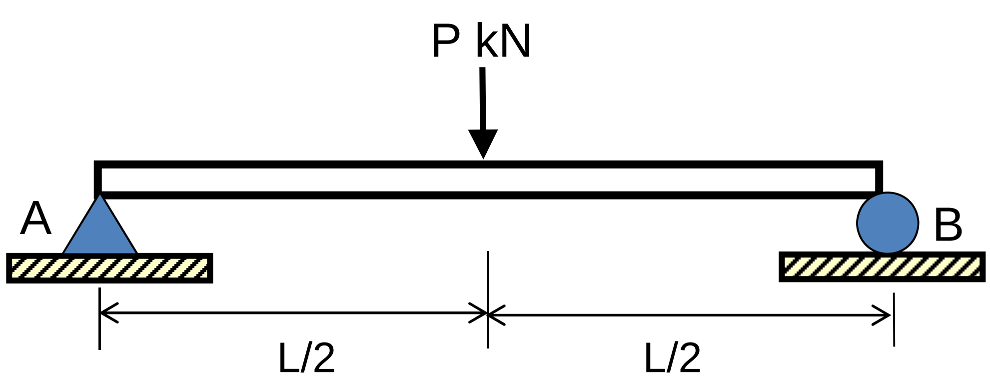


- Positive shear force diagram drawn above the beam
- Positive bending moment diagram drawn below the beam

Example 4

a) Calculate the shear force and bending moment for the beam subjected to a concentrated load as shown in the figure, and draw the shear force diagram (SFD) and bending moment diagram (BMD).

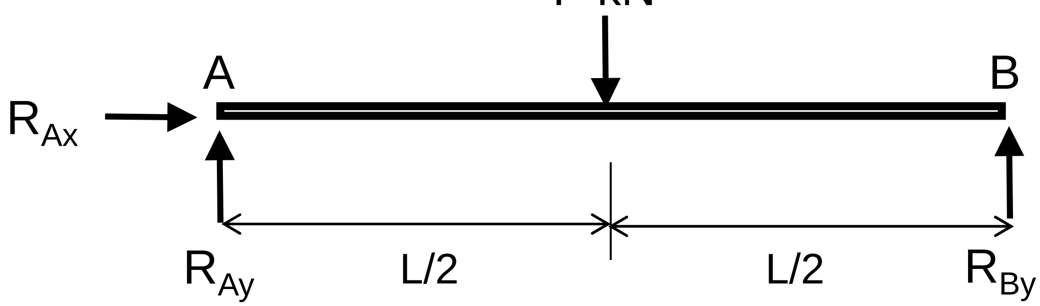
b) If P = 20 kN and L = 6 m, draw the SFD and BMD for the beam.



Example 4 (cont.)

Solution

a)



By taking the moment at A,

$\Sigma M_A = 0$

$-R_B \times L + P \times L/2 = 0$

$R_B = P/2$ kN

$\Sigma F_y = 0$

$R_A + R_B = P$

$R_A = P - P/2$

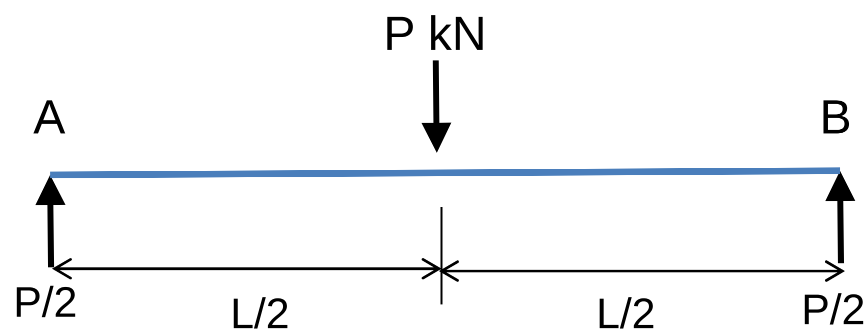
$R_A = P/2$ kN

$\Sigma F_x = 0$

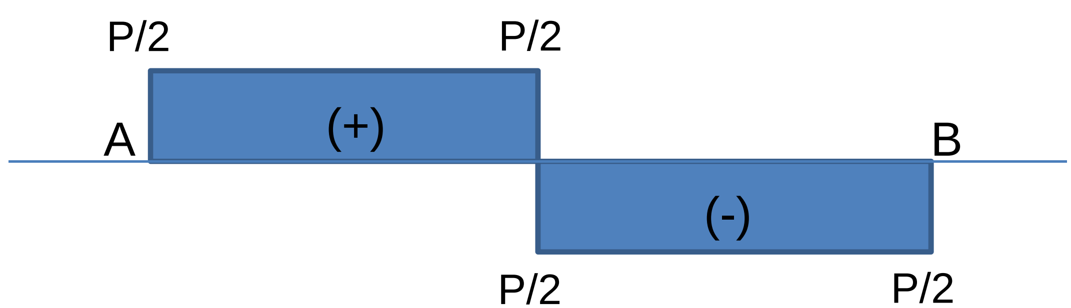
$R_{Ax} = 0$

Example 4 (cont.)

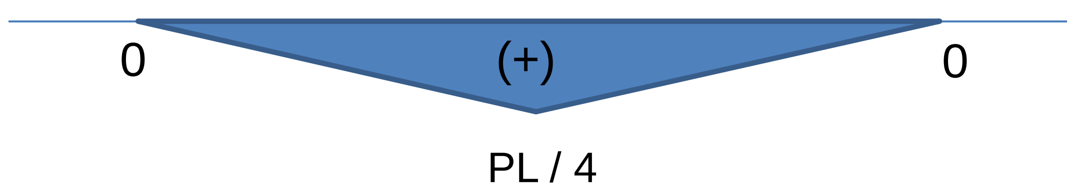
Solution (cont.)



SFD



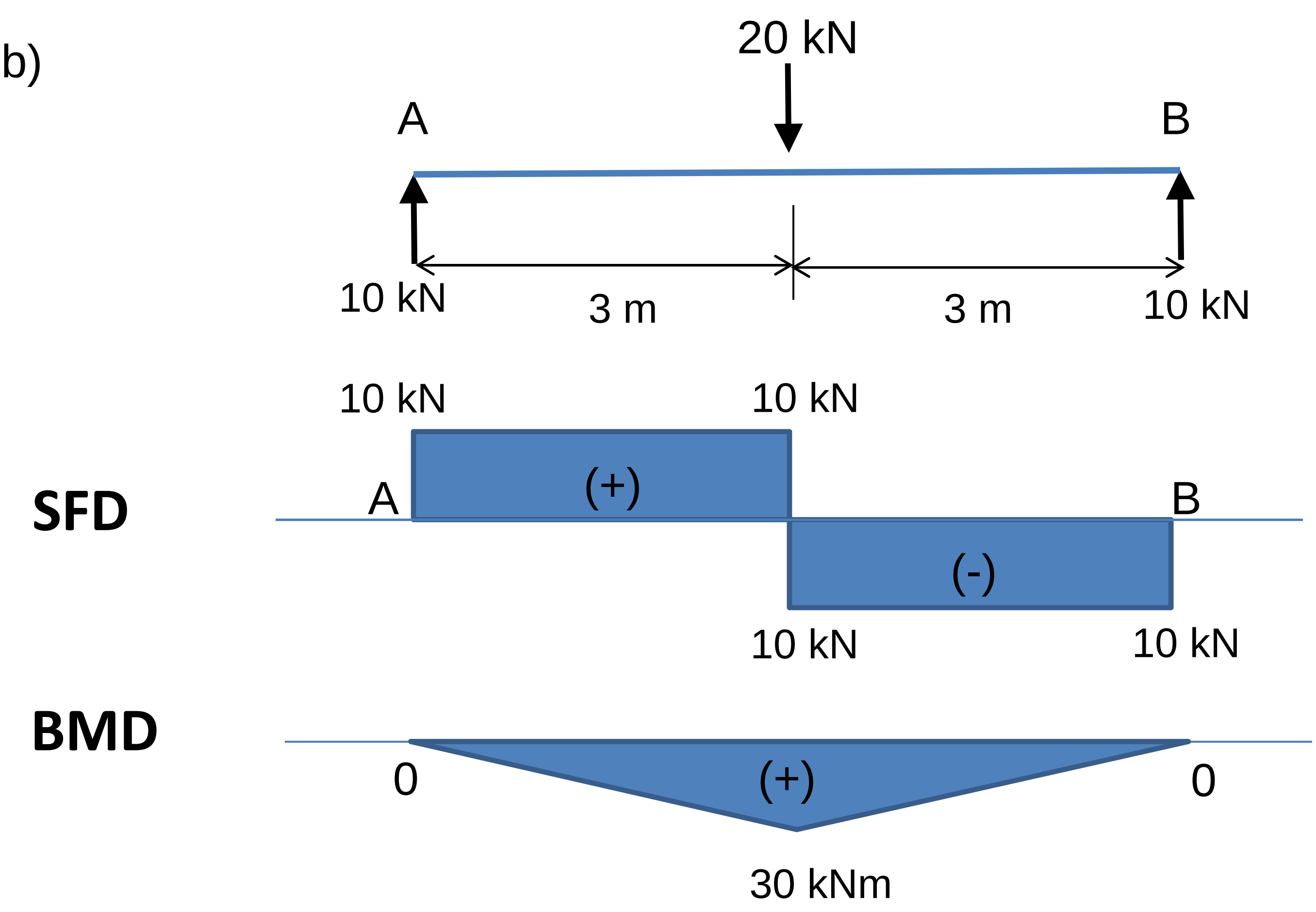
BMD



Example 4 (cont.)

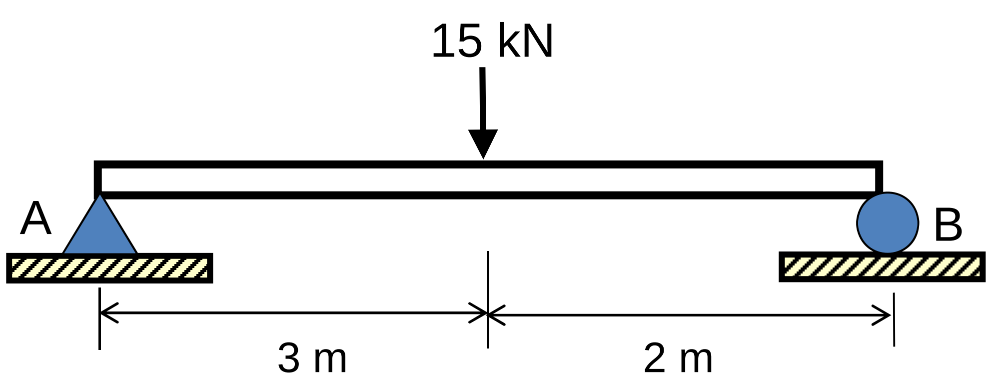
Solution (cont.)

b)



Example 5

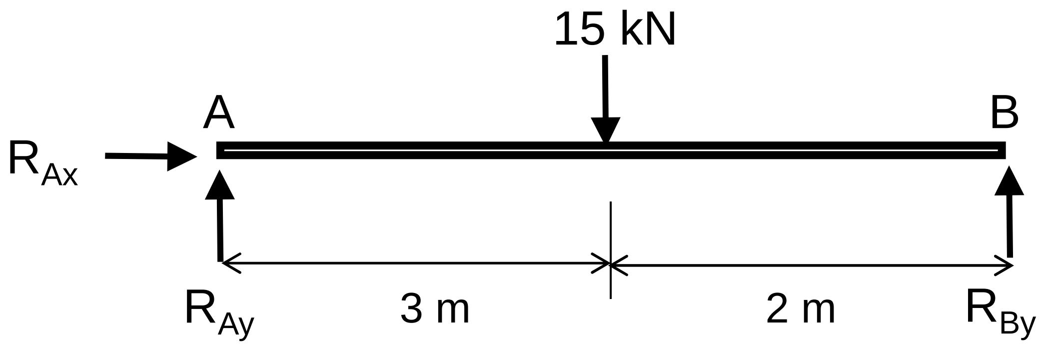
Calculate the shear force and bending moment for the beam subjected to a concentrated load as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 5 (cont.)

Solution

a)



By taking the moment at A,

$\Sigma M_A = 0$

$-R_B \times 5 + 15 \times 3 = 0$

$R_B = 9$ kN

$\Sigma F_y = 0$

$R_A + R_B = 15$

$R_A = 15 - 9$

$R_A = 6$ kN

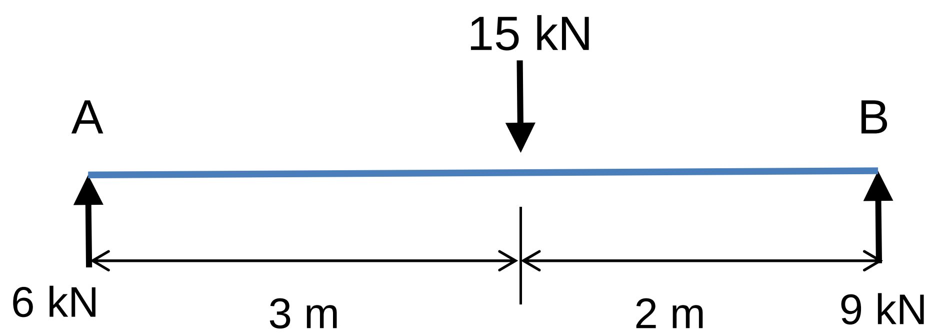
$\Sigma F_x = 0$

$R_{Ax} = 0$

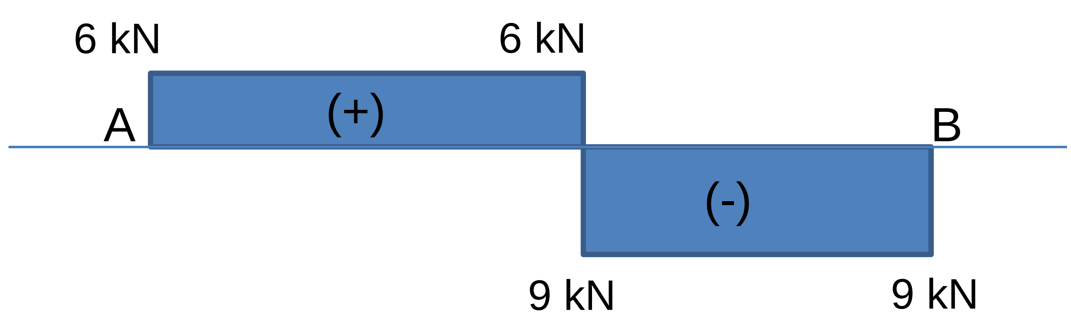
Example 5 (cont.)

Solution (cont.)

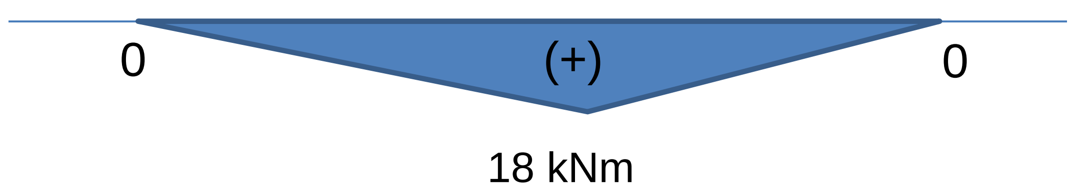
b)



SFD

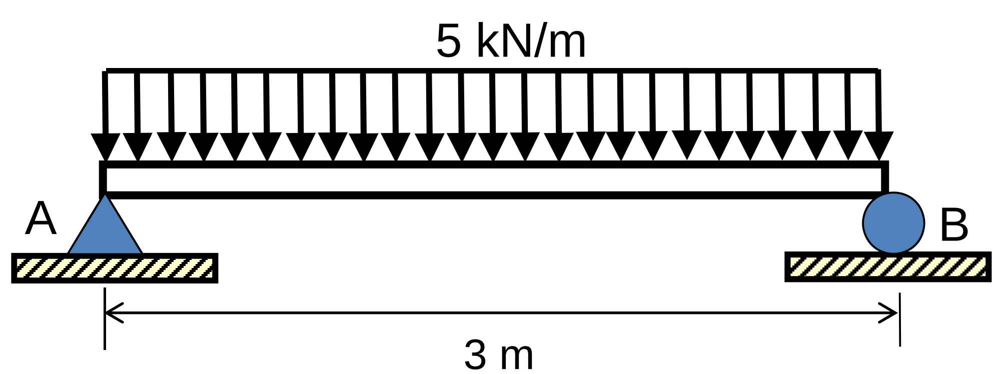


BMD



Example 6

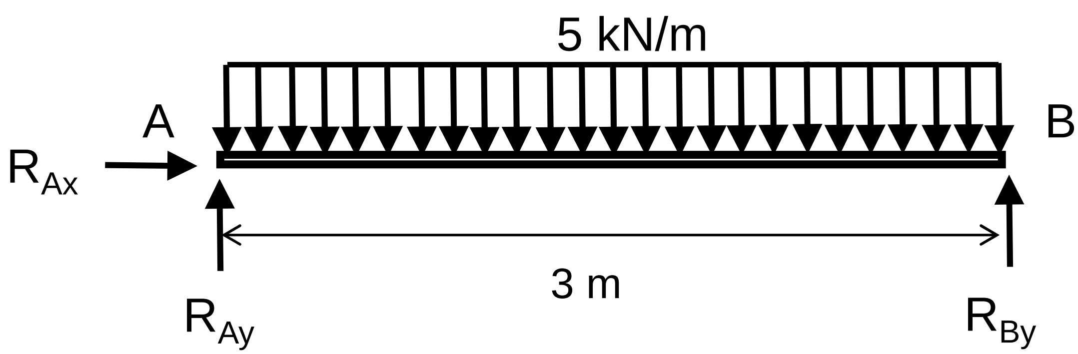
Calculate the shear force and bending moment for the beam subjected to an uniformly distributed load as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 6 (cont.)

Solution

a)



By taking the moment at A,

$\Sigma M_A = 0$

$-R_B \times 3 + 5 \times 3 \times 3/2 = 0$

$R_B = 7.5$ kN

$\Sigma F_y = 0$

$R_A + R_B = 5 \times 3$

$R_A = 15 - 7.5$

$R_A = 7.5$ kN

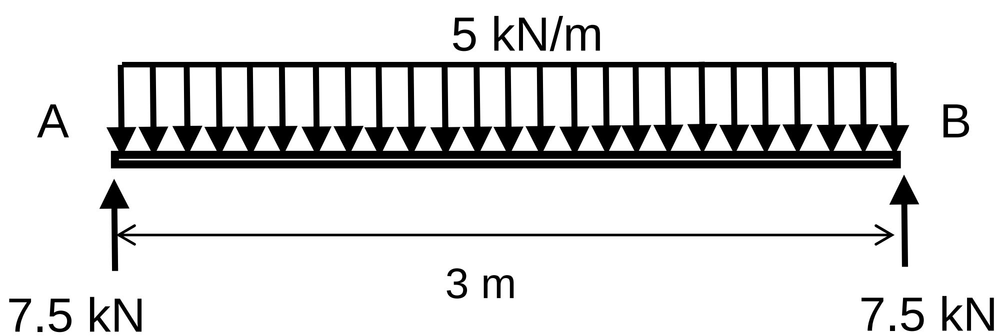
$\Sigma F_x = 0$

$R_{Ax} = 0$

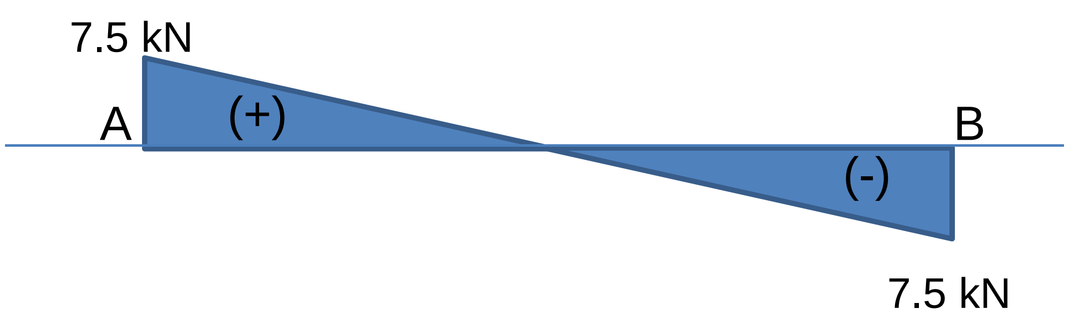
Example 6 (cont.)

Solution (cont.)

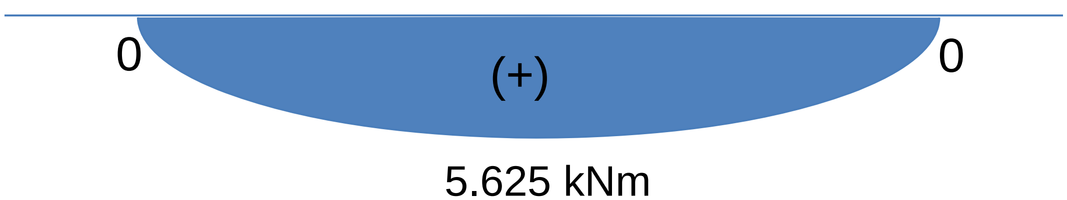
a)



SFD

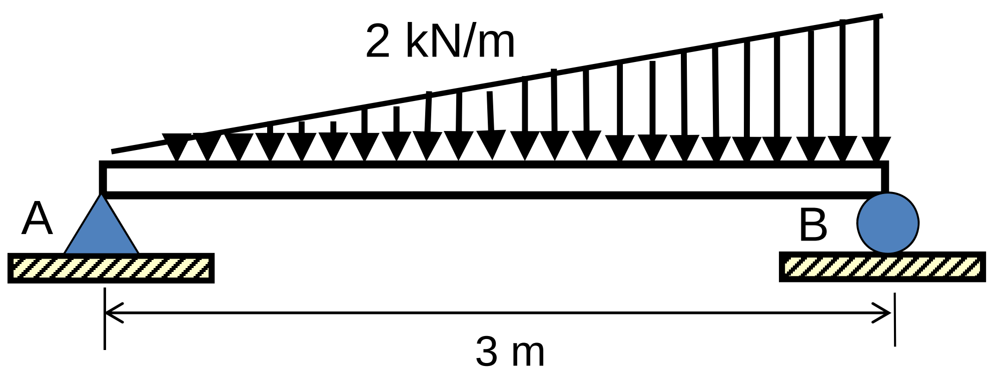


BMD



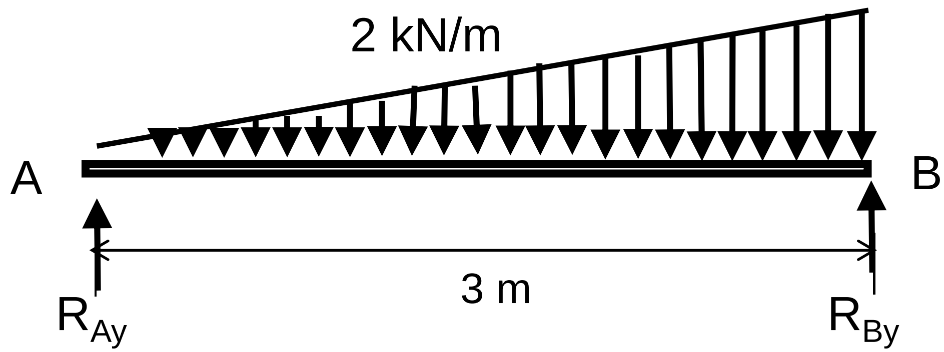
Example 7

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 7 (cont.)

Solution



By taking the moment at A,

$\Sigma M_A = 0$

$2 \times 3/2 \times 3 \times 2/3 - R_B \times 3 = 0$

$R_B = 2$ kN

$\Sigma F_y = 0$

$R_A + R_B = 2 \times 3/2$

$R_A = 3 - 2$

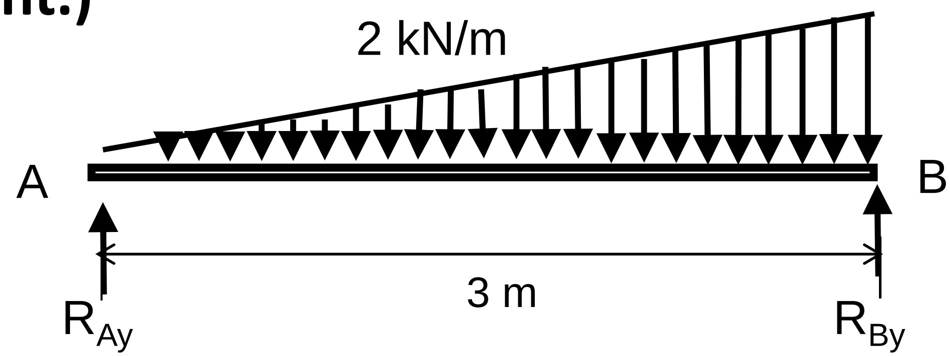
$R_A = 1$ kN

$\Sigma F_x = 0$

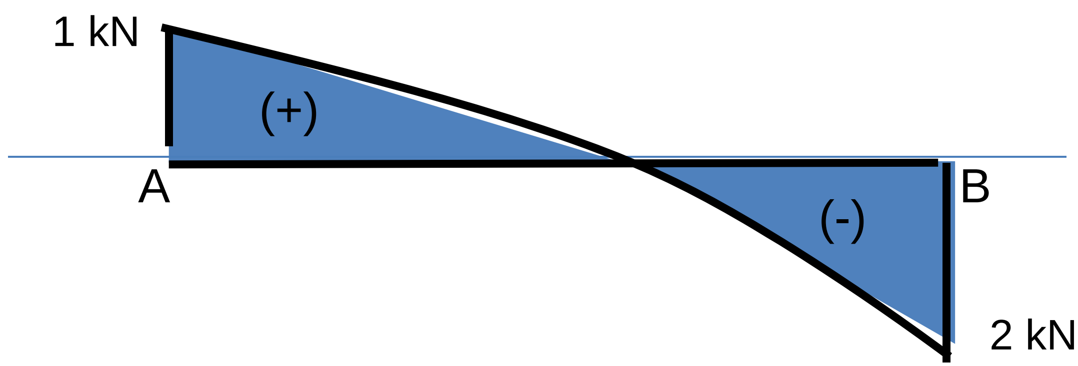
$R_{Ax} = 0$

Example 7 (cont.)

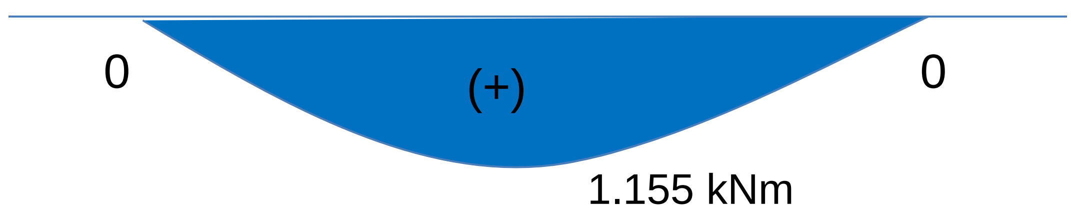
Solution (cont.)



SFD

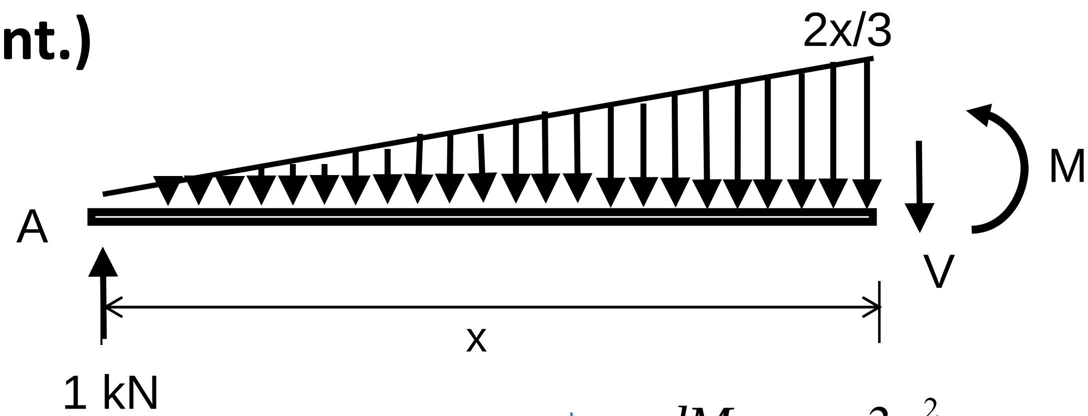


BMD



Example 7 (cont.)

Solution (cont.)



$1 - 2x/3(x)(1/2) - V = 0$

$V = 1 - 2x^2/6$

If $x = 0$, $V = 1$ kN and $x = 3$, $V = -2$ kN

$-\ M + 1 \times x - 2x/3(x)(1/2)\ (x/3) = 0$

$M = x - x^3/9$

M = maximum when $\dfrac{dM}{dx} = 0$

$$\frac{dM}{dx} = 1 - \frac{3x^2}{9} = 0$$

$$x^2 = \frac{9}{3}$$

$$x = \frac{3}{\sqrt{3}} = 1.732m$$

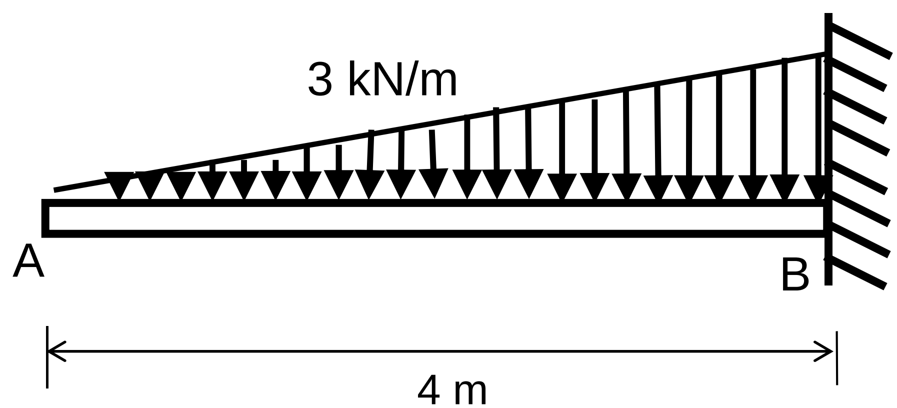
Therefore, M maximum

$M = (1.732) - (1.732)^3/9$

$M = 1.155$ kNm

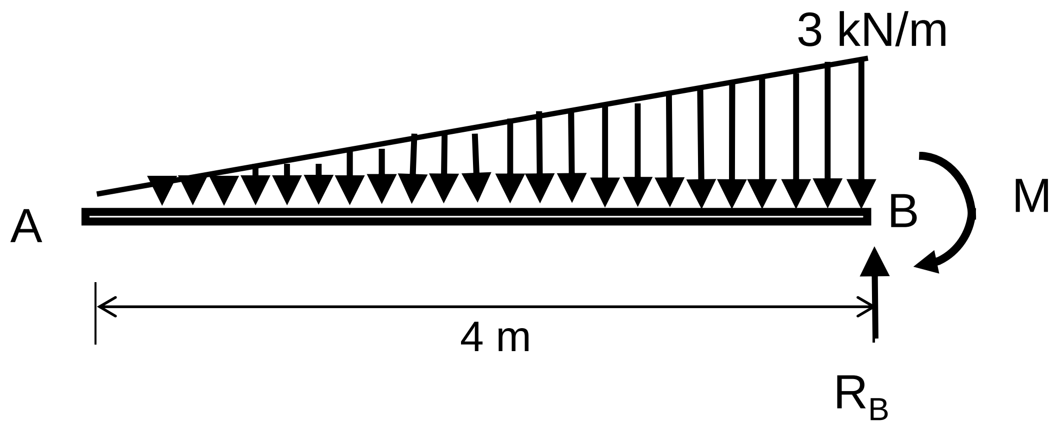
Example 8

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 8 (cont.)

Solution



By taking the moment at B,

$\Sigma M_B = 0$

$M_B = 3 \times 4/2 \times 4/3$

$M_B = 8$ kNm

$\Sigma F_y = 0$

$R_B = 3 \times 4/2$

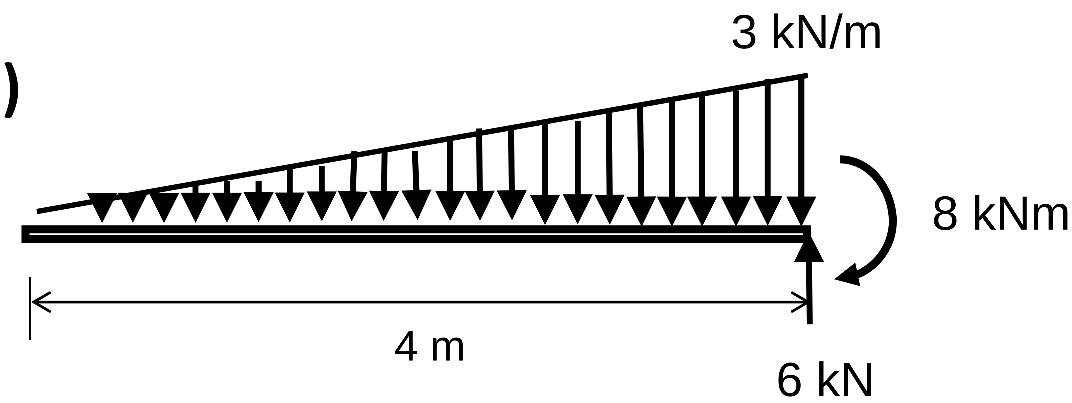
$R_B = 6$ kN

$\Sigma F_x = 0$

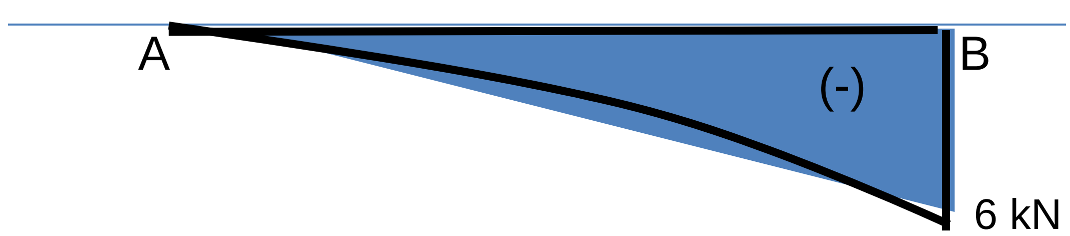
$R_{Bx} = 0$

Example 8 (cont.)

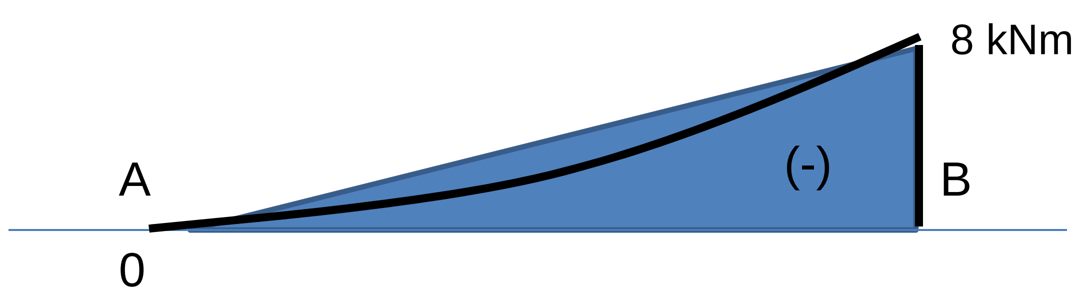
Solution (cont.)



SFD

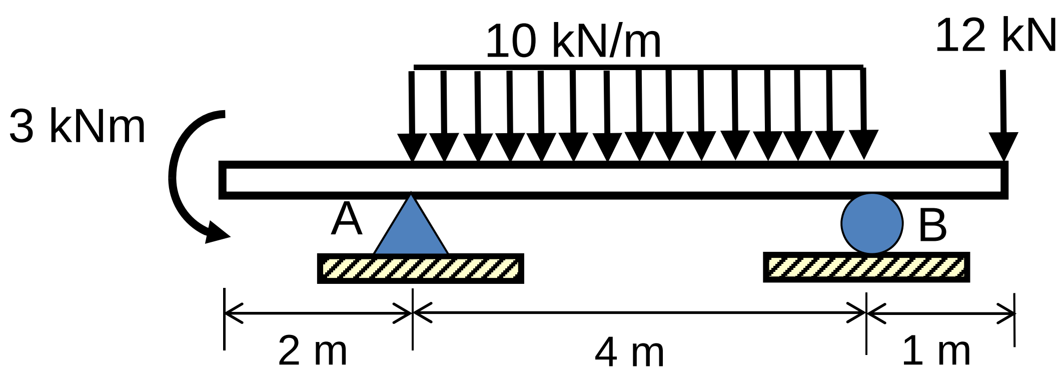


BMD



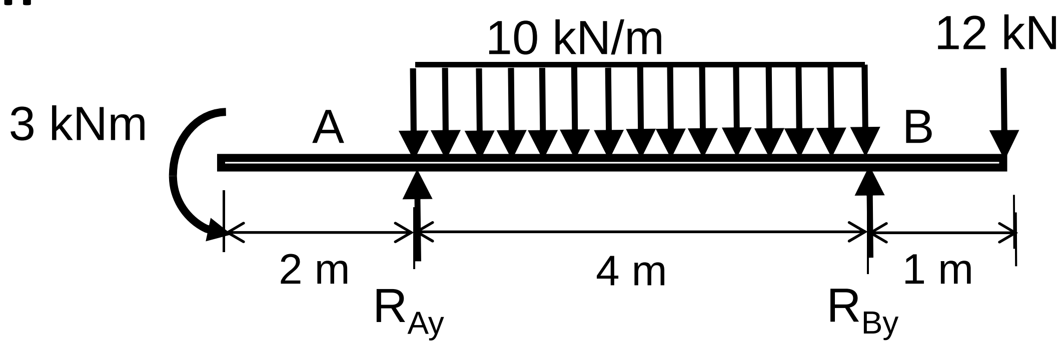
Example 9

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 9 (cont.)

Solution



By taking the moment at A,

$\Sigma M_A = 0$

$-R_B \times 4 - 3 + 10 \times 4 \times 4/2 + 12 \times 5 = 0$

$R_B = 34.25$ kN

$\Sigma F_y = 0$

$R_A + R_B = 10 \times 4 + 12$

$R_A = 52 - 34.25$

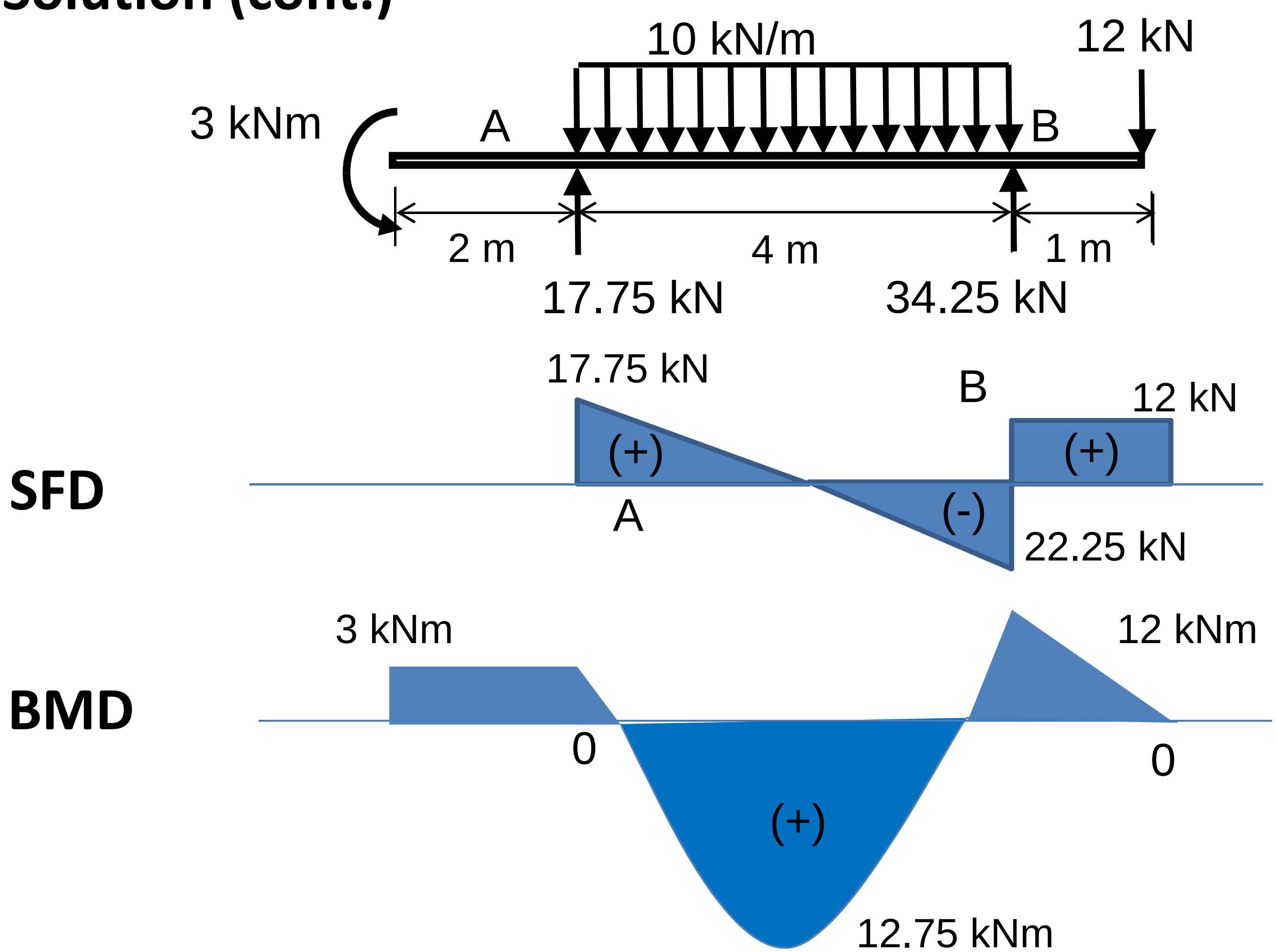
$R_A = 17.75$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

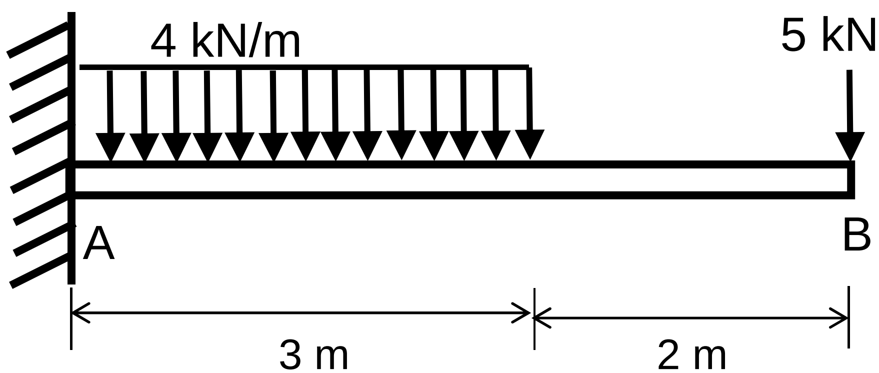
Example 9 (cont.)

Solution (cont.)



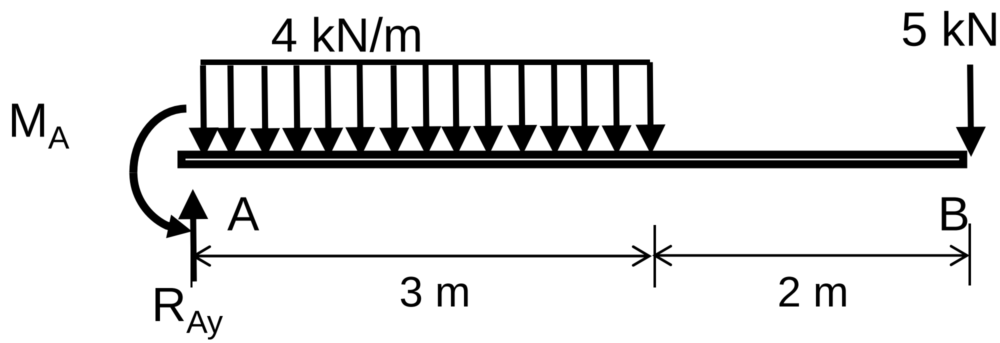
Example 10

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 10 (cont.)

Solution



By taking the moment at A,

$\Sigma M_A = 0$

$-M_A + 4 \times 3 \times 3/2 + 5 \times 5 = 0$

$M_A = 43$ kNm

$\Sigma F_y = 0$

$R_A = 4 \times 3 + 5$

$R_A = 12 + 5$

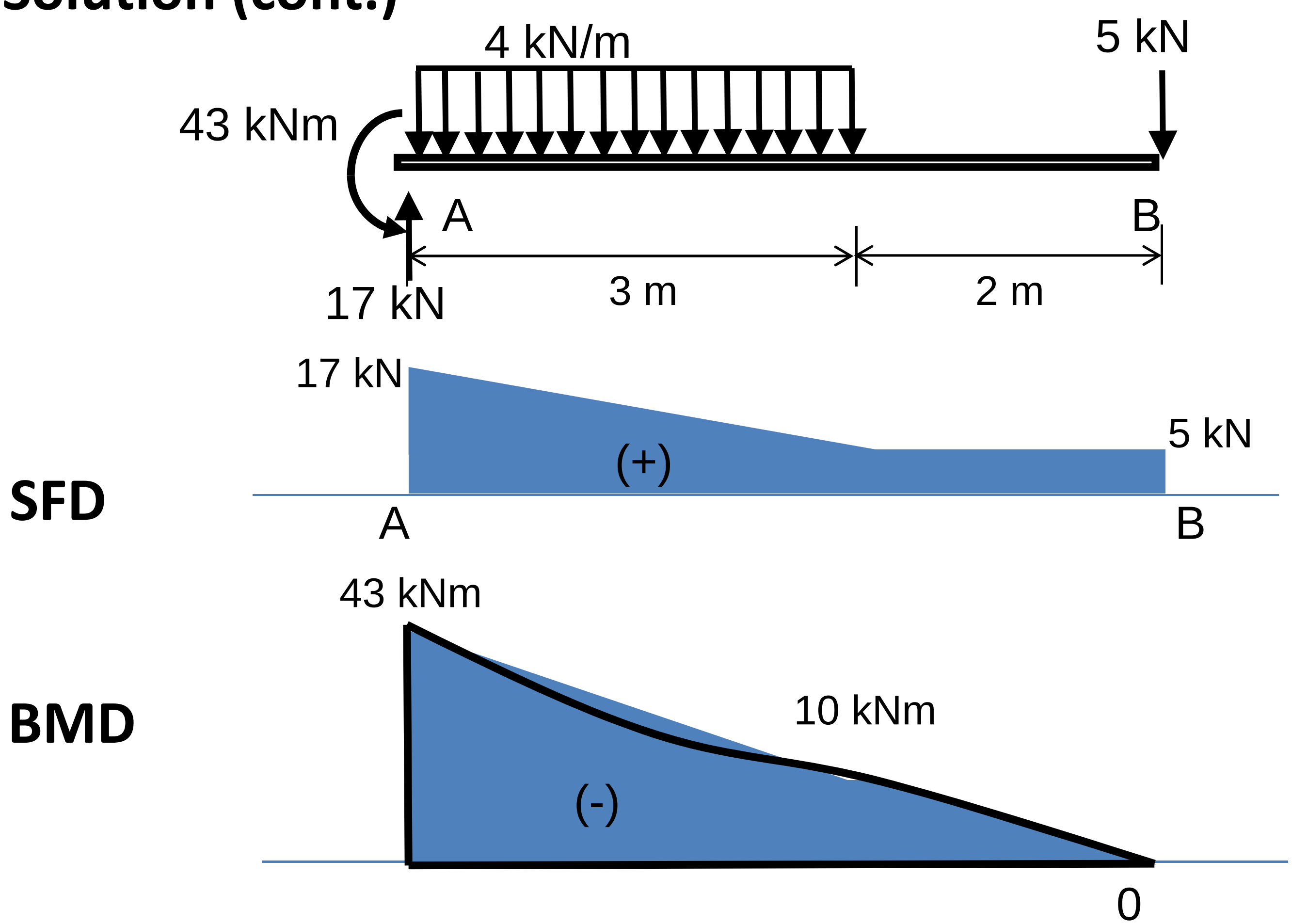
$R_A = 17$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

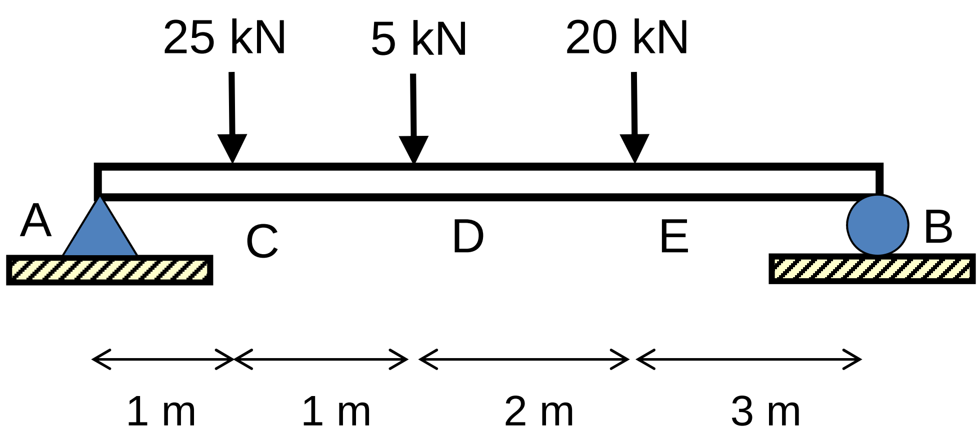
Example 10 (cont.)

Solution (cont.)



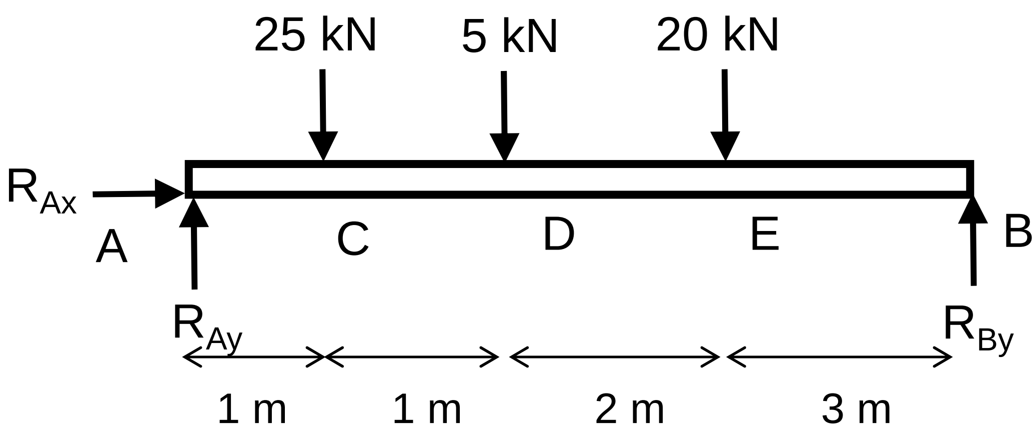
Example 11

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 11 (cont.)

Solution



By taking the moment at A,

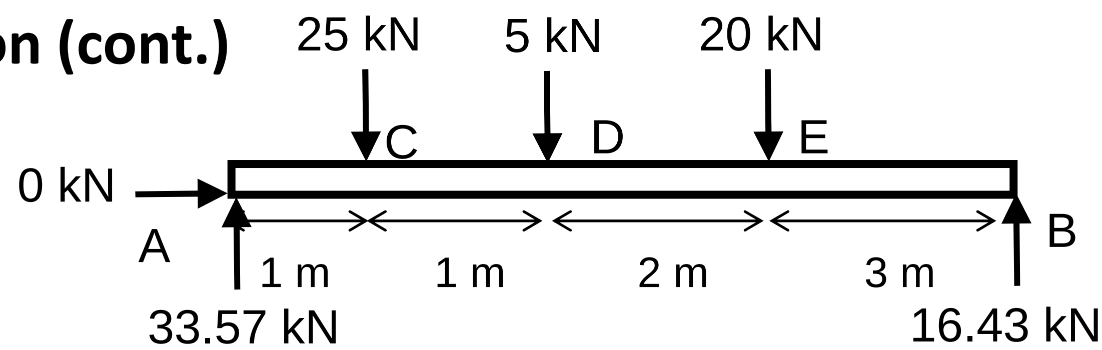
$$\Sigma M_A = 0$$
$$25 \times 1 + 5 \times 2 + 20 \times 4 - R_{By} \times 7 = 0$$
$$R_{By} = 16.43 \text{ kN}$$

$$\Sigma F_y = 0$$
$$R_{Ay} + R_{By} = 25 + 5 + 20$$
$$R_{Ay} = 50 - 16.43$$
$$R_{Ay} = 33.57 \text{ kN}$$

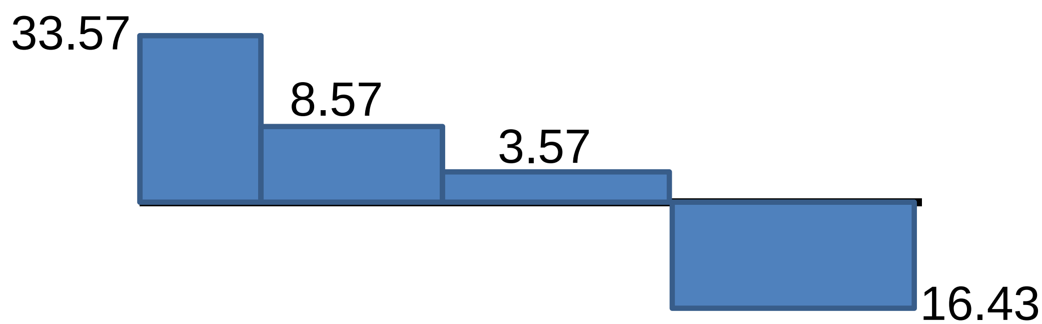
$$\Sigma F_x = 0$$
$$R_{Ax} = 0$$

Example 11 (cont.)

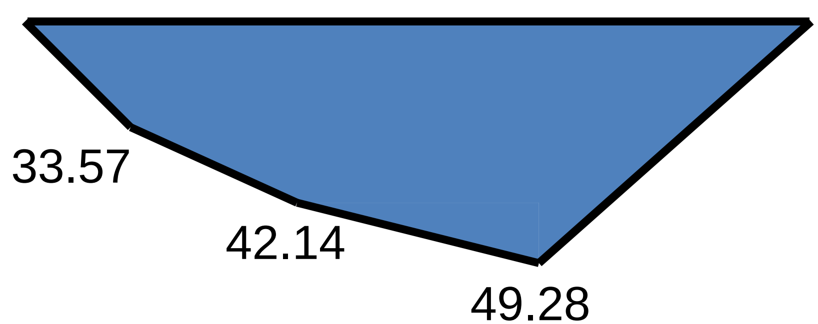
Solution (cont.)



SFD

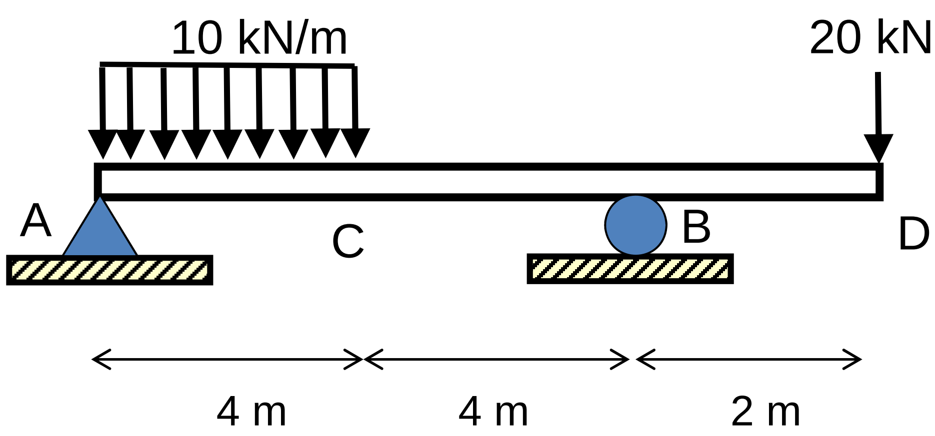


BMD



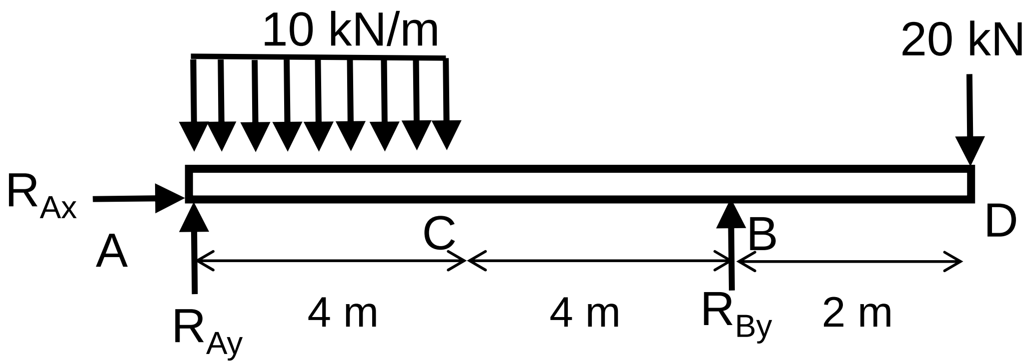
Example 12

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 12 (cont.)

Solution



By taking the moment at A,

$\Sigma M_A = 0$

$10 \times 4 \times 2 + 20 \times 10 - \mathrm{R_{By}} \times 8 = 0$

$R_{By} = 35$ kN

$\Sigma F_y = 0$

$R_{Ay} + \mathrm{R_{By}} = 10 \times 4 + 20$

$R_{Ay} = 60 - 35$

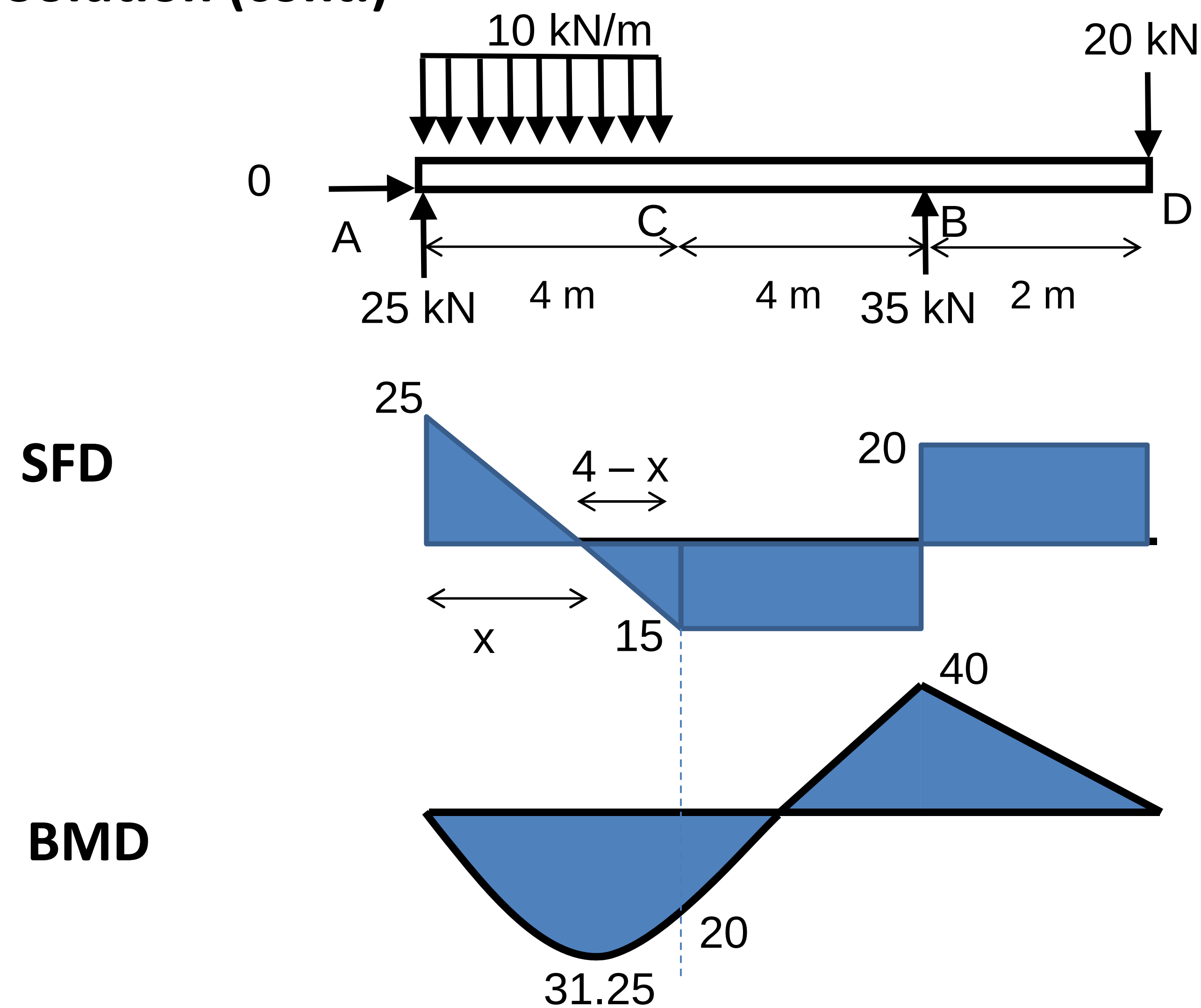
$R_{Ay} = 25$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

Example 12 (cont.)

Solution (cont.)



$$\frac{x}{25} = \frac{4-x}{15}$$

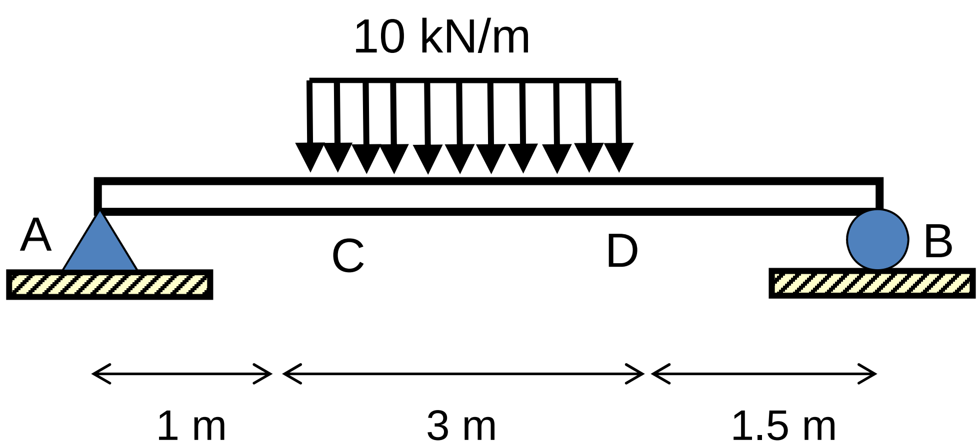
$$15x = 100 - 25x$$

$$40x = 100$$

$$x = 2.5$$

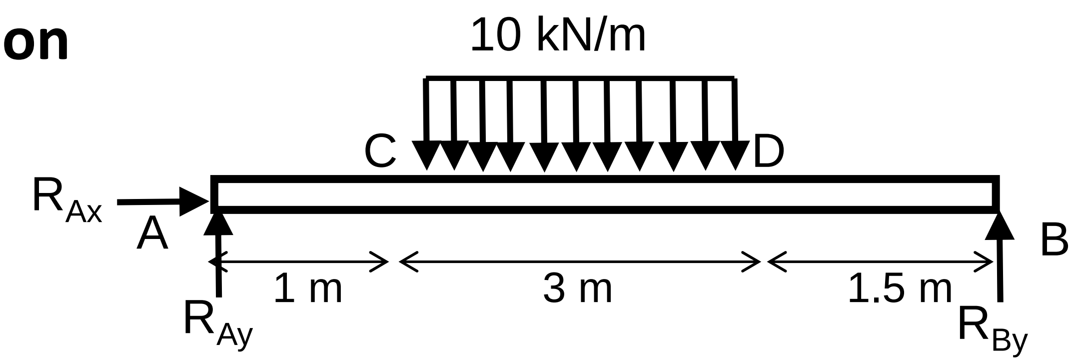
Example 13

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 13 (cont.)

Solution



By taking the moment at A,

$\Sigma M_A = 0$

$10 \times 3 \times 2.5 - R_{By} \times 5.5 = 0$

$R_{By} = 13.64$ kN

$\Sigma F_y = 0$

$R_{Ay} + R_{By} = 10 \times 3$

$R_{Ay} = 30 - 13.64$

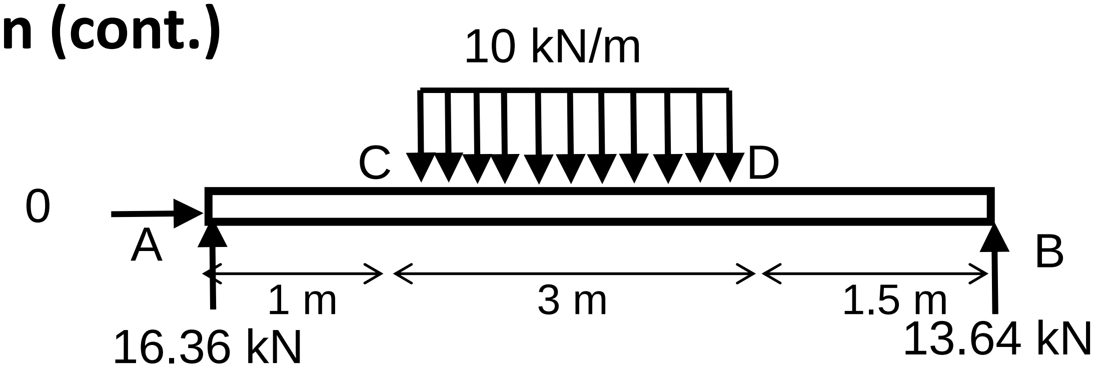
$R_{Ay} = 16.36$ kN

$\Sigma F_x = 0$

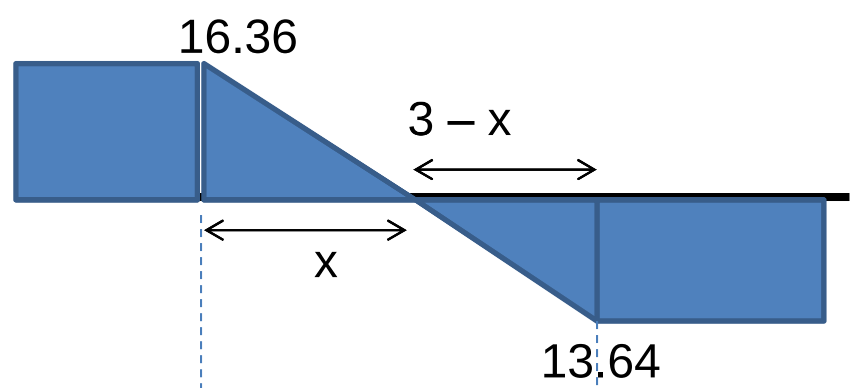
$R_{Ax} = 0$

Example 13 (cont.)

Solution (cont.)



SFD



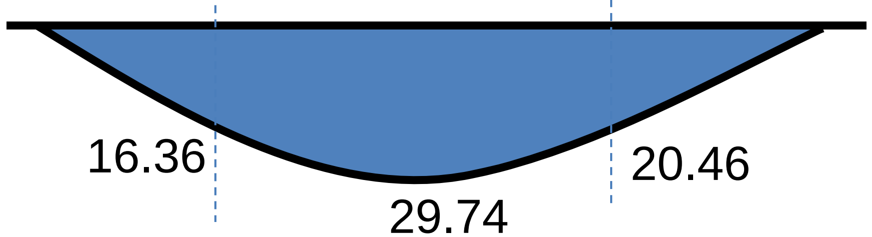
$$\frac{x}{16.36} = \frac{3-x}{13.64}$$

$$13.64x = 49.08 - 16.36x$$

$$30x = 49.08$$

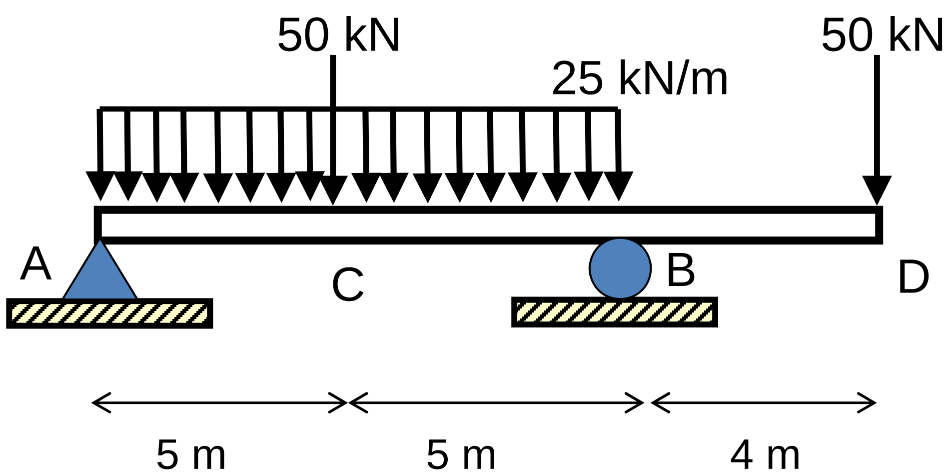
$$x = 1.636$$

BMD



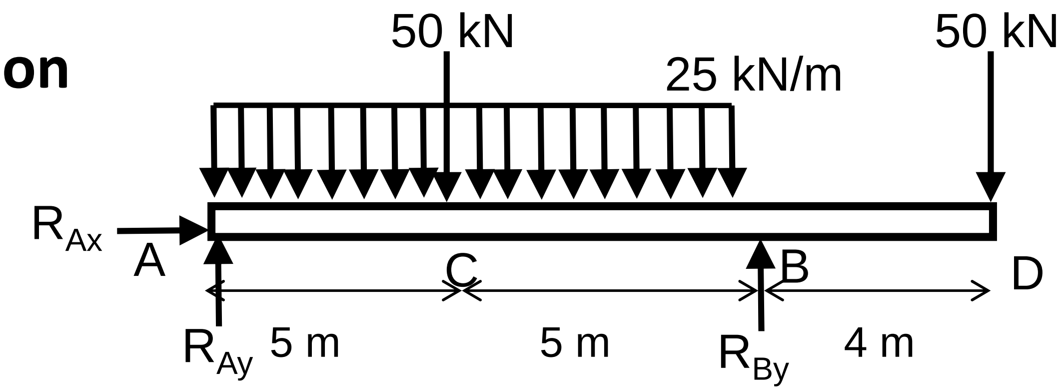
Example 14

Calculate the shear force and bending moment for the beam subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 14 (cont.)

Solution



By taking the moment at A,

$\Sigma M_A = 0$

$25 \times 10 \times 5 + 50 \times 5 + 50 \times 14 - R_{By} \times 10 = 0$

$R_{By} = 220$ kN

$\Sigma F_x = 0$

$R_{Ax} = 0$

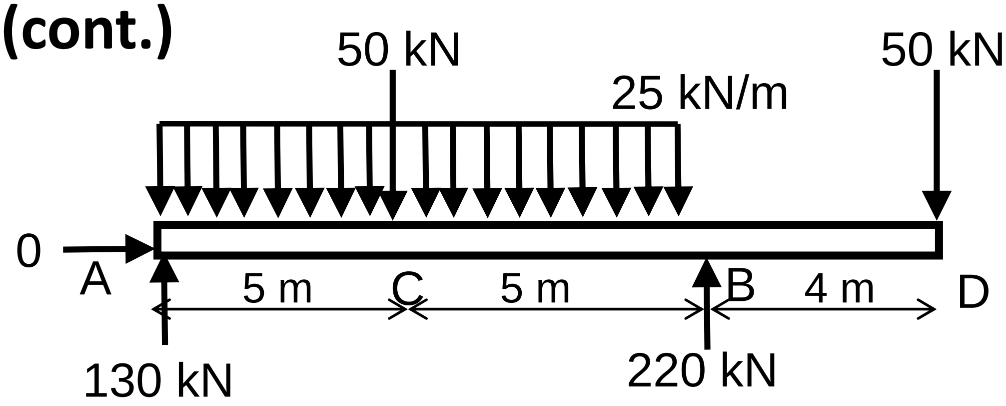
$\Sigma F_y = 0$

$R_{Ay} + R_{By} = 25 \times 10 + 50 + 50$

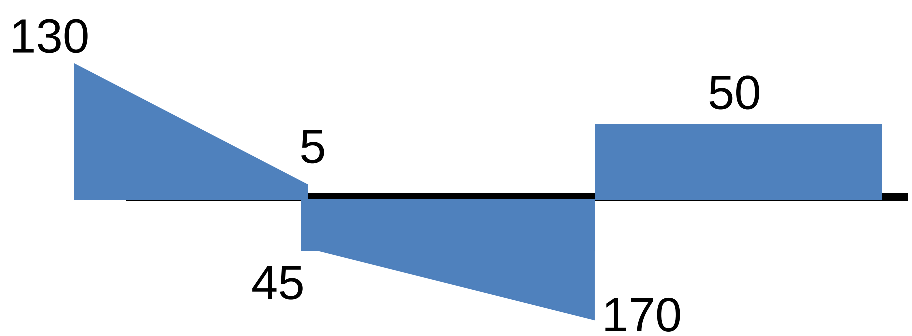
$R_{Ay} = 350 - 220 = 130$ kN

Example 14 (cont.)

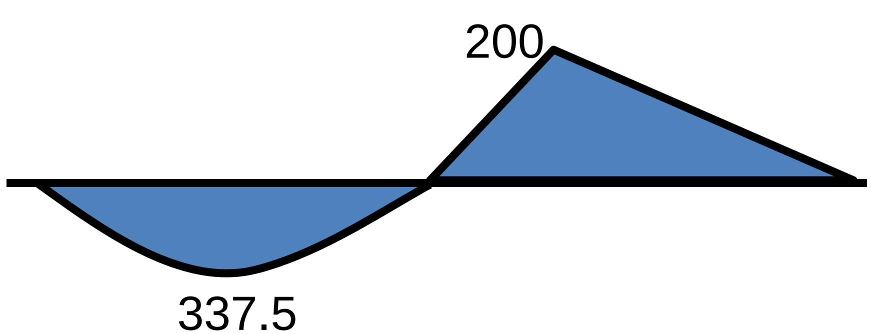
Solution (cont.)



SFD



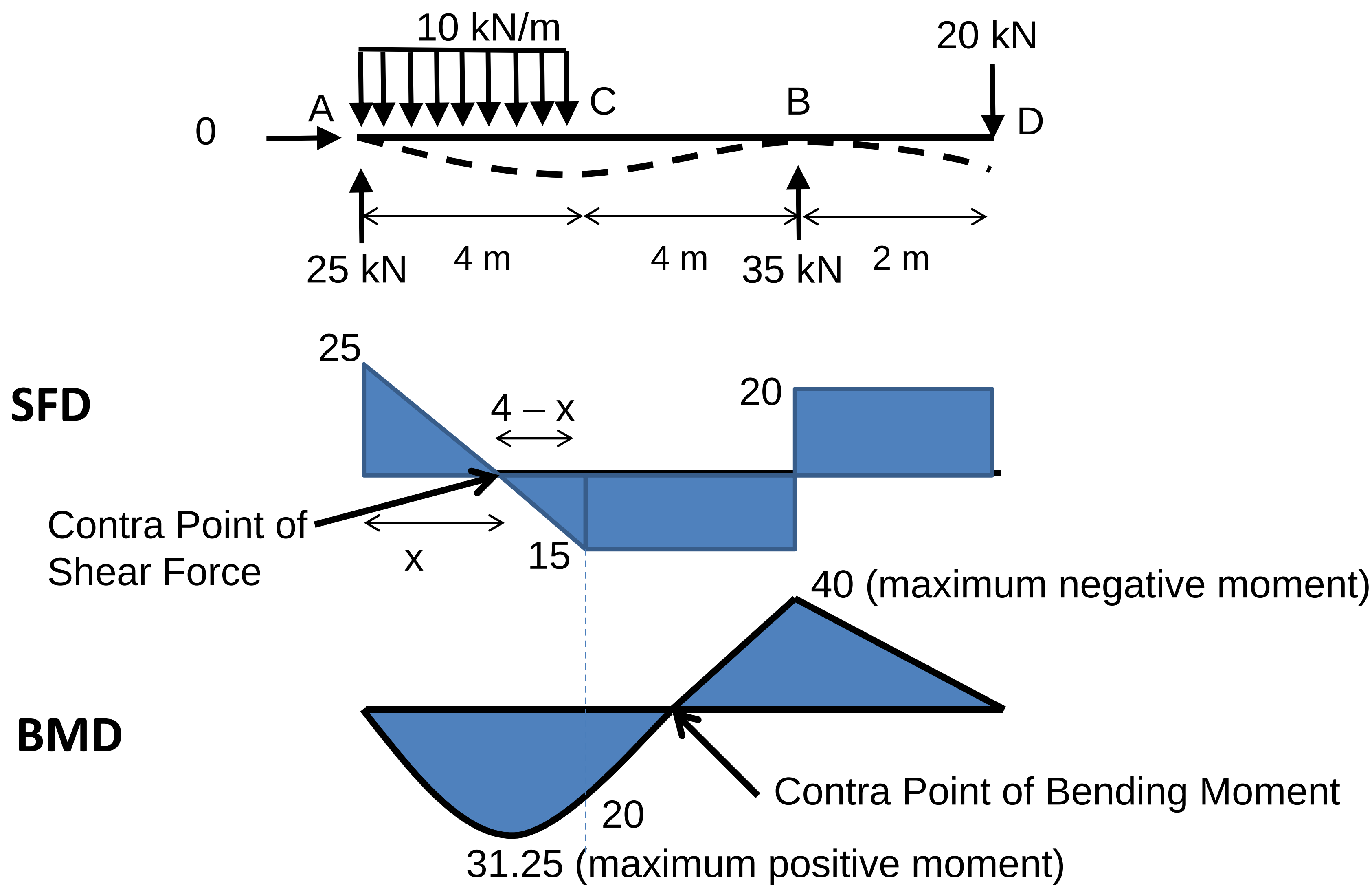
BMD



Contra Point of SF and BM

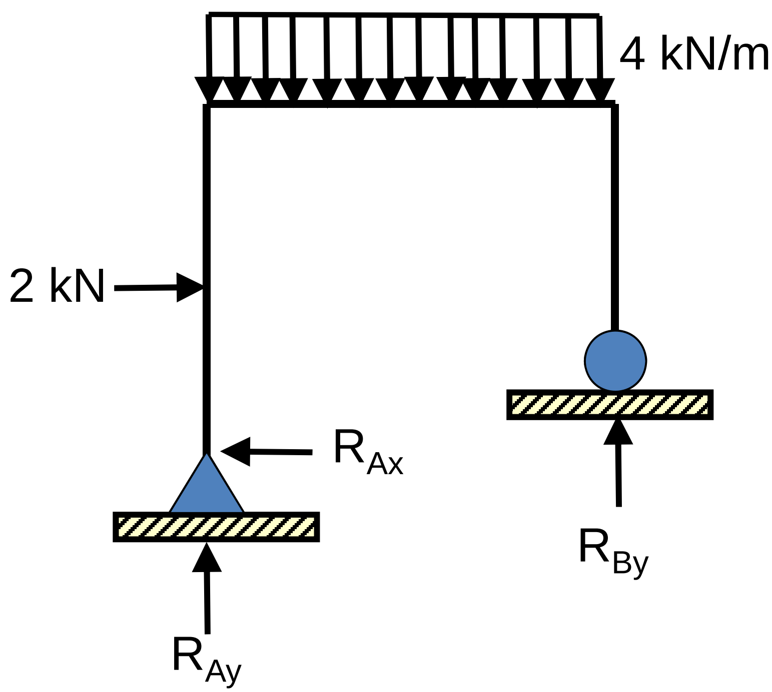
- Contra point is a place where positive shear force/bending moment shifting to the negative region or vice-versa.
- Contra point for shear: ***V = 0***
- Contra point for moment: ***M = 0***
- When shear force is ***zero***, the moment is ***maximum***.
- ***Maximum shear force*** usually occur at the ***support / concentrated load***.

Contra Point of SF and BM



Statically Determinate Frames

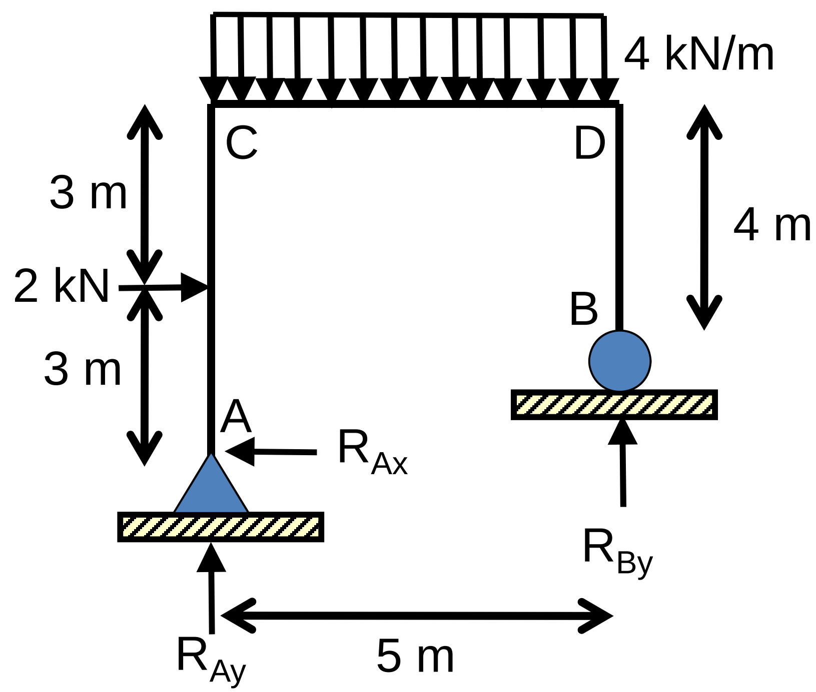
- For a frame to be statically determinate, the number of unknown (reactions) must be able to solved using the equations of equilibrium.



$$\Sigma M_A = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_x = 0$$

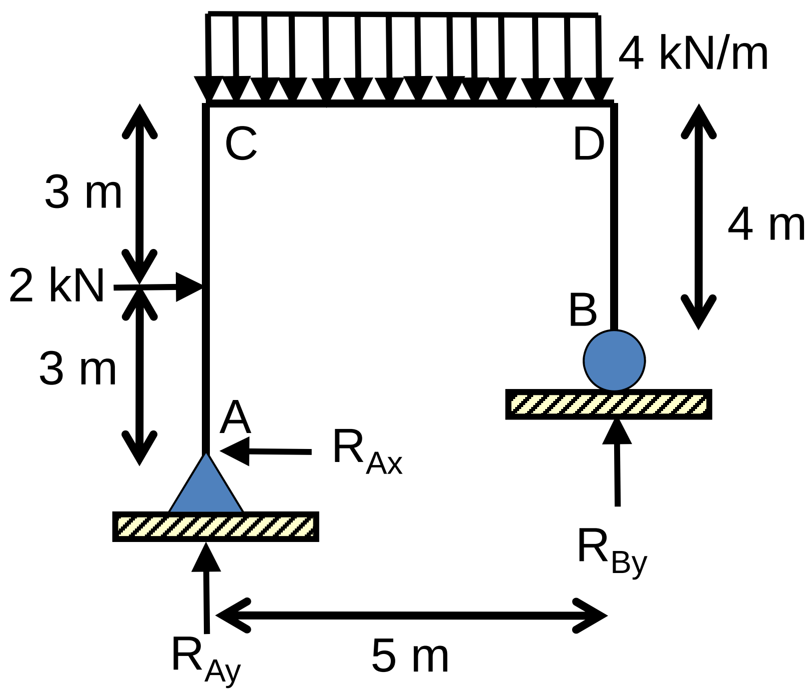
Example 15

Calculate the shear force and bending moment for the frame subjected to the loads as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



Example 15 (cont.)

Solution



$$\Sigma M_A = 0$$

$$4 \times 5 \times 2.5 + 2 \times 3 - R_{By} \times 5 = 0$$

$$R_{By} = 11.2 \text{ kN}$$

$$\Sigma F_y = 0$$

$$R_{Ay} + R_{By} = 4 \times 5$$

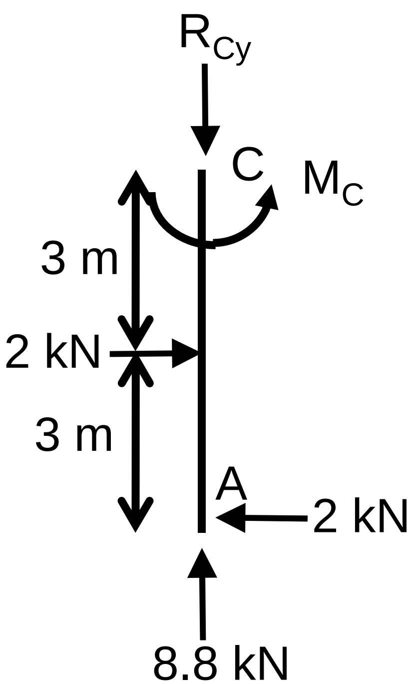
$$R_{Ay} = 20 - 11.2 = 8.8 \text{ kN}$$

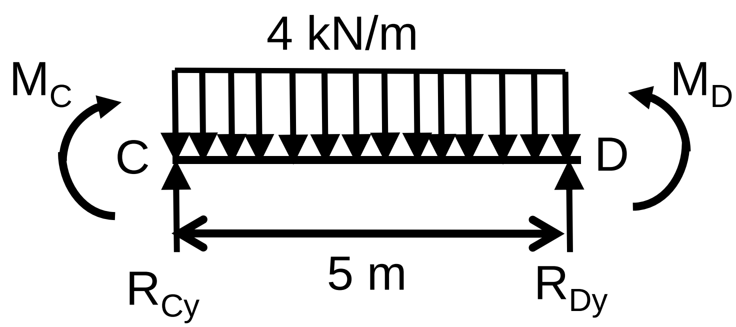
$$\Sigma F_x = 0$$

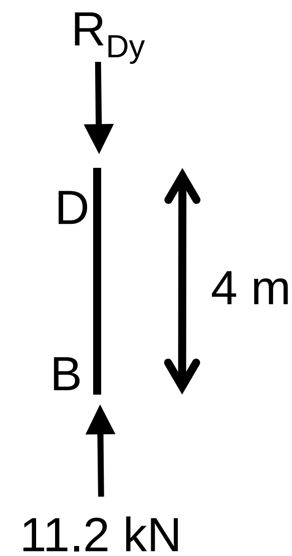
$$R_{Ax} = 2 \text{ kN}$$

Example 15 (cont.)

Solution (cont.)







$\Sigma F_y = 0: R_{Cy} + R_{Dy} = 4 \times 5$

$R_{Dy} = 20 - 8.8 = 11.2$ kN

$\Sigma M_C = 0:$

$M_C + 4 \times 5 \times 2.5 - R_{Dy} \times 5 - M_D = 0$

$M_D = 0$ kNm

$\Sigma M_A = 0: 2 \times 3 - M_C = 0$

$M_C = 6$ kNm

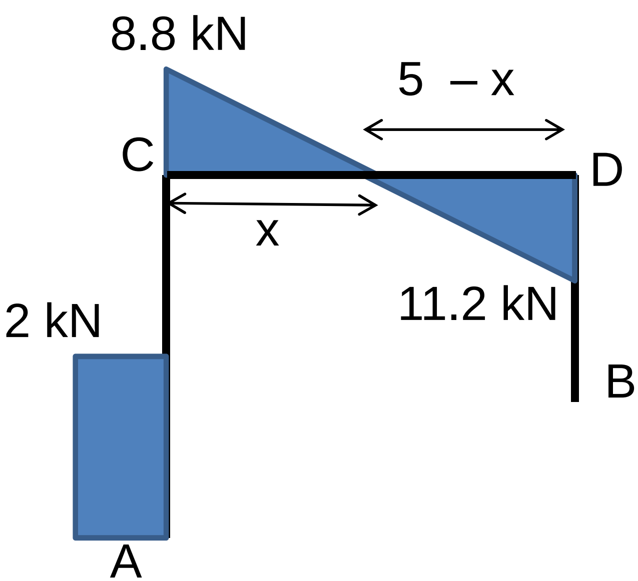
$\Sigma F_y = 0: R_{cy} = 8.8$ kN

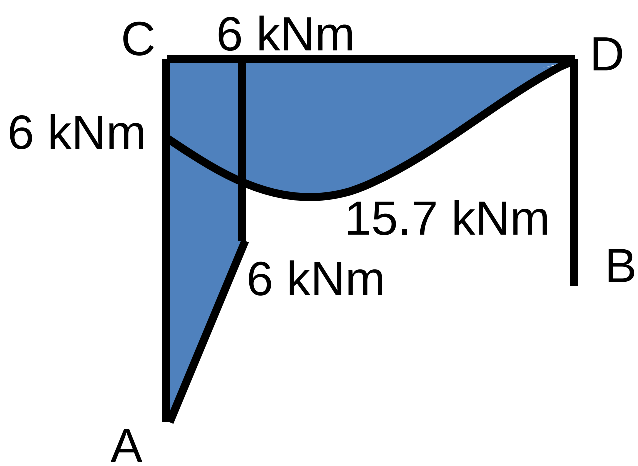
$\Sigma F_y = 0:$

$R_{Dy} = 11.2$ kN

Example 15 (cont.)

Solution





$$\frac{x}{8.8} = \frac{5-x}{11.2}$$

$$11.2x = 44 - 8.8x$$

$$20x = 44$$

$$x = 2.2$$

$$M_{max} = 8.8 \times 2.2 \times 0.5 + 6 = 15.7 \text{ kNm}$$

References

1. Hibbeler, R.C., Mechanics Of Materials, 8th Edition in SI units, Prentice Hall, 2011.
2. Gere dan Timoshenko, Mechanics of Materials, 3rd Edition, Chapman & Hall.
3. Yusof Ahmad, 'Mekanik Bahan dan Struktur' Penerbit UTM 2001