SAB2223 Mechanics of Materials and Structures

TOPIC 1
STRESS AND STRAIN

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STRESS AND STRAIN

Introduction

MECHANICS :

Physical science that covers the problems of bodies at rest or in motion due to the **forces acting on the bodies**.

Can be divided into two areas:

1. Statics - Study of bodies at rest or with uniform velocity.
2. Dynamics - Study of bodies in motion.

Introduction

- Mechanics of materials is a study of the relationship between the **external loads** on a body and the intensity of the **internal loads** within the body/material (if on structures – mechanics of structures).
- This subject also involves the **deformations** and stability of a body when subjected to external forces.

Tension – Elongation

Compression – Shortening

Torsion – Twisting

Bending Moment - Rotation

Equilibrium of a Deformable Body

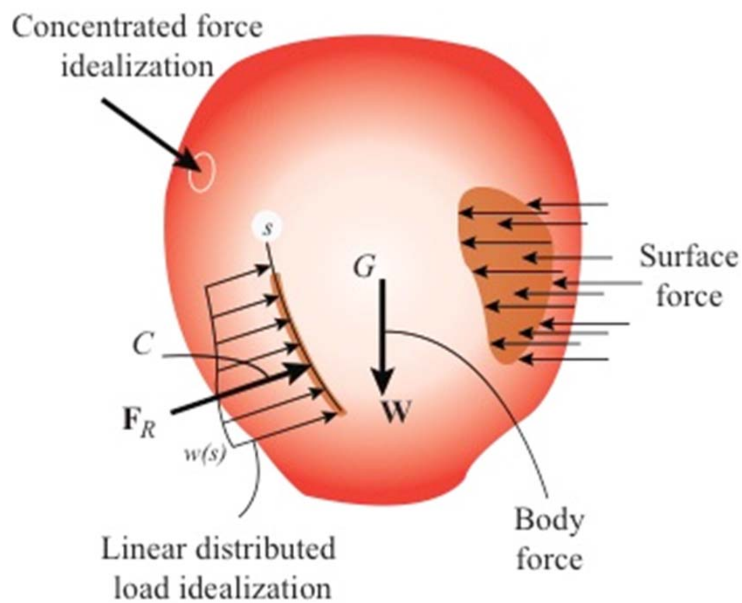
External Forces

1. Surface Forces

   - caused by direct contact of other body's surface

2. Body Forces

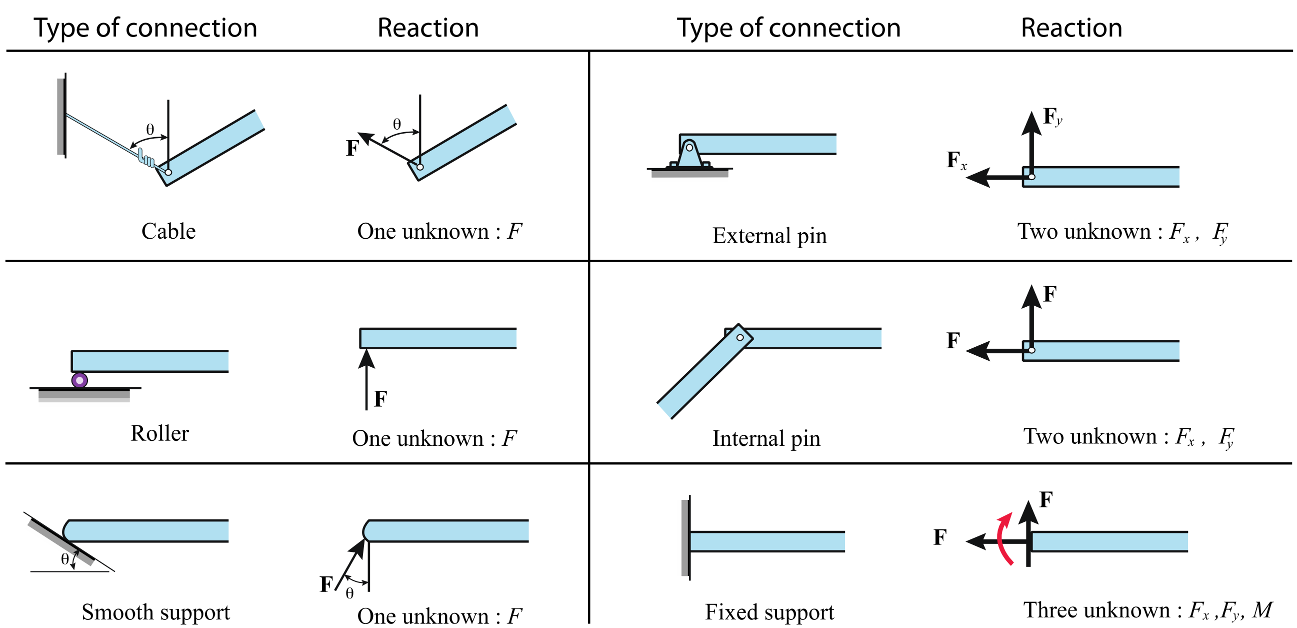
   - other body exerts a force without contact



Equilibrium of a Deformable Body

Reactions

- Surface forces developed at the supports/points of contact between bodies.



Equilibrium of a Deformable Body

Equations of Equilibrium

- Equilibrium of a body requires a ***balance of forces*** and a ***balance of moments***

$$\sum \mathrm{F} = 0 \qquad \sum \mathrm{M}_O = 0$$

- For a body with *x*, *y*, *z* coordinate system with origin *O,*

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$$
$$\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$$

- ***Best way to account for these forces is to draw the body's free-body diagram (FBD).***

Equilibrium of a Deformable Body

Internal Resultant Loadings

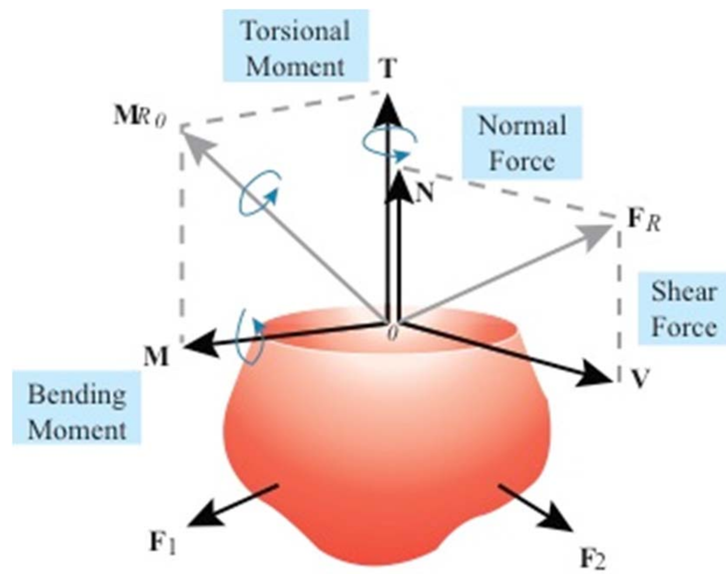
- Objective of FBD is to determine the resultant force and moment acting within a body.
- In general, there are 4 different types of resultant loadings:

  a) Normal force, **N**

  b) Shear force, **V**

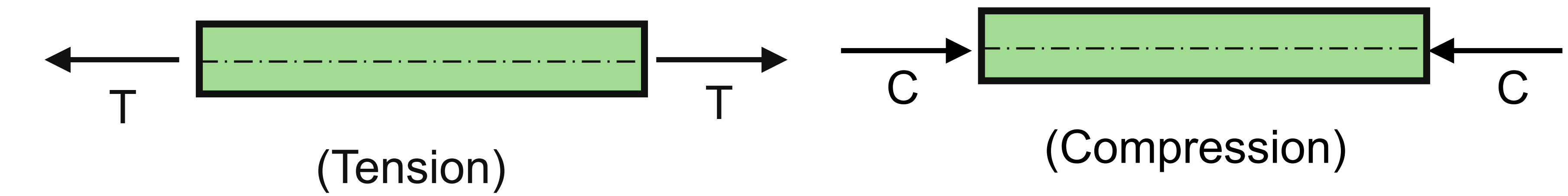
  c) Torsional moment or torque, **T**

  d) Bending moment, **M**



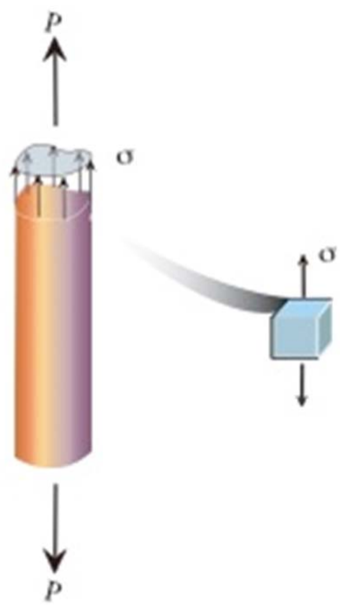
Types of Forces and Stress

NORMAL FORCE - The force that acts perpendicular to a certain surface and passes through the symmetrical axis of the body.



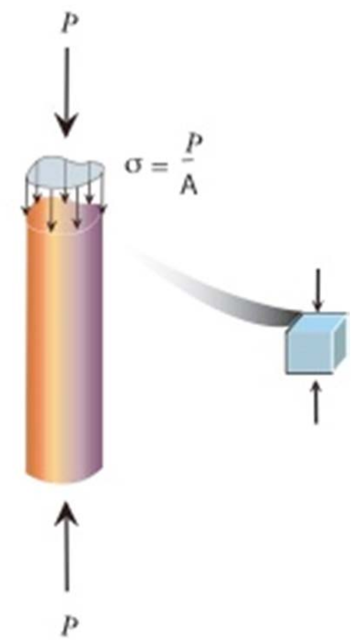
Types of Forces and Stress

Normal Stress – The intensity of force, or force per unit area.



Tension

$$\sigma = \frac{P}{A}$$



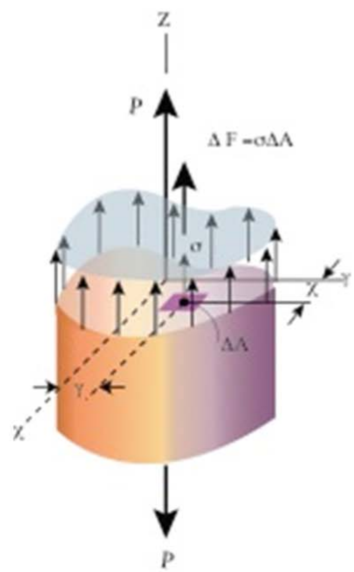
Compression

Tensile Stress (+ ve)

Compressive Stress (- ve)

Average Normal Stress

- When a ***cross-sectional area*** bar is subjected to axial force through the centroid, it is only subjected to normal stress.
- Stress is assumed to be averaged over the area.

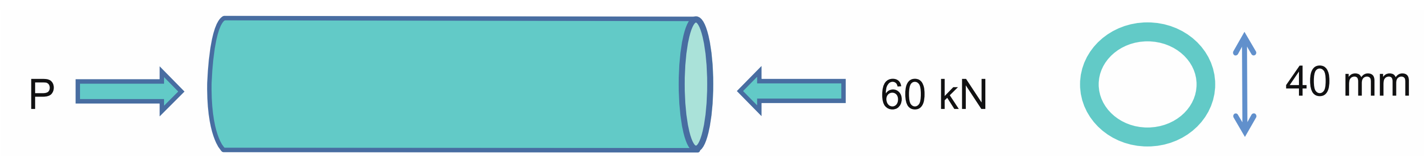


$$\sigma = \frac{P}{A}$$

σ = average normal stress
P = resultant normal force
A = cross sectional area of bar

Example 1

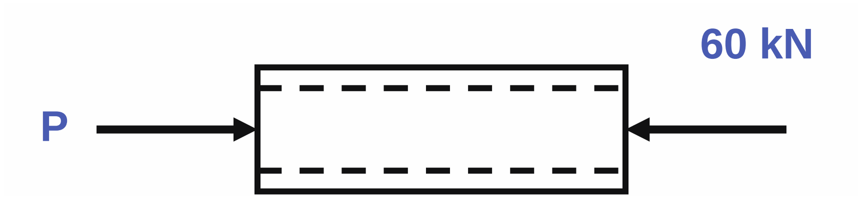
A galvanized pipe has a constant diameter of 40 mm and a thickness of 10 mm subject to a compression force of 60 kN as shown in the figure. Determine the normal stress in the pipe.



Example 1 (cont.)

Solution

(Always draw the Free Body Diagram)



$\xrightarrow{+}$ $$\sum F_x = 0; \; -P - 60 = 0 \quad \therefore \; P = -60\,kN(C)$$

Cross-Sectional Area, $$A = \frac{\pi(40^2 - 20^2)}{4} = 942.5\,mm^2$$

Normal Stress, $$\sigma = \frac{P}{A} = \frac{60}{942.5}$$

$$= 0.064\,kN/mm^2$$

Example 2

An aluminum rod subjected to a tension force of 50 kN. If the maximum allowable stress for aluminum ($\sigma_{aluminum}$) is 100 N/mm^2, determine the size of the rod that is safe to be used.

Example 2 (cont.)

Solution

Internal force P = 50 kN.

Normal Stress, $\sigma = \dfrac{P}{A} \leq 100\, N/mm^2$

Cross-sectional area, $A \geq \dfrac{P}{100}$

$$\geq \frac{50\, x\, 10^3}{100}$$

$$\geq 500\, mm^2$$

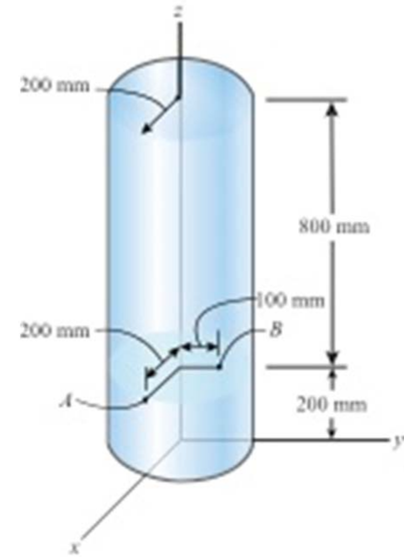
Area of rod, $A = \dfrac{\pi D^2}{4} \geq 500\, mm^2$

Diameter, $D \geq \sqrt{\dfrac{500\, x\, 4}{\pi}}$

$$\geq 25.2\, mm$$

Example 3

The casting is made of steel that has a specific weight of $\gamma_{st} = 80\ \text{kN/m}^3$. Determine the average compressive stress acting at points *A* and *B*.



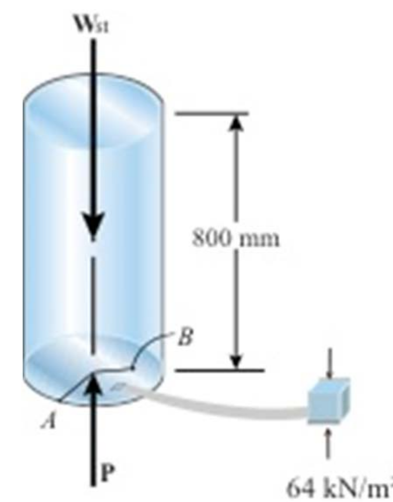
Solution:

By drawing a free-body diagram of the top segment, the internal axial force *P* at the section is

$$+\uparrow \sum F_z = 0; \qquad P - W_{st} = 0$$

$$P - (80)(0.8)\pi(0.2)^2 = 0$$

$$P = 8.042\ \text{kN}$$



The average compressive stress becomes

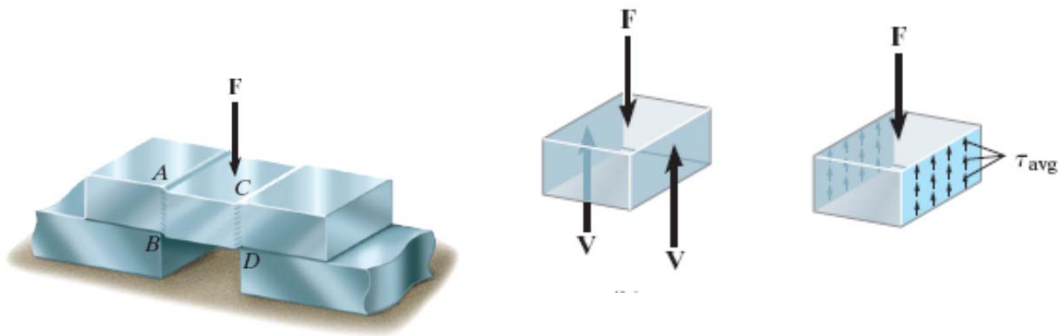
$$\sigma = \frac{P}{A} = \frac{8.042}{\pi(0.2)^2} = 64.0\ \text{kN/m}^2 \quad \text{(Ans)}$$

Average Shear Stress

- The ***average shear stress*** distributed over each sectioned area that develops a shear force.

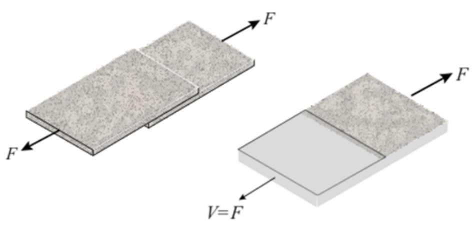
$$\tau_{avg} = \frac{V}{A}$$

τ = average shear stress
P = internal resultant shear force
A = area at that section

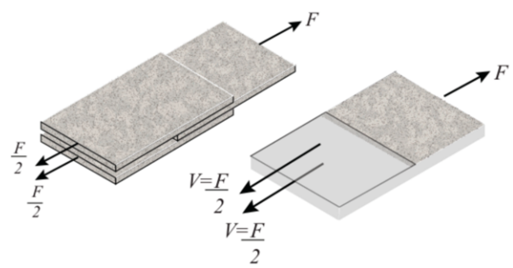


- 2 different types of shear:

a) Single Shear

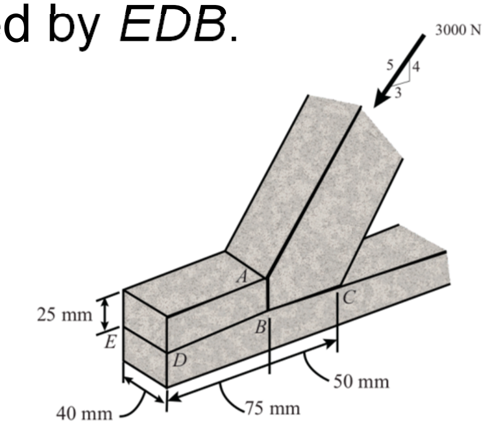


b) Double Shear



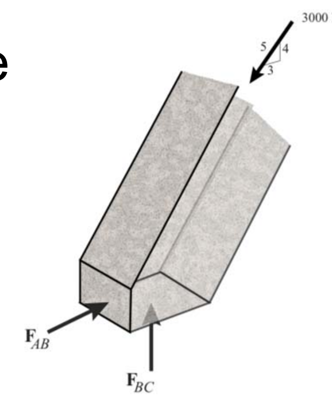
Example 4

The inclined member is subjected to a compressive force of 3000 N. Determine the average compressive stress along the smooth areas of contact defined by *AB* and *BC*, and the average shear stress along the horizontal plane defined by *EDB*.



Solution:

The compressive forces acting on the areas of contact are



$$+\rightarrow \sum F_x = 0; \quad F_{AB} - 3000\left(\tfrac{3}{5}\right) = 0 \Rightarrow F_{AB} = 1800 \text{ N}$$

$$+\uparrow \sum F_y = 0; \quad F_{BC} - 3000\left(\tfrac{4}{5}\right) = 0 \Rightarrow F_{BC} = 2400 \text{ N}$$

Example 4 (cont.)

Solution:

The shear force acting on the sectioned horizontal plane *EDB* is

$$+\rightarrow \sum F_x = 0; \quad V = 1800\ \text{N}$$

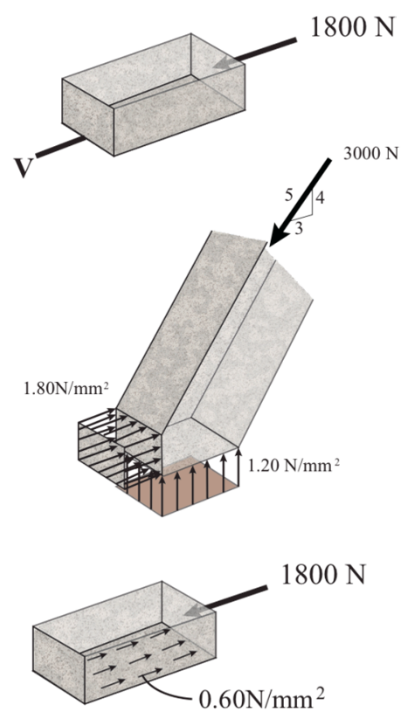
Average compressive stresses along the AB and BC planes are

$$\sigma_{AB} = \frac{1800}{(25)(40)} = 1.80\ \text{N/mm}^2 \ \ (\text{Ans})$$

$$\sigma_{BC} = \frac{2400}{(50)(40)} = 1.20\ \text{N/mm}^2 \ \ (\text{Ans})$$

Average shear stress acting on the BD plane is

$$\tau_{avg} = \frac{1800}{(75)(40)} = 0.60\ \text{N/mm}^2 \ \ (\text{Ans})$$



Allowable Stress

- Many unknown factors that influence the actual stress in a member.
- A *factor of safety* is needed to obtained allowable load.
- The ***factor of safety*** (F.S.) is a ratio of the failure load divided by the allowable load

$$F.S = \frac{F_{fail}}{F_{allow}}$$

$$F.S = \frac{\sigma_{fail}}{\sigma_{allow}}$$

$$F.S = \frac{\tau_{fail}}{\tau_{allow}}$$

Example 5

The rigid bar *AB* supported by a steel rod *AC* having a diameter of 20 mm and an aluminum block having a cross sectional area of 1800 mm^2. The 18-mm-diameter pins at *A* and *C* are subjected to *single shear*. If the failure stress for the steel and aluminum is $(\sigma_{st})_{fail} = 680\text{ MPa}$ and $(\sigma_{al})_{fail} = 70\text{ MPa}$ respectively, and the failure shear stress for each pin is $\tau_{fail} = 900\text{ MPa}$, determine the largest load *P* that can be applied to the bar. Apply a factor of safety of F.S. = 2.

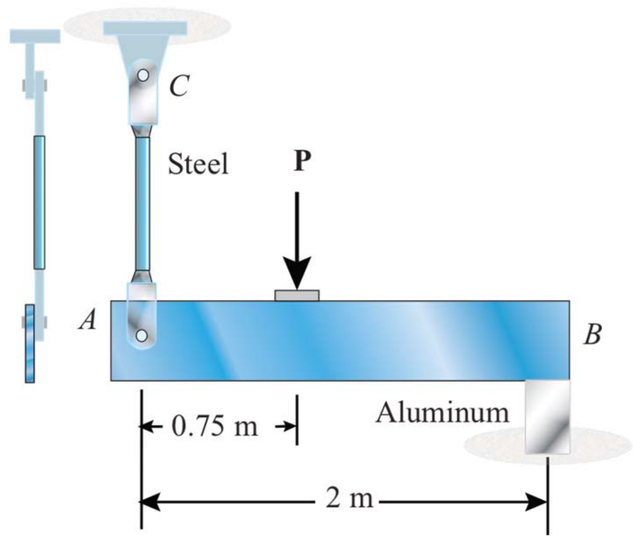
Solution:

The allowable stresses are

$$(\sigma_{st})_{allow} = \frac{(\sigma_{st})_{fail}}{F.S.} = \frac{680}{2} = 340\text{ MPa}$$

$$(\sigma_{al})_{allow} = \frac{(\sigma_{al})_{fail}}{F.S.} = \frac{70}{2} = 35\text{ MPa}$$

$$\tau_{allow} = \frac{\tau_{fail}}{F.S.} = \frac{900}{2} = 450\text{ MPa}$$



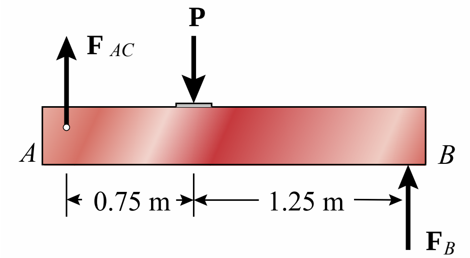
Example 5 (cont.)

Solution:

There are three unknowns and we apply the equations of equilibrium,

$$\curvearrowleft + \sum M_B = 0; \quad P(1.25) - F_{AC}(2) = 0 \qquad (1)$$

$$\curvearrowleft + \sum M_A = 0; \quad F_B(2) - P(0.75) = 0 \qquad (2)$$



We will now determine each value of *P* that creates the allowable stress in the rod, block, and pins, respectively.

For rod AC, $F_{AC} = (\sigma_{st})_{allow}(A_{AC}) = 340(10^6)\left[\pi(0.01)^2\right] = 106.8\text{ kN}$

Using Eq. 1, $P = \dfrac{(106.8)(2)}{1.25} = 171\text{ kN}$

For block B, $F_B = (\sigma_{al})_{allow} A_B = 35(10^6)\left[1800(10^{-6})\right] = 63.0\text{ kN}$

Using Eq. 2, $P = \dfrac{(63.0)(2)}{0.75} = 168\text{ kN}$

Example 5 (cont.)

Solution:

For pin A or C, $V = F_{AC} = \tau_{allow} A = 450\left(10^6\right)\left[\pi\left(0.009\right)^2\right] = 114.5\ \text{kN}$

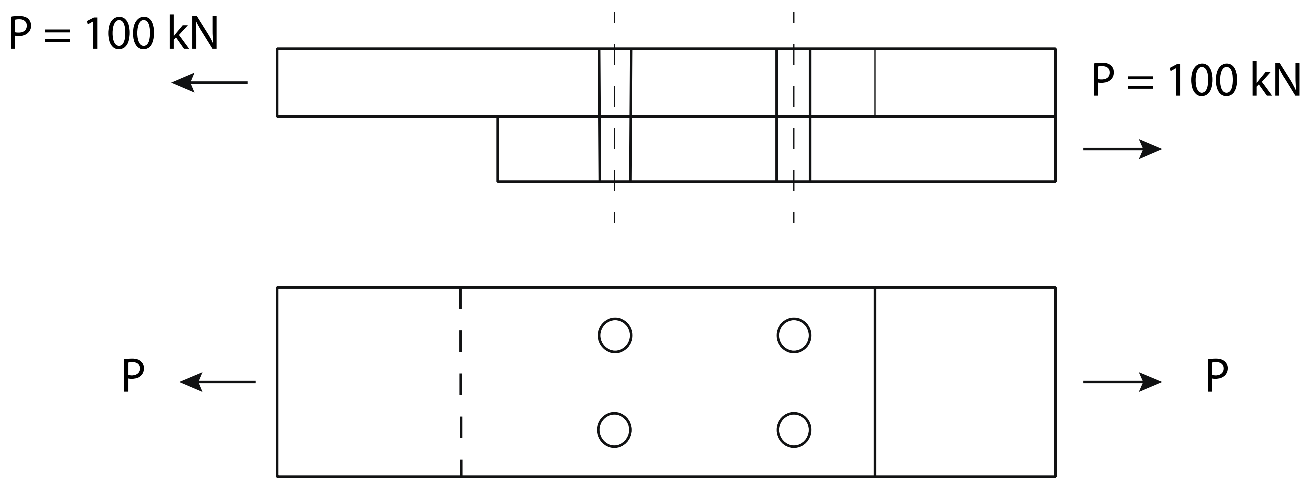
Using Eq. 1, $P = \dfrac{(114.5)(2)}{1.25} = 183\ \text{kN}$

When *P* reaches its *smallest value* (168 kN), it develops the allowable normal stress in the aluminium block. Hence,

$$P = 168\ \text{kN (Ans)}$$

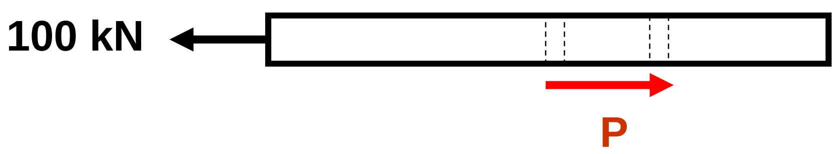
Example 6

The figure below shows two pieces of wood connected together using 4 numbers of bolts with 24 mm diameter. Calculate the average shear stress in each bolt.



Example 6 (cont.)

Solution



$$\xrightarrow{+} \quad \sum F_x = 0; \ -100 + P = 0 \quad \therefore \ P = 100\,kN$$

Shearing area,

$$A = 4\left(\frac{\pi \times 24^2}{4}\right) = 1809.6\,mm^2$$

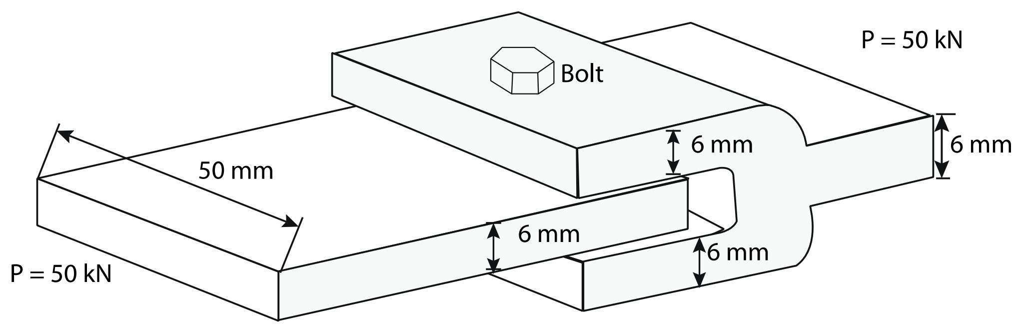
Shear Stress,

$$\tau = \frac{P}{A} = \frac{100 \times 10^3}{1809.6} = 55.26\,N/mm^2$$

Example 7

A coupling type of steel connection was designed to carry 50 kN. If the bolt diameter is 15 mm, calculate

a) Average shear stress in bolt
b) Maximum tensile stress in plate
c) Maximum tensile stress in coupling

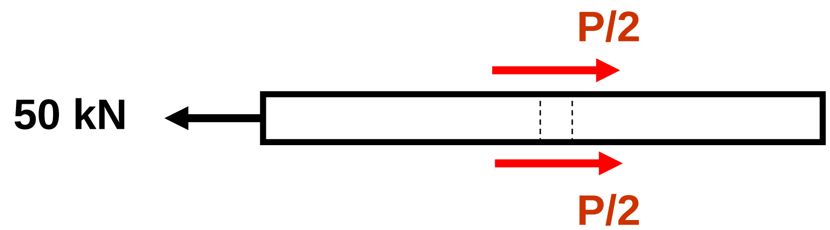


UTM

Example 7 (cont.)

Solution

(a) Double Shear:



$\overset{+}{\rightarrow}$

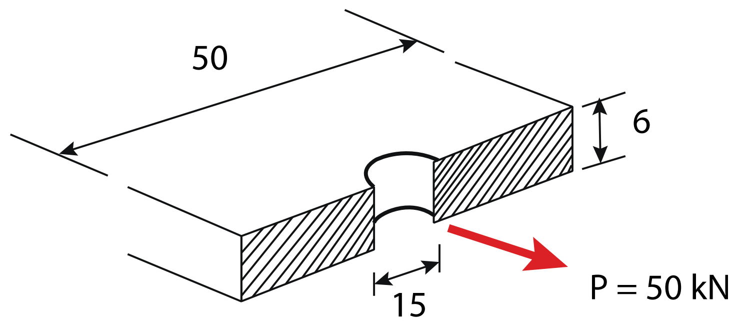
$$\sum F_x = 0; \ -50 + \frac{P}{2} + \frac{P}{2} = 0 \quad \therefore \ P = 50\,kN$$

Total shearing area, $A = 2\left(A_{bolt}\right) = 2\left(\frac{\pi \times 15^2}{4}\right) = 353.43\,mm^2$

Shear Stress, $\tau = \frac{P}{A} = \frac{50 \times 10^3}{353.43} = 141.5\,N/mm^2$

Example 7 (cont.)

Solution (cont.)



(b) Tensile stress In the plate:

Area, $A = (50 - 15) \times 6 = 210\, mm^2$

Normal Stress, $\sigma = \dfrac{P}{A} = \dfrac{50 \times 10^3}{210} = 238.1\, N/mm^2$

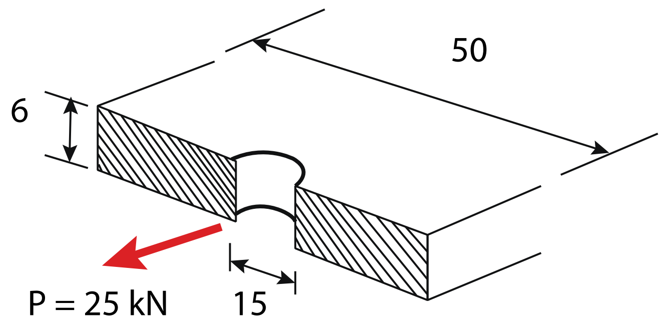
Example 7 (cont.)

Solution (cont.)

(c) Maximum tensile stress In the coupling:

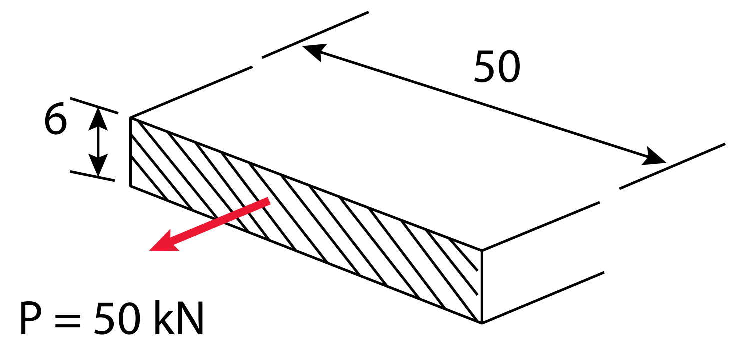
Area, $A = (50 - 15) \times 6 = 210\, mm^2$

Normal Stress, $\sigma = \dfrac{P}{A} = \dfrac{25 \times 10^3}{210} = 119.05\, N/mm^2$



Area, $A = 50 \times 6 = 300\, mm^2$

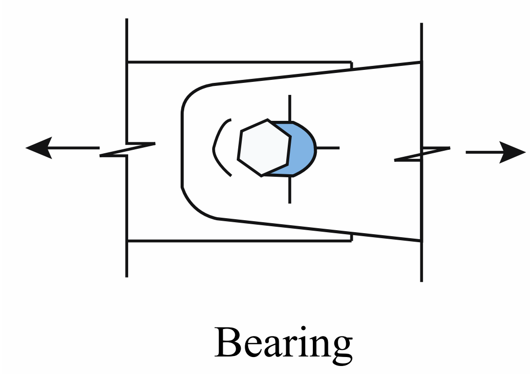
Normal Stress, $\sigma = \dfrac{P}{A} = \dfrac{50 \times 10^3}{300} = 166.7\, N/mm^2$

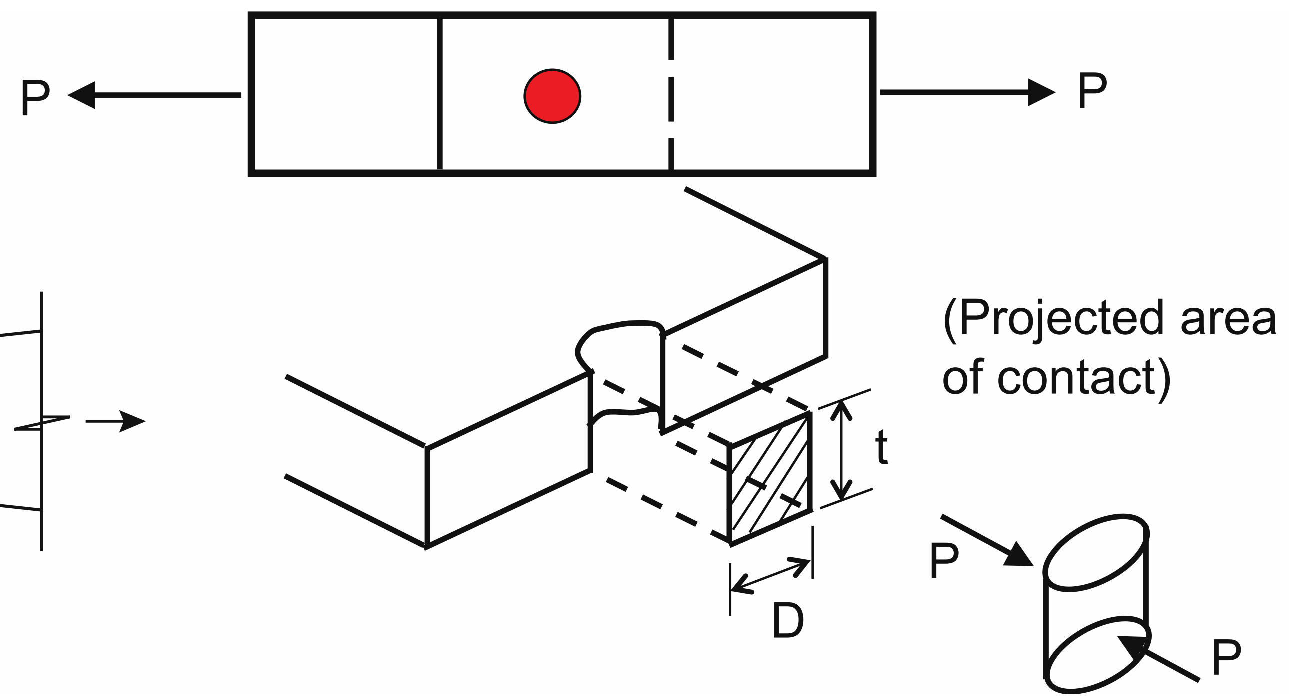


$\therefore$ Maximum Normal Stress, $\sigma = 166.7\ N/mm^2$.

Types of Forces and Stress

BEARING STRESS – Is a stress due to the pressure between two rigid bodies.





Types of Forces and Stress

- the pressure occurs at the surface or area of contact between the bolt and the plate.

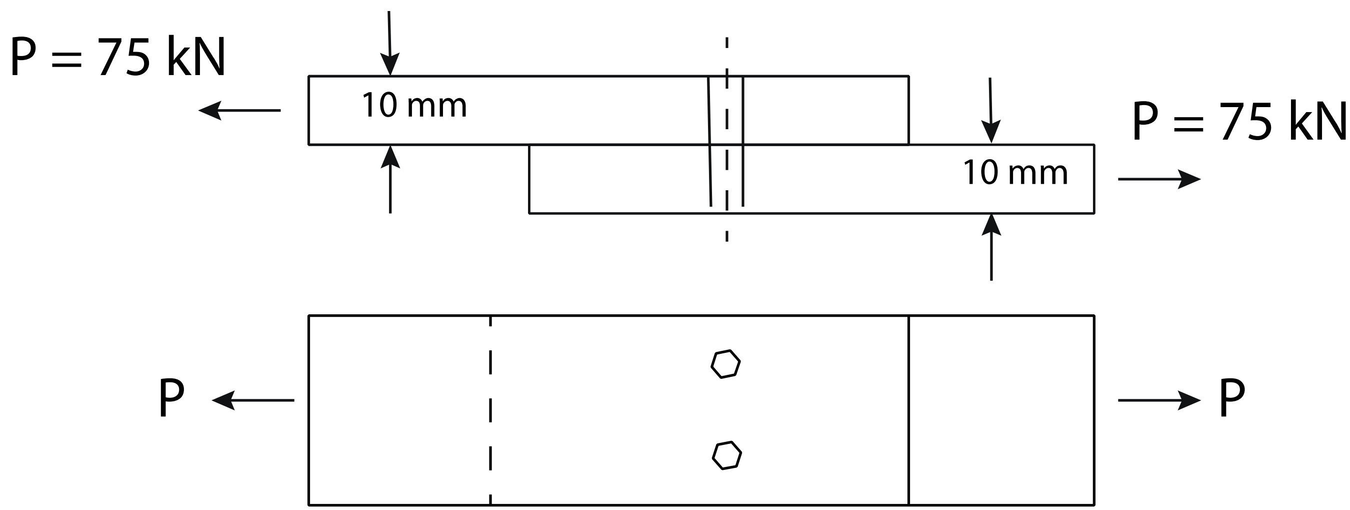
- the area of pressure is the projected area of contact (curve part).

- Bearing stress,

$$\sigma_b = \frac{P}{A} = \frac{P}{tD}$$

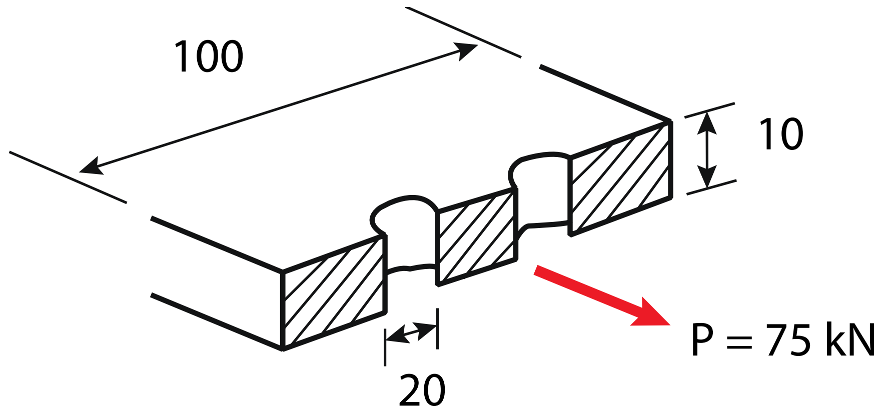
Example 8

The figure below shows two pieces of 10 mm wood connected together using 2 numbers of bolts with 20 mm diameter. Calculate the shear bearing in each member.



Example 8 (cont.)

Solution



Bearing area for each member,

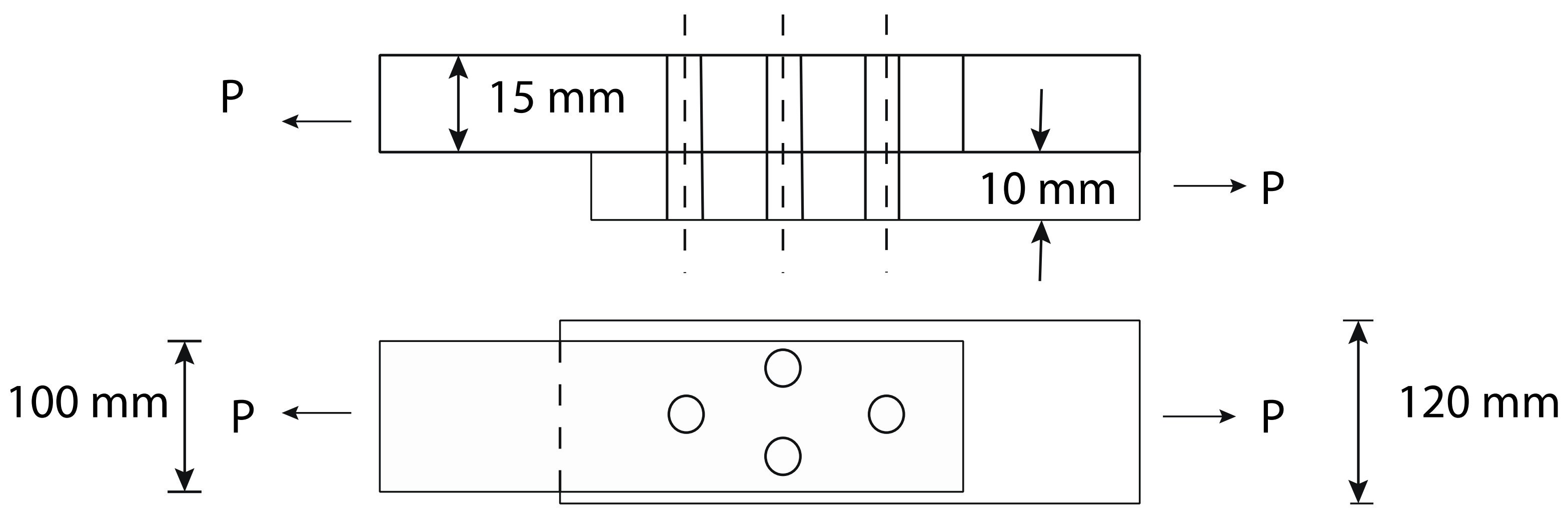
$$A = 2(20 \times 10) = 400\,mm^2$$

Bearing Stress,

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{400} = 187.5\,N/mm^2$$

Example 9

The figure below shows two plates connected together using 4 numbers of bolts with 20 mm diameter. Determine the maximum load, P if the allowable shear stress, tensile stress and bearing stress are 80 N/mm², 100 N/mm² and 140 N/mm² respectively. Assume the loading distributed equally in each bolt.



Example 9 (cont.)

Solution:

Shearing Stress:

Single shear area, $A = 4\left(\frac{\pi \times 20^2}{4}\right) = 1256.8\, mm^2$

$\therefore$ Shearing Stress, $\tau = \frac{P}{A} \leq 80\, N/mm^2$

$$\therefore P \leq \frac{80 \times 1256.6}{10^3} = 100.53\, kN$$

Tensile Stress:

Plate 1 Critical area is at *a-a*, $A_{a-a} = (100 - 40)15 = 900\, mm^2$

Tensile Stress, $\sigma = \frac{P}{A} \leq 100\, N/mm^2$

$$\therefore P \leq (100 \times 900) \div 10^3 = 90\, kN$$

Example 9 (cont.)

Solution (cont.)

Plate 2 Area at *a-a*, $A_{a-a} = (120 - 40)10 = 800\,mm^2$

Tensile Stress, $\sigma = \dfrac{P}{A} \leq 100\,N/mm^2$

$$\therefore P \leq (100 \times 800) \div 10^3 = 80\,kN$$

Bearing Stress:

Plate 1 Bearing area , $A = 4(20 \times 15) = 1200\,mm^2$

Bearing Stress, $\sigma = \dfrac{P}{A} \leq 140\,N/mm^2$

$$\therefore P \leq (140 \times 1200) \div 10^3 = 168\,kN$$

Example 9 (cont.)

Solution (cont.)

Plate 2 Bearing area , $A = 4(20 \times 10) = 800\, mm^2$

Bearing Stress, $\sigma = \dfrac{P}{A} \leq 140\, N/mm^2$

$$\therefore P \leq (140 \times 800) \div 10^3 = 112\, kN$$

$\therefore$ The maximum P is 80 kN.

Strain

Normal Strain

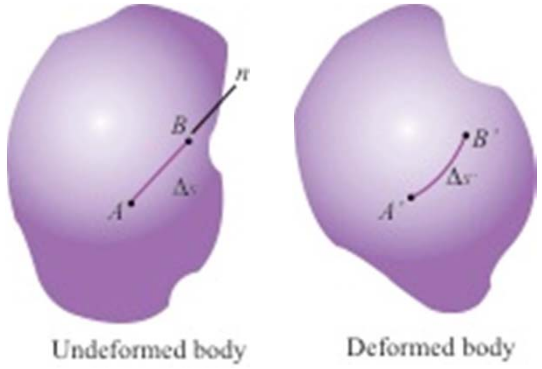
- The elongation / contraction of a line segment per unit of length is referred to as ***normal strain***.
- Average normal strain is defined as

$$\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

- If the normal strain is known, then the approximate final length is

$$\Delta s' \approx (1 + \varepsilon)\Delta s$$

+ε ➔line elongate
-ε ➔line contracts



Example 10

A 600 mm bar with a constant cross sectional area subjected to an elongation of 0.15 mm. Calculate the strain value.

Solution

Strain, $\varepsilon = \frac{\delta}{L} = \frac{0.15}{600} = 2.5 \times 10^{-4}$

Strain

Units

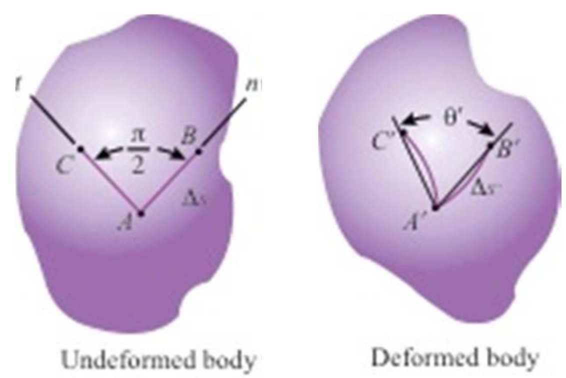
- Normal strain is a *dimensionless quantity* since it is a ratio of two lengths.

Shear Strain

- Change in angle between 2 line segments that were *perpendicular* to one another refers to ***shear strain***.

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

θ<90 ➔+shear strain
θ>90 ➔-shear strain



Example 11

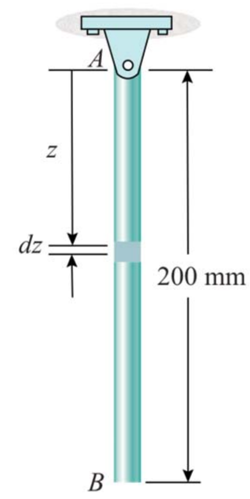
The slender rod creates a normal strain in the rod of $\varepsilon_z = 40\left(10^{-3}\right)z^{1/2}$ where *z* is in meters. Determine (a) displacement of end *B* due to the temperature increase, and (b) the average normal strain in the rod.

Solution:

Part (a)

Since the normal strain is reported at each point along the rod, it has a deformed length of

$$dz' = \left[1 + 40\left(10^{-3}\right)z^{1/2}\right]dz$$



The sum along the axis yields the *deformed length* of the rod is

$$z' = \int_0^{0.2} \left[1 + 40\left(10^{-3}\right)z^{1/2}\right]dz = 0.20239\ \text{m}$$

The displacement of the end of the rod is therefore

$$\Delta_B = 0.20239 - 0.2 = 0.00239\text{m} = 2.39\text{mm} \downarrow \text{(Ans)}$$

Example 11 (cont.)

Solution:

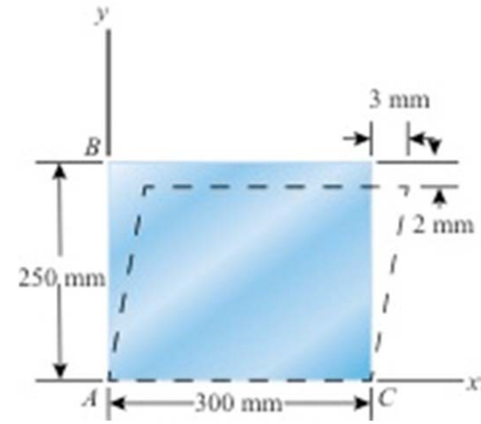
Part (b)

Assumes the rod has an original length of 200 mm and a change in length of 2.39 mm. Hence,

$$\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{2.39}{200} = 0.0119 \text{ mm/mm (Ans)}$$

Example 12

The plate is deformed into the dashed shape. If, in this deformed shape, horizontal lines on the plate remain horizontal and do not change their length, determine (a) the average normal strain along the side *AB*, and (b) the average shear strain in the plate relative to the *x* and *y* axes.



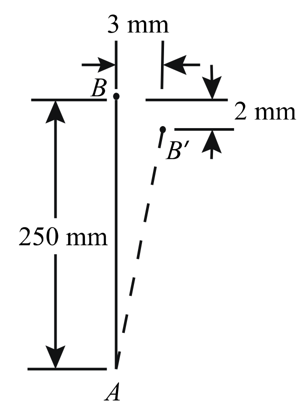
Solution:

Part (a)

Line *AB*, coincident with the *y* axis, becomes line after deformation, thus the length of this line is

$$AB' = \sqrt{(250-2)^2 + 3^2} = 248.018\ \text{mm}$$

The average normal strain for *AB* is therefore



$$(\varepsilon_{AB})_{avg} = \frac{AB' - AB}{AB} = \frac{248.018 - 250}{240} = -7.93(10^{-3})\,\text{mm/mm}\ (\text{Ans})$$

The negative sign indicates the strain causes a contraction of *AB*.

Example 12 (cont.)

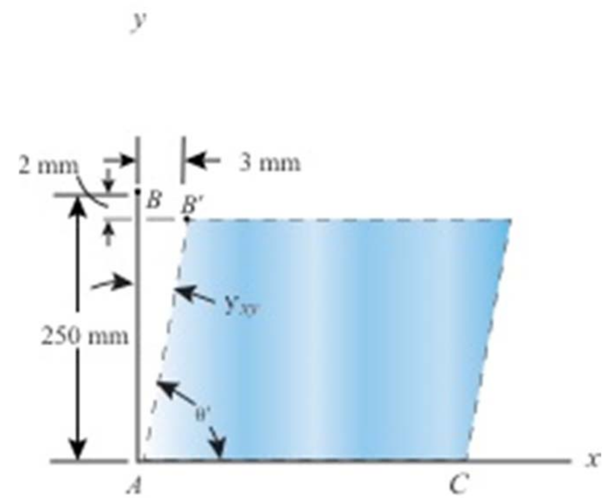
Solution:

Part (b)

As noted, the once 90° angle *BAC* between the sides of the plate, referenced from the *x*, *y* axes, changes to θ' due to the displacement of *B* to *B'*.

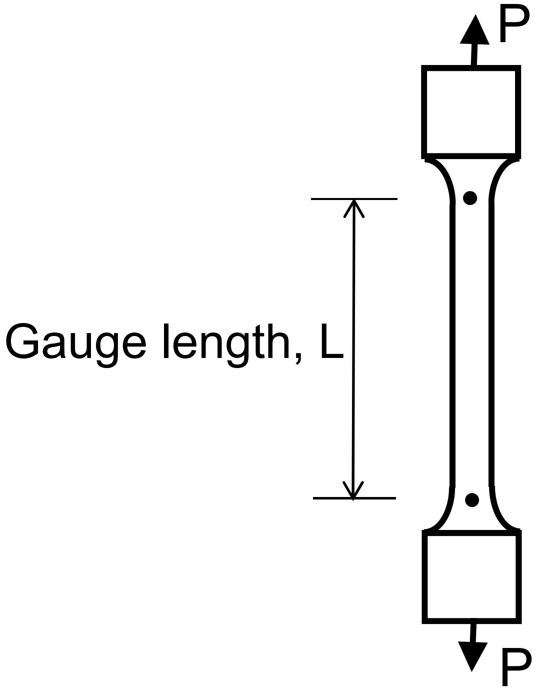
Since $\gamma_{xy} = \frac{\pi}{2} - \theta'$ then γ_{xy} is the angle shown in the figure. Thus,

$$\gamma_{xy} = \tan^{-1}\left(\frac{3}{250-2}\right) = 0.121\,\text{rad (Ans)}$$



Stress – Strain Relationship

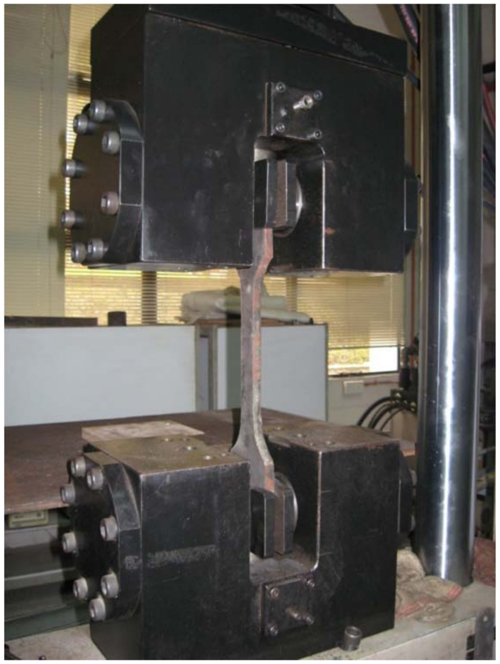
- By conducting a tensile test and by plotting a stress-strain curve, the mechanic behaviour of a particular material can be determined.



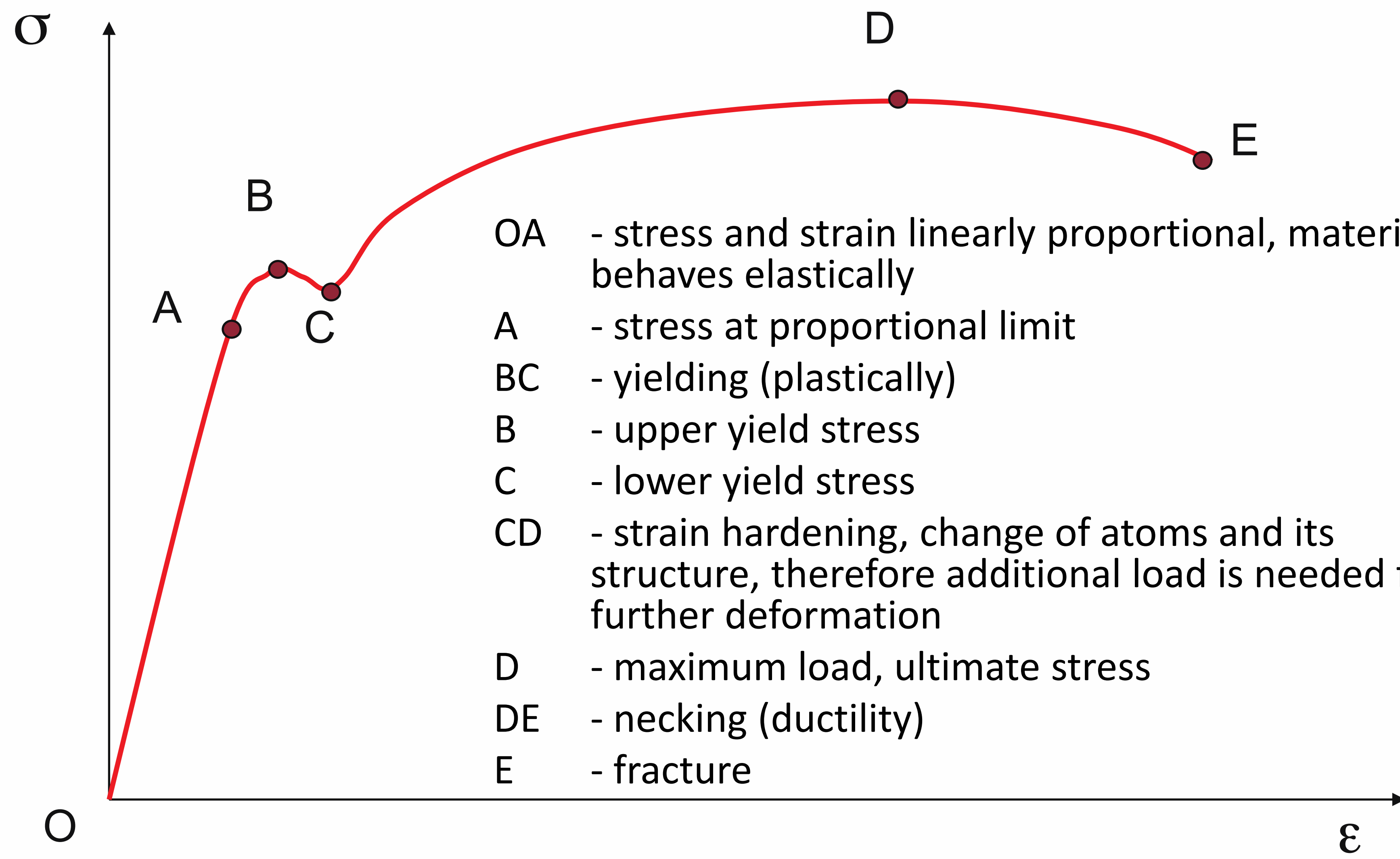
Stress, $\sigma = \dfrac{P}{A}$

Strain, $\varepsilon = \dfrac{\delta}{L}$

Stress – Strain Relationship



Stress – Strain Curve



OA	- stress and strain linearly proportional, material behaves elastically
A	- stress at proportional limit
BC	- yielding (plastically)
B	- upper yield stress
C	- lower yield stress
CD	- strain hardening, change of atoms and its structure, therefore additional load is needed for further deformation
D	- maximum load, ultimate stress
DE	- necking (ductility)
E	- fracture

Example 13

Diameter specimen = 11.28 mm

Length = 56 mm

Minimum diameter after failure = 6.45 mm

Load (kN)	Elongation (x 10^{-3} mm)	Stress, σ (N/mm^2)	Strain, ε
2.47	5.60		
4.97	11.90		
7.40	18.20		
…	…		
35.70	13440.00		
28.00	14560.00		

Example 13 (cont.)

Solution

Stress, $\sigma = \dfrac{P}{A}$ Strain, $\varepsilon = \dfrac{\delta}{L}$ L = 56 mm $A = \left(\dfrac{\pi \times 11.28^2}{4}\right) = 99.95\,mm^2$

Load (kN)	Elongation (x 10^{-3} mm)	Stress, σ (N/mm^2)	Strain, ε
2.47	5.60	24.72	0.0001
4.97	11.90	49.73	0.0002
7.40	18.20	74.05	0.0003
…	…	…	…
35.70	13440.00	357.24	0.2400
28.00	14560.00	280.19	0.2600

Example 13 (cont.)

Solution

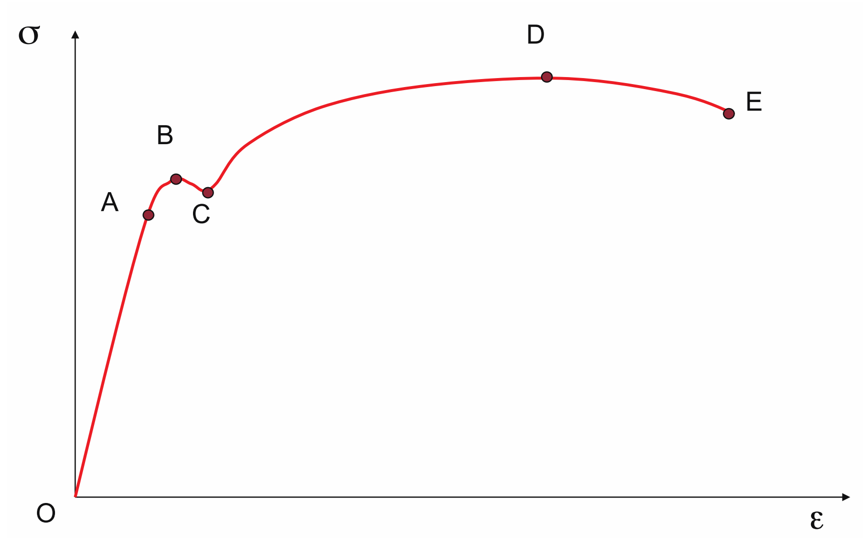
a) $E = 210.7\ kN/mm^2$

b) $\sigma_D = 400.27\ N/mm$

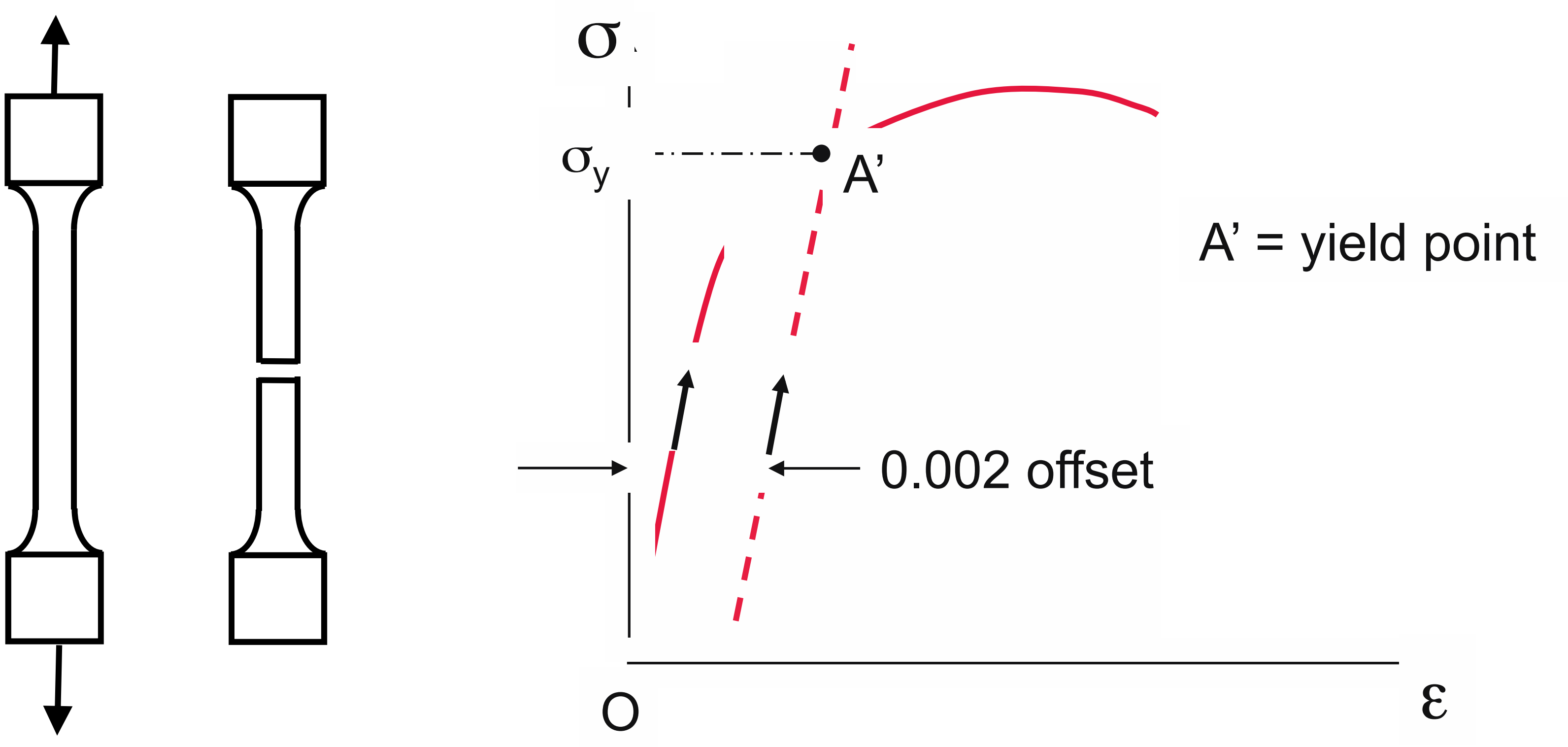
c) $\sigma_B = 333\ N/mm^2$, $\sigma_C = 312\ N/mm^2$

d) % decrease in diameter = 57.18 %

e) % elongation = 26 %



- For a brittle material such as aluminum or glass, the yield point is not clearly identified.



- By using the offset method, the yield point (yield stress) can be taken at $\varepsilon = 0.002$ (0.2%).

Ductile and Brittle Materials

Ductile Materials

- Material that can subjected to large strains before it ruptures is called a ***ductile material***.

Brittle Materials

- Materials that exhibit little or no yielding before failure are referred to as ***brittle materials***.

Hooke's Law : Elastic Modulus, *E*

- Most of the materials used for a structure posses an elastic behaviour.

- The linear part of the stress-strain curve (i.e. OA for steel's) can be expressed as:

$$\sigma = E\varepsilon$$

Elastic Modulus, E

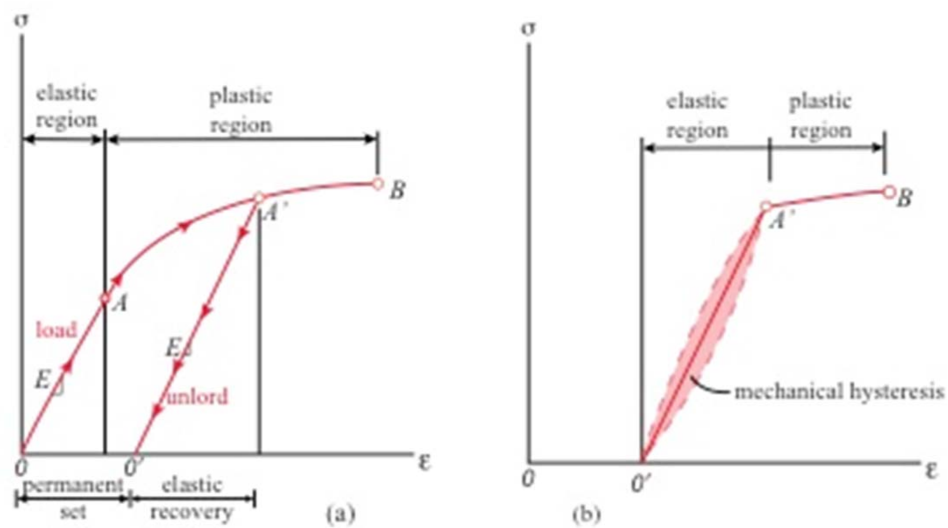
Hence, $$E = \frac{\sigma}{\varepsilon}$$

- a constant
- the slope of the linearly elastic part
- Elastic Modulus
- Young Modulus
- kN/mm^2

Hooke's Law

Strain Hardening

- When ductile material is loaded into the *plastic region* and then unloaded, *elastic strain is recovered.*
- The *plastic strain remains* and material is subjected to a ***permanent set***.



Linear Deformation

From Hooke's Law,

$$\sigma = E\varepsilon$$

$$\frac{P}{A} = E\left(\frac{\delta}{L}\right)$$

$$\therefore Deformation,\ \delta = \frac{PL}{AE}$$

Example 14

A steel rod subjected to a loading of 200 kN is having an linear deformation of 0.8 mm. With σ = 275 N/mm^2 and E = 205 N/mm^2, calculate the minimum rod diameter and length that is suitable to be used.

Example 14 (cont.)

Solution

Internal force P = 200 kN.

Stress, $\sigma = \dfrac{P}{A} \leq 275\, N/mm^2$

$$\therefore A \geq \frac{200\,x\,10^3}{275}$$
$$\geq 727.3\, mm^2$$

Area of rod, $A = \dfrac{\pi D^2}{4} \geq 727.3\, mm^2$

$$\therefore D \geq \sqrt{\frac{727.3\,x\,4}{\pi}}$$
$$\geq 30.4\, mm$$

Elongation, $\delta = \dfrac{PL}{AE} = \dfrac{\sigma L}{E} \leq 0.8\, mm^2$

$$\therefore L \geq \frac{(205\,x\,10^3)(0.8)}{275}$$
$$\geq 596.4\, mm$$

Linear Deformation

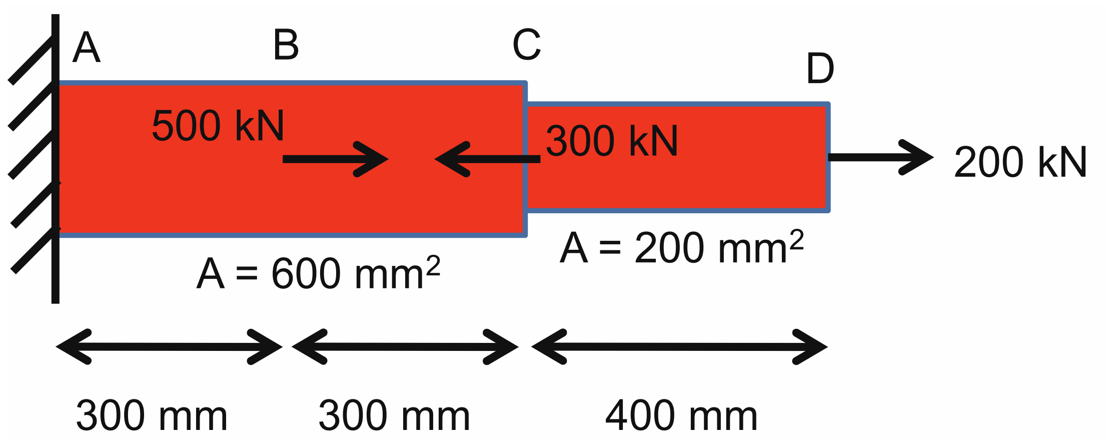
If the material is subjected to loads at several points, or consists of several cross-sectional areas, or make of different materials, the material have to be divided into appropriate segments.

Hence;

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Example 15

Calculate the linear deformation occurred on the rod as shown in the figure. ($E = 200\ kN/mm^2$)



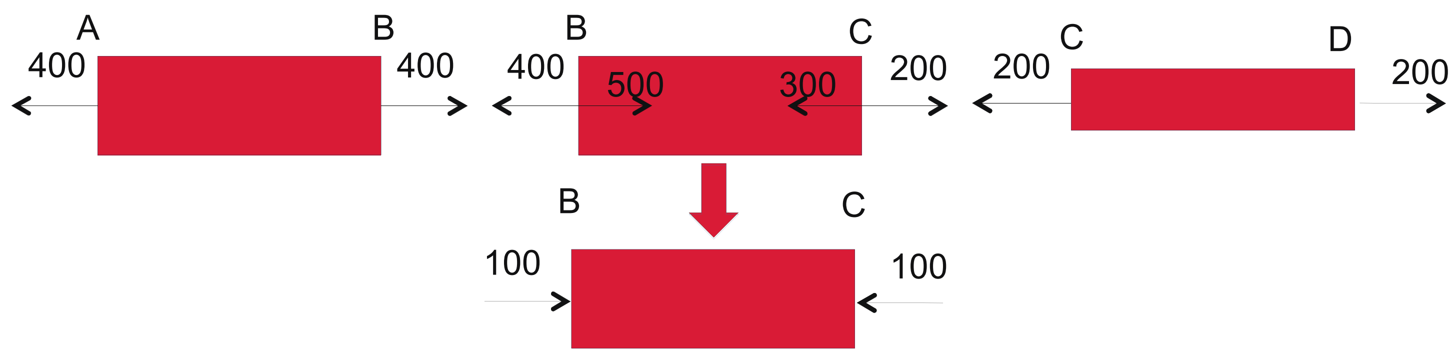
Example 15 (cont.)

Solution

Calculate the reaction at A, R_A.

$$\xrightarrow{+} \sum F_x = 0; \; 500 - 300 + 200 - R_A = 0$$

$$\therefore R_A = 400\,kN \leftarrow$$



Tension (+)
L_{AB} = 300 mm
A_{AB} = 600 mm^2
P_{AB} = 400 kN

Compression (-)
L_{BC} = 300 mm
A_{BC} = 600 mm^2
P_{BC} = -100 kN

Tension (+)
L_{CD} = 400 mm
A_{CD} = 200 mm^2
P_{CD} = 200 kN

Example 15 (cont.)

Solution (cont.)

Elongation,

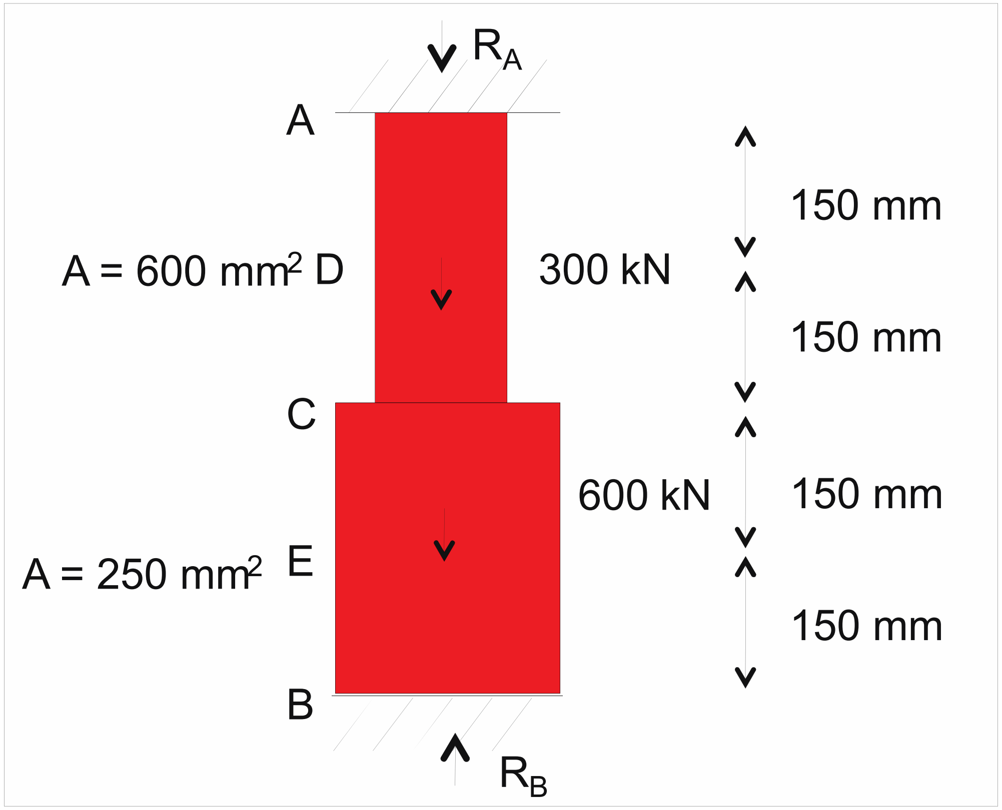
$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{P_{AB} L_{AB}}{A_{AB} E_{AB}} + \frac{P_{BC} L_{BC}}{A_{BC} E_{BC}} + \frac{P_{CD} L_{CD}}{A_{CD} E_{CD}}$$

$$= \frac{1}{200}\left(\frac{400 \times 300}{600} - \frac{100 \times 300}{600} + \frac{200 \times 400}{200} \right)$$

$$= 2.75\ mm$$

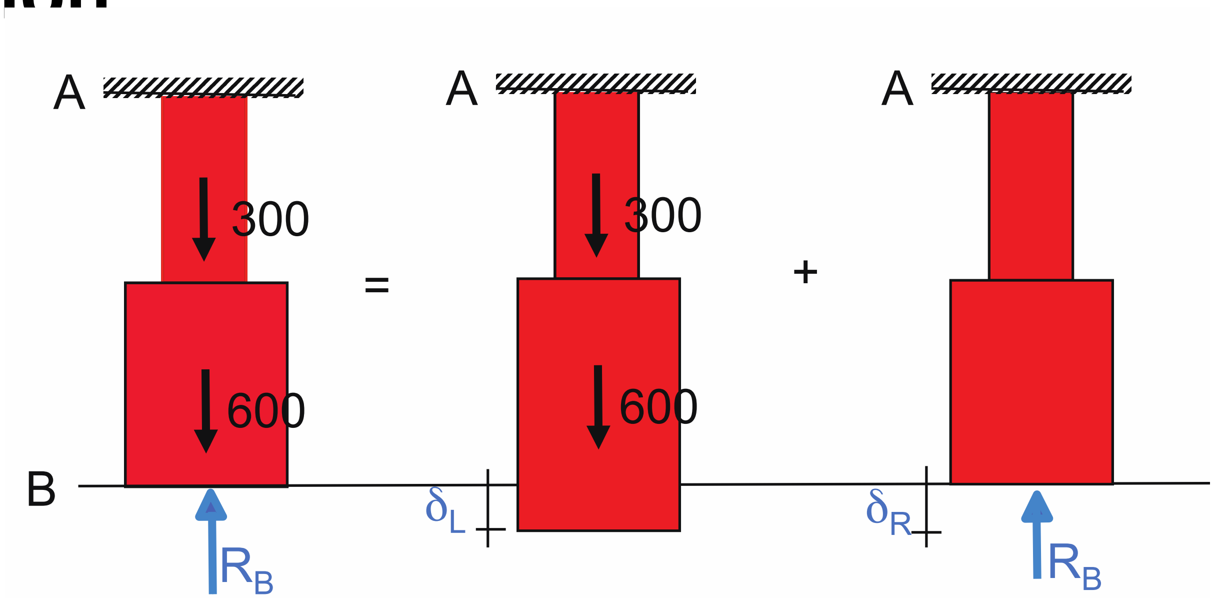
Example 16

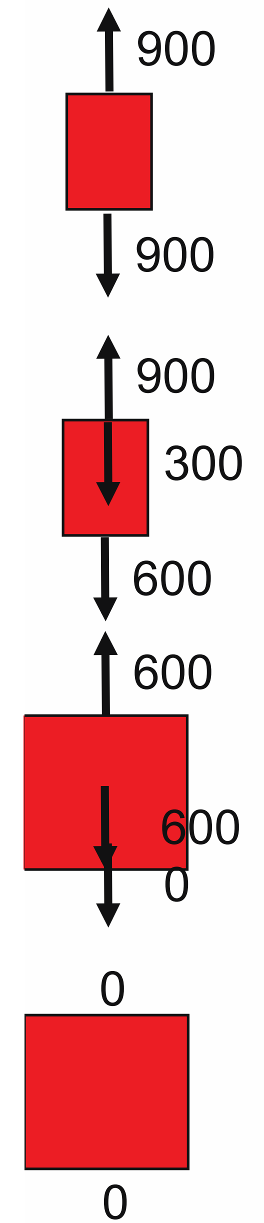
Calculate the reaction on A and B for steel bar and loading given in the figure.



Example 16 (cont.)

Solution





The total elongation is determined by removing one support i.e at B.

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

$$= \frac{1}{E}\left(0 + \frac{600 \times 150}{400} + \frac{600 \times 150}{250} + \frac{900 \times 150}{250}\right) \quad = \frac{1125}{E}$$

Example 16 (cont.)

Solution

A force, R_B is needed to return the bar to its initial state. Let say, the shortening due to the force R_B is δ_R. Therefore:

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i}$$

$$= \frac{R_B}{E}\left(\frac{300}{400} + \frac{300}{250}\right) = \frac{1.95 R_B}{E}$$

It can be seen that, $\delta_R = \delta_L$

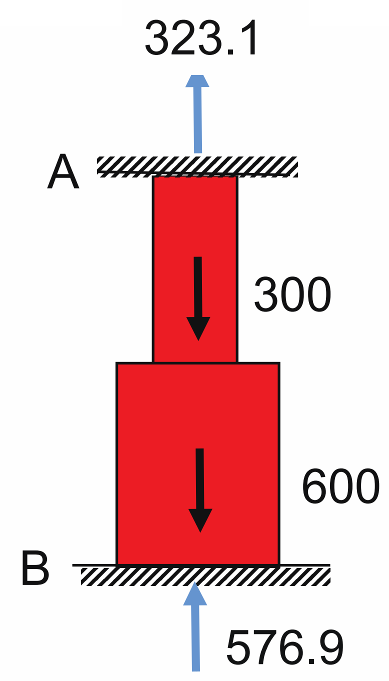
$$\frac{1.95 R_B}{E} = \frac{1125}{E}$$

$$\therefore R_B = \frac{1125}{1.95} = 576.9\ kN$$

Reaction at A:

$$\sum F_x = 0;\ R_A - 300 - 600 + 576.9 = 0$$

$$\therefore R_A = 323.1 kN$$

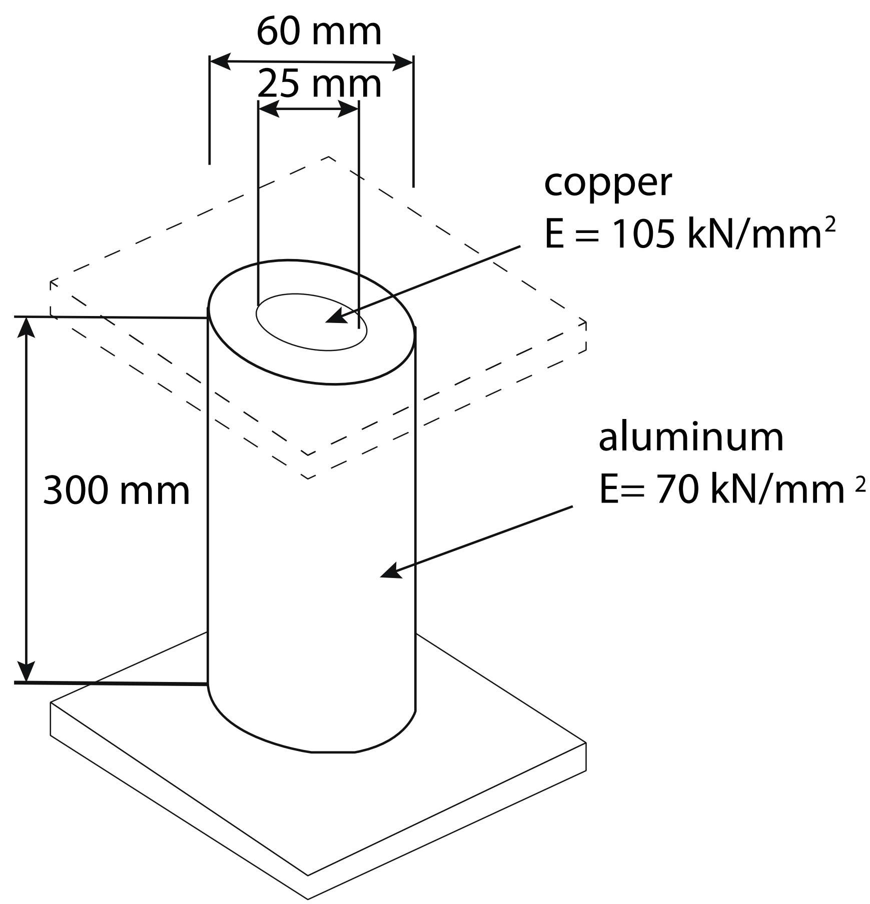


Example 17

An axial load of 200 kN is applied to the end of a member as shown in the figure. Calculate:

a) Normal stress on aluminum

b) Deformation of the member



Example 17 (cont.)

Solution

Shortening for both material is the same, therefore:

$$\delta_{aluminum} = \delta_{copper}$$

$$\frac{P_a L_a}{A_a E_a} = \frac{P_c L_c}{A_c E_c} \qquad - (1)$$

The total axial load applied on both material is 200 kN, therefore:

$$P_a + P_c = 200$$

$$P_c = 200 - P_a \qquad - (2)$$

Example 17 (cont.)

Solution (cont.)

Replace equation (2) into equation (1), thus

$$\frac{P_a L_a}{A_a E_a} = \frac{(200 - P_a) L_c}{A_c E_c}$$

$$\frac{P_a \times 300}{(\frac{\pi \times 60^2}{4} - \frac{\pi \times 25^2}{4}) \times 70} = \frac{(200 - P_a) \times 300}{(\frac{\pi \times 25^2}{4}) \times 105}$$

$$0.0018 P_a = 1.164 - 0.0058 P_a$$

$$0.0076 P_a = 1.164$$

$$P_a = 153.16 \text{ kN}$$

$$P_c = 200 - P_a$$

$$P_c = 200 - 153.16$$

$$P_c = 46.84 \text{ kN}$$

Example 17 (cont.)

Solution (cont.)

a) Normal stress on aluminum:

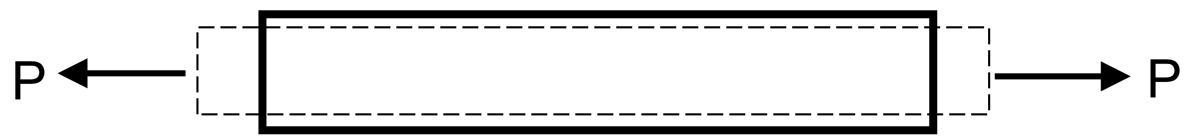
$$\sigma_a = \frac{P_a}{A_a} = \frac{153.16 \times 10^3}{\left(\frac{\pi \times 60^2}{4} - \frac{\pi \times 25^2}{4}\right)} = 65.55 N/mm^2$$

b) Deformation of the member:

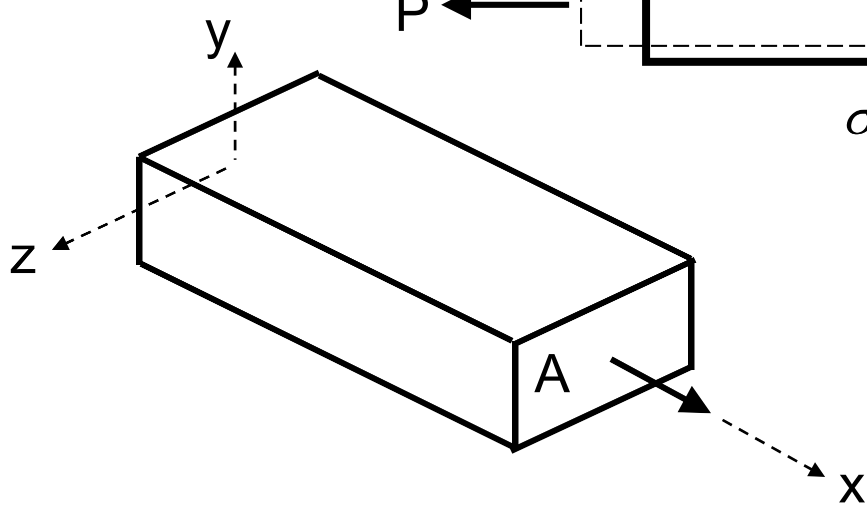
$$\delta_a = \frac{P_a L_a}{A_a E_a} = \frac{153.16 \times 300}{\left(\frac{\pi \times 60^2}{4} - \frac{\pi \times 25^2}{4}\right) \times 70} = 0.28 mm$$

Poisson's Ratio

- For most of the engineering materials, the axial elongation due to an axial load, *P* occurs simultaneously with the lateral shrinkage.



$\sigma_y = \sigma_z = 0$



$$\sigma_x = \frac{P}{A}$$

$$\varepsilon_x = \frac{\sigma_x}{E} \quad \text{(axial strain)}$$

Poisson's Ratio

For isotropic material (the material that has only one behaviour or property),

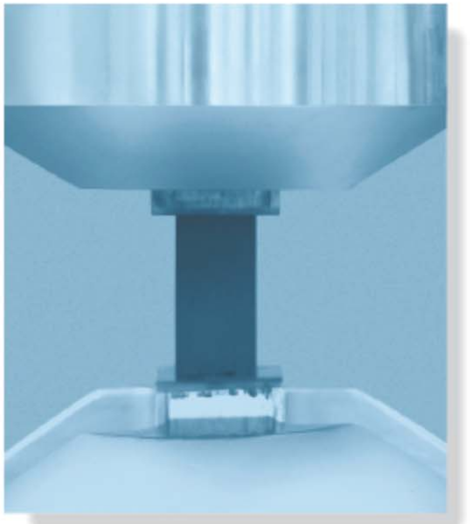
$\varepsilon_y = \varepsilon_z$ (lateral strain)

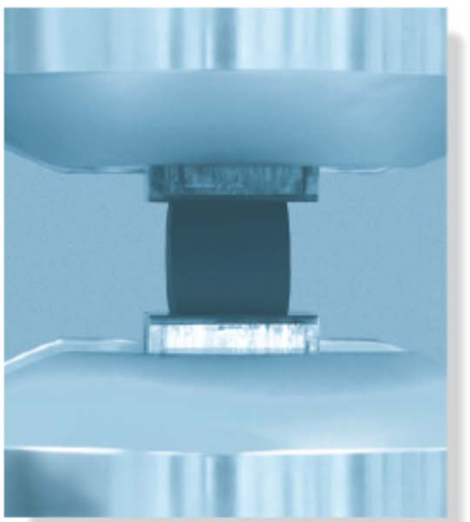
- Poisson Ratio,

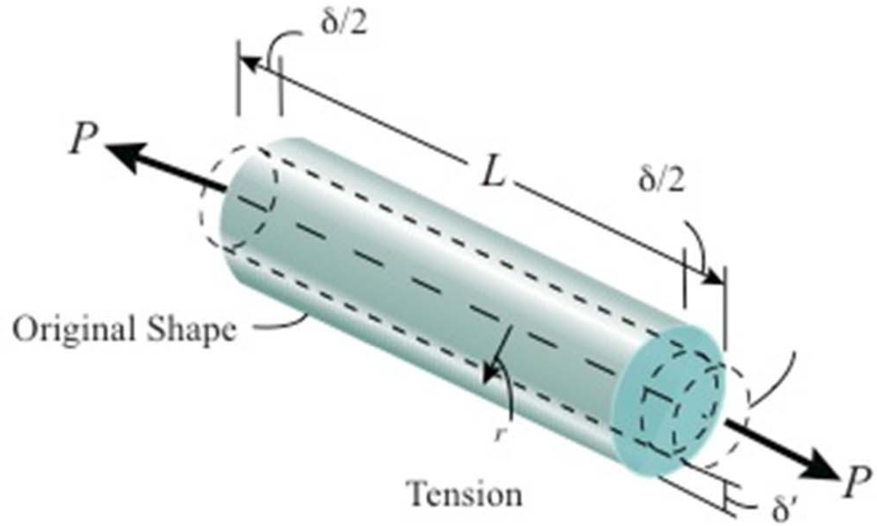
$$\nu = \frac{lateral \ \ strain}{axial \ \ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\therefore \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\frac{\nu\sigma_x}{E}$$

Poisson's Ratio



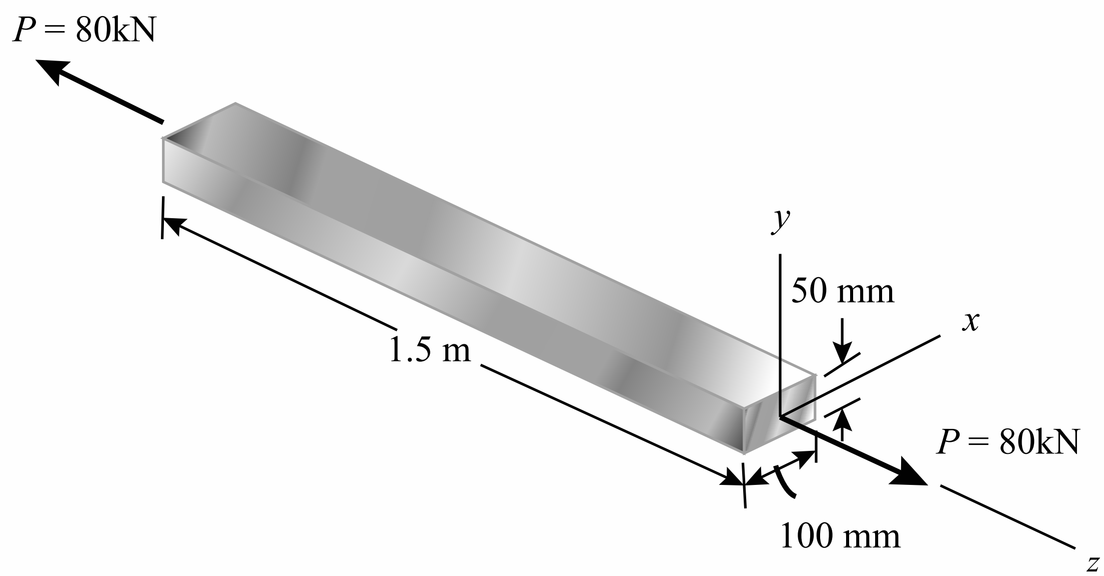




$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

Example 18

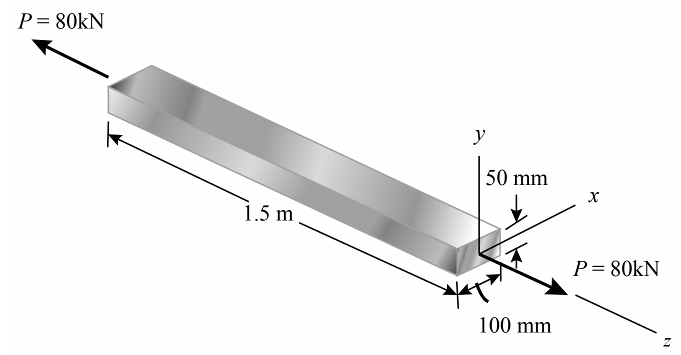
A bar made of A-36 steel has the dimensions shown below. If an axial force of P = 80kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.



Example 18 (Cont.)

Solutions

- The normal stress in the bar is



$$\sigma_z = \frac{P}{A} = \frac{80(10^3)}{(0.1)(0.05)} = 16.0(10^6)\,\text{Pa}$$

- From the table for A-36 steel, E_{st} = 200 GPa

$$\varepsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)}{200(10^6)} = 80(10^{-6})\,\text{mm/mm}$$

- The axial elongation of the bar is therefore

$$\delta_z = \varepsilon_z L_z = [80(10^{-6})(1.5)] = 120\,\mu\text{m} \quad (\text{Ans})$$

Example 18 (cont.)

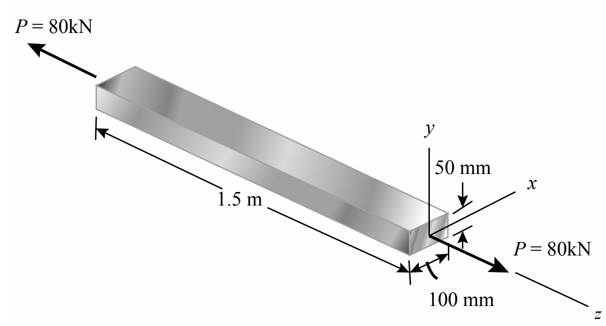
Solutions

- The contraction strains in *both* the *x* and *y* directions are

$$\varepsilon_x = \varepsilon_y = -\nu_{st}\varepsilon_z = -0.32\left[80\left(10^{-6}\right)\right] = -25.6\ \mu\text{m/m}$$

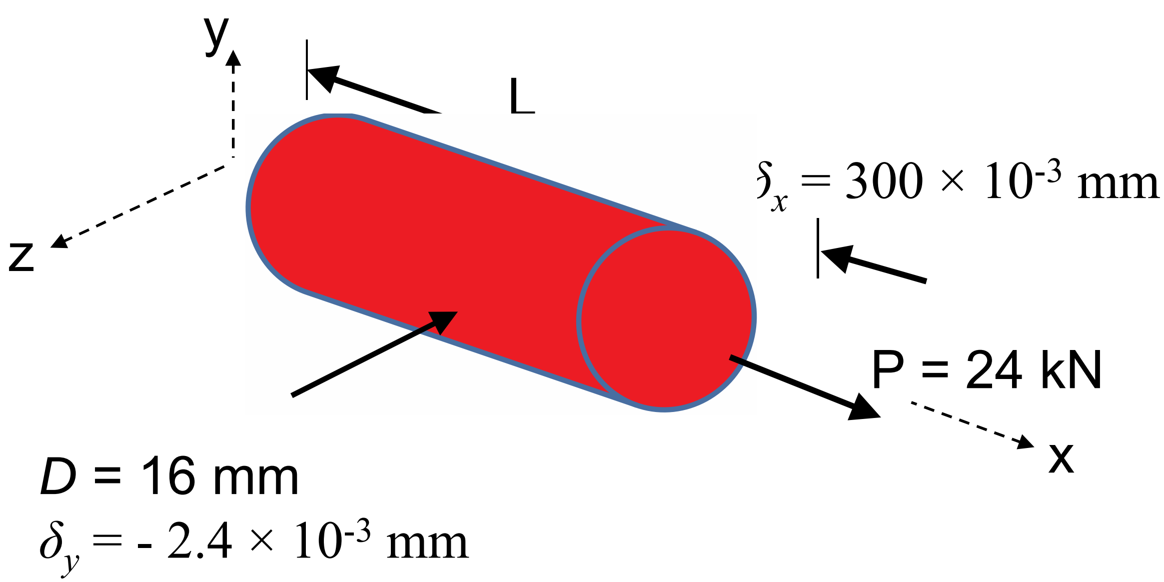
- The changes in the dimensions of the cross section are

$$\delta_x = \varepsilon_x L_x = -\left[25.6\left(10^{-6}\right)(0.1)\right] = -2.56\mu\text{m}\ \ (\text{Ans})$$
$$\delta_y = \varepsilon_y L_y = -\left[25.6\left(10^{-6}\right)(0.-05)\right] = -1.28\mu\text{m}\ \ (\text{Ans})$$



Example 19

A rod with 16 mm diameter and 500 mm in length was found to have an elongation of 300 × 10^{-3} mm and a contraction in diameter of 2.4 × 10^{-3} mm when subjected to an axial load of 24 kN. Calculate the Elastic Modulus, *E* and poisson ratio, *v* of that material.



Example 19 (cont.)

Area of the rod, $$A = \frac{\pi D^2}{4}$$

Stress in x-direction, $$\sigma_x = \frac{P}{A}$$

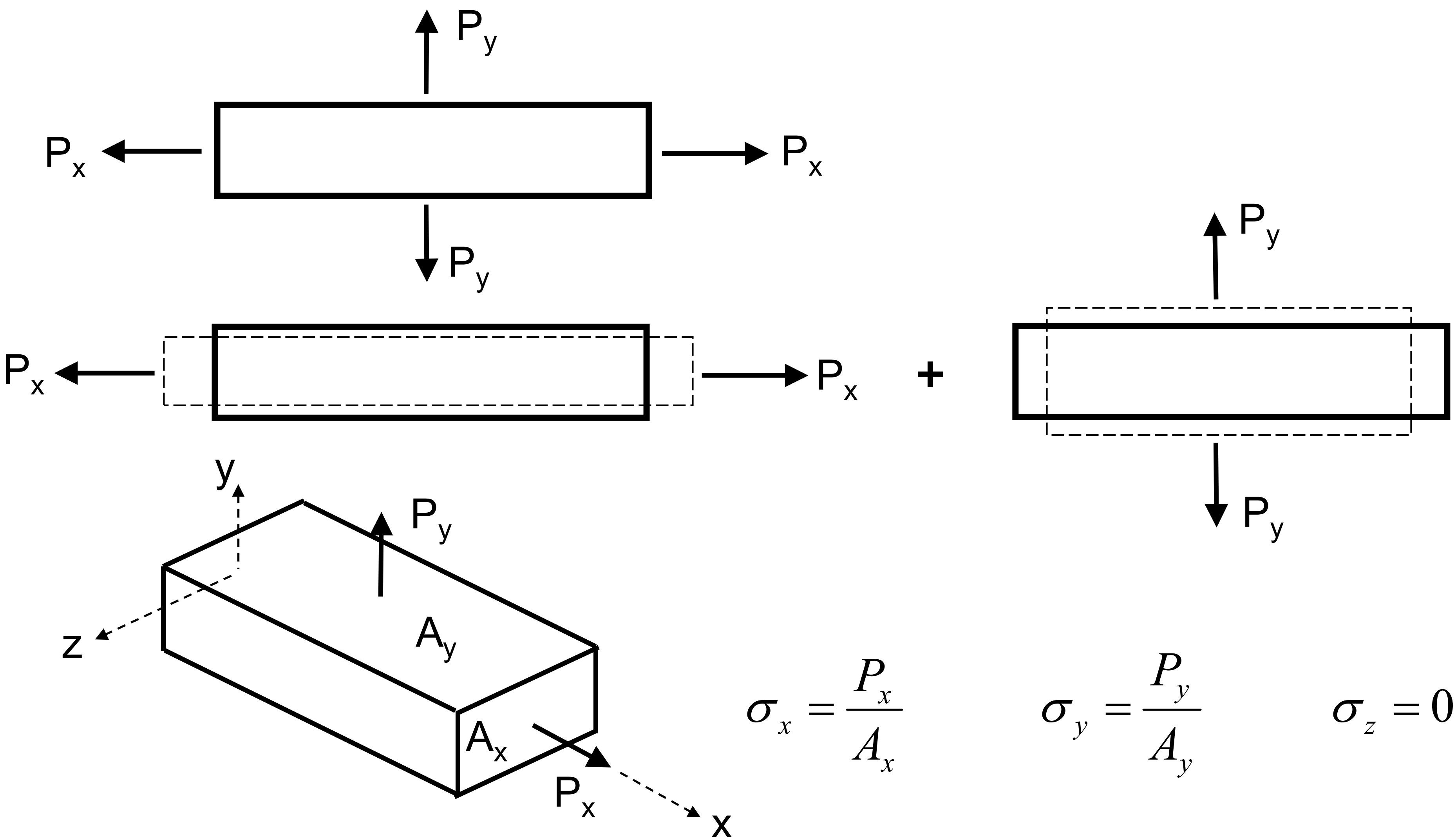
Strain in x-direction, $$\varepsilon_x = \frac{\delta_x}{L}$$

Strain in y-direction, $$\varepsilon_y = \frac{\delta_y}{D}$$

Elastic Modulus, $$E = \frac{\sigma_x}{\varepsilon_x}$$

Poisson Ratio, $$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

Bi-Axial Loads



$$\sigma_x = \frac{P_x}{A_x} \qquad \sigma_y = \frac{P_y}{A_y} \qquad \sigma_z = 0$$

Bi-Axial Loads

Strain in *x*-direction due to σ_x ,

$$\varepsilon_{xx} = \frac{\sigma_x}{E}$$

Strain in *x*-direction due to σ_y ,

$$\varepsilon_{xy} = -\frac{v\sigma_y}{E}$$

Therefore, the total strain in *x*-direction ,

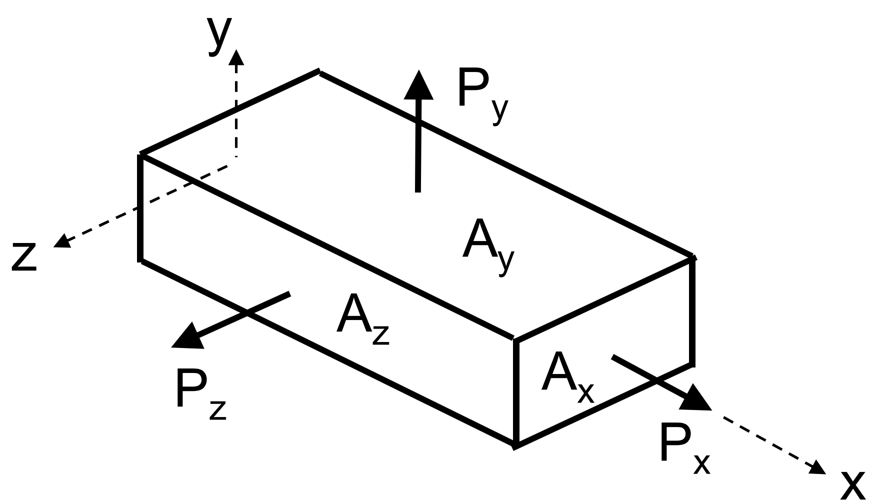
$$\varepsilon_x = \varepsilon_{xx} + \varepsilon_{xy} = \frac{\sigma_x}{E} - \frac{v\sigma_y}{E} = \frac{1}{E}(\sigma_x - v\sigma_y)$$

Bi-Axial Loads

In the same way, the total strain in *y*-direction ,

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{v\sigma_x}{E} = \frac{1}{E}(\sigma_y - v\sigma_x)$$

Tri-Axial Loads



$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right)$$

$$= \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right)$$

Tri-Axial Loads

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x - \nu\sigma_z\right)$$

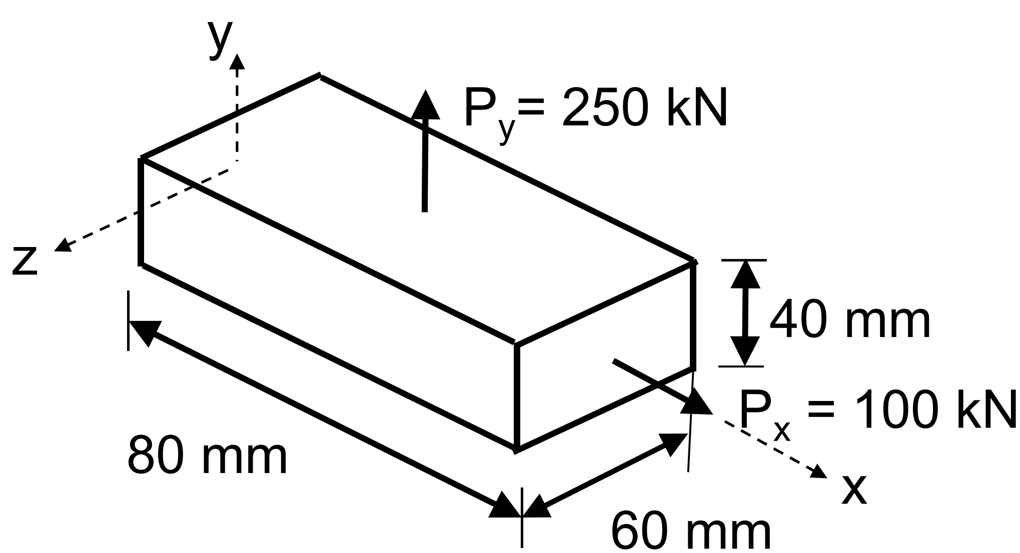
$$= \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_x - \nu\sigma_y\right)$$

$$= \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right)$$

Example 20

A block is subjected to 100kN force in x-direction and 250kN in y-direction as shown in the figure. Calculate the deformation in x-, y- and z-direction. E = 200 kN/mm^2 and v = 0.25.



UTM

Example 20 (cont.)

Solution

$$\sigma_x = \frac{P_x}{A_x} = \frac{100 \times 10^3}{40 \times 60} = 41.67\, N/mm^2$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{250 \times 10^3}{60 \times 80} = 52.08\, N/mm^2$$

$$\sigma_z = 0$$

Example 20 (cont.)

Solution (cont.)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v(\sigma_y + \sigma_z))$$

$$= \frac{1}{200\times10^3}(41.67 - 0.25(52.08 + 0)) = 1.43\times10^{-4}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v(\sigma_x + \sigma_z))$$

$$= \frac{1}{200\times10^3}(52.08 - 0.25(41.67 + 0)) = 2.08\times10^{-4}$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - v(\sigma_x + \sigma_y))$$

$$= \frac{1}{200\times10^3}(0 - 0.25(41.67 + 52.08)) = -1.17\times10^{-4}$$

Example 20 (cont.)

Solution (cont.)

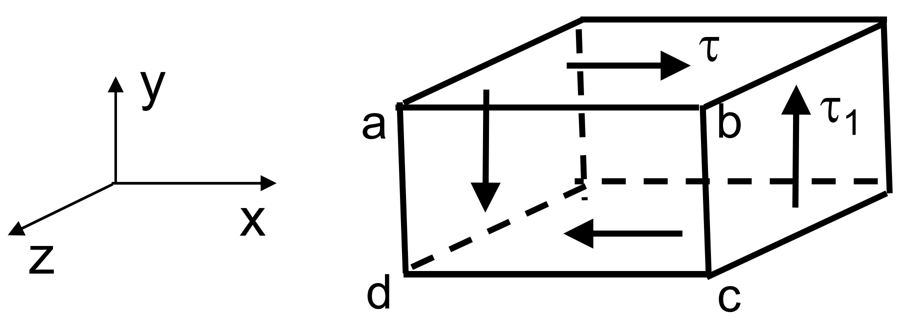
$$\delta_x = \varepsilon_x L_x = 1.43 \times 10^{-4} \times 80 = 0.0115\, mm$$

$$\delta_y = \varepsilon_y L_y = 2.08 \times 10^{-4} \times 40 = 0.0083\, mm$$

$$\delta_z = \varepsilon_z L_z = -1.17 \times 10^{-4} \times 60 = -0.0070\, mm$$

Shearing Stress and Shearing Strain

- Observing a certain body subjected to shearing stress acting parallel or tangent to the surface of the body.
- An element subjected to shearing stress:

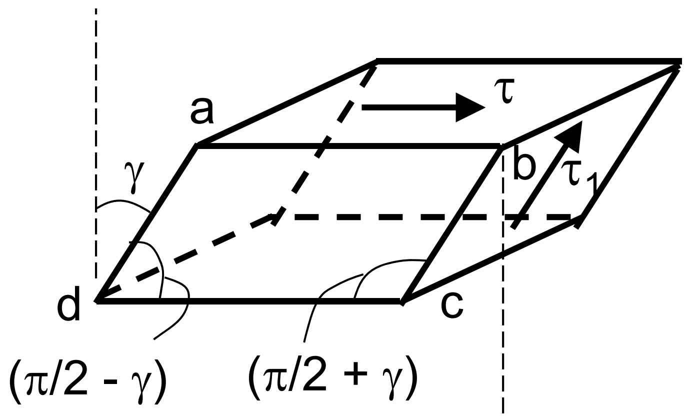


It is proven that $\tau = \tau_1$.

(These are +ve sign conventions)

Shearing Stress and Shearing Strain

- The deformation due to the shearing stress:



The shearing strain of the element is the angle at b or d which is γ (in radian).

- The shearing stress-strain relationship is:

$$\tau = G\gamma$$

where G = Shear Elastic Modulus (Modulus of Rigidity)

Shearing Stress and Shearing Strain

- The relationship between Modulus of Rigidity , *G* and Modulus of Elasticity, *E* is :

$$G = \frac{E}{2(1+\nu)}$$

Shear Stress-Strain Diagram

- Strength parameter G – Shear modulus of elasticity or the modules of rigidity
- G is related to the modulus of elasticity E and Poisson's ratio *v*.

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+v)}$$

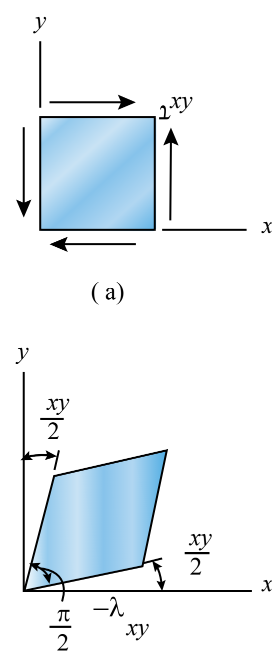


Fig. 3-23

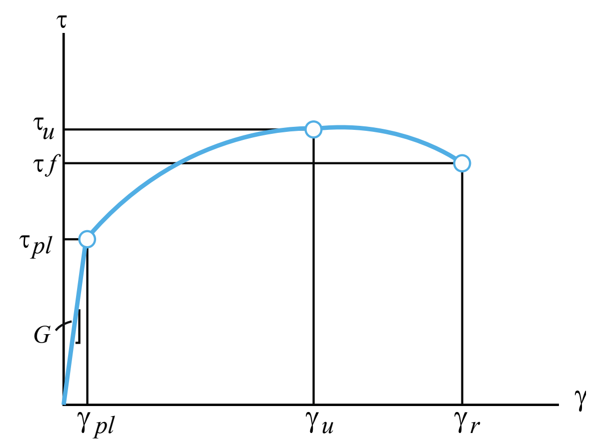
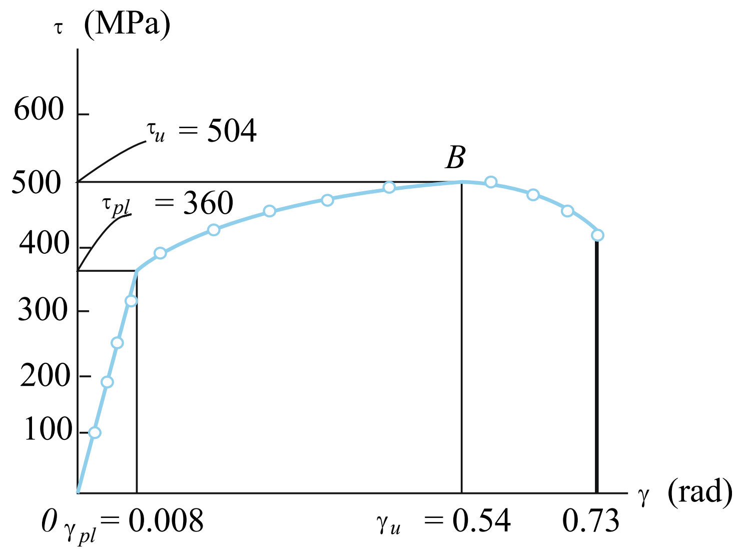


Fig. 3-24

Example 21

A specimen of titanium alloy is tested in torsion and the shear stress– strain diagram is shown below. Determine the shear modulus *G*, the proportional limit, and the ultimate shear stress. Also, determine the maximum distance *d* that the top of a block of this material, could be displaced horizontally if the material behaves elastically when acted upon by a shear force **V**. What is the magnitude of **V** necessary to cause this displacement?



Example 21 (cont.)

Solutions

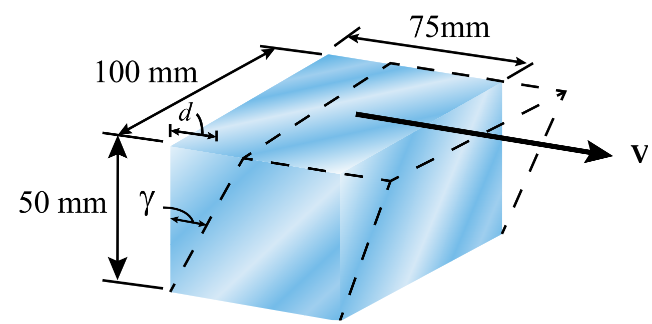
- By inspection, the graph ceases to be linear at point *A*. Thus, the proportional limit is

$$\tau_{pl} = 360\ \text{MPa (Ans)}$$

- This value represents the maximum shear stress, point *B*. Thus the ultimate stress is

$$\tau_u = 504\ \text{MPa (Ans)}$$

- Since the angle is small, the top of the will be displaced horizontally by



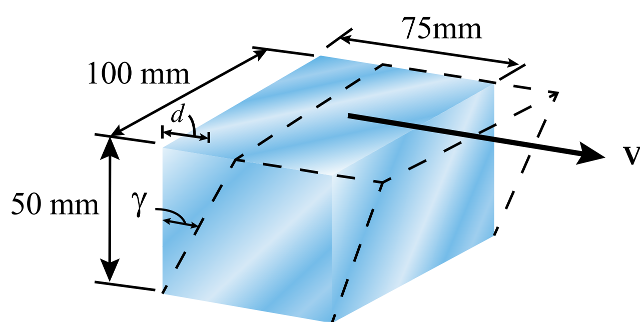
$$\tan(0.008\ \text{rad}) \approx 0.008 = \frac{d}{50\ \text{mm}} \Rightarrow d = 0.4\ \text{mm}$$

Example 21 (cont.)

Solutions

- The shear force *V* needed to cause the displacement is

$$\tau_{avg} = \frac{V}{A}; \qquad 360\text{ MPa} = \frac{V}{(75)(100)} \Rightarrow V = 2700\text{ kN (Ans)}$$



Example 22

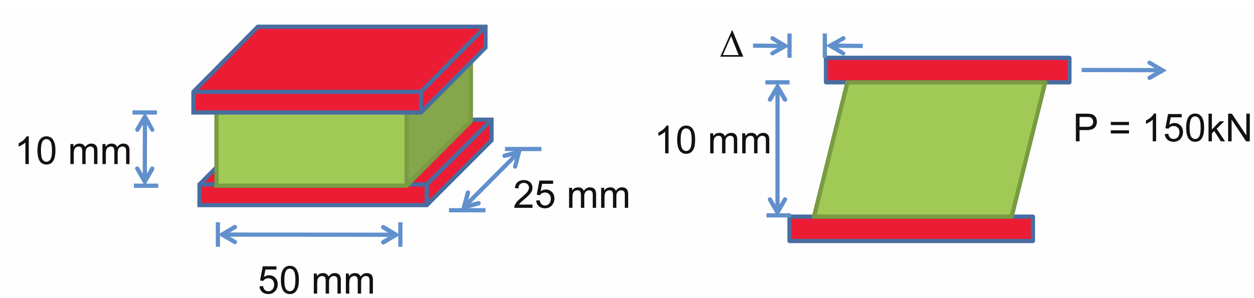
A rubber type of material with 10mm thickness has a G value of 0.75 N/mm^2 placed between two plate to absorb a shear force, P of 150kN as shown in the figure.

a) Determine shear stress, τ

b) Determine shear strain, γ for the material

c) Determine Elastic Modulus, *E* if *v* = 0.5

d) Lateral movement of the plate, Δ



Example 22 (cont.)

Solution

a) $\tau = \frac{P}{A} = \frac{150}{50 \times 25} = 0.12 N / mm^2$

b) $\tau = G\gamma$

$$\gamma = \frac{\tau}{G} = \frac{0.12}{0.75} = 0.16 Rad$$

c) $G = \frac{E}{2(1+v)}$

$$E = 0.75 \times 2 \times (1 + 0.5) = 2.25 N / mm^2$$

d) $\tau = G\gamma$

$$\frac{\Delta}{10} = \frac{\tau}{G}$$

$$\Delta = \frac{0.12 \times 10}{0.75} = 1.6 mm$$

Elastic Deformation of an Axially Loaded Member

$$\sigma = \frac{P(x)}{A(x)} \quad \text{and} \quad \varepsilon = \frac{d\delta}{dx}$$

- Provided these quantities do not exceed the proportional limit, we can relate them using Hooke's Law, i.e. σ = E ε

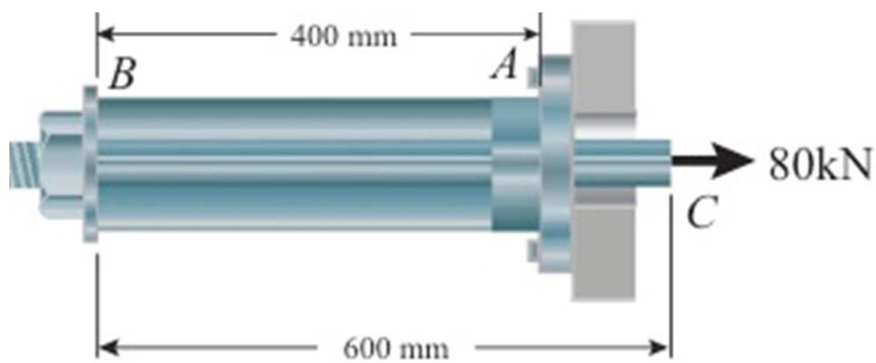
$$\frac{P(x)}{A(x)} = E\left(\frac{d\delta}{dx}\right)$$

$$d\delta = \frac{P(x)dx}{A(x)E}$$

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

Example 23

The assembly shown below consists of an aluminum tube *AB* having a cross-sectional area of 400 mm^2. A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end *C* of the rod. Take E_{st} = 200 GPa, E_{al} = 70 GPa.



(a)

Example 23 (cont.)

Solutions

- Find the displacement of end *C* with respect to end *B*.

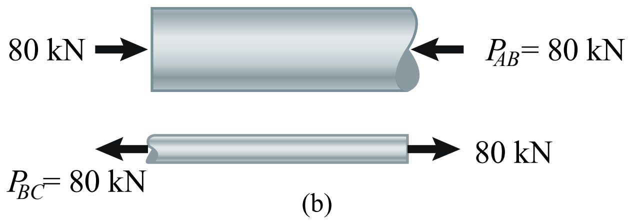
$$\delta_{C/B} = \frac{PL}{AE} = \frac{\left[+80\left(10^3\right)\right](0.6)}{\pi(0.005)\left[200\left(10^9\right)\right]} = +0.003056\,m \rightarrow$$

- Displacement of end *B* with respect to the *fixed* end *A,*

$$\delta_B = \frac{PL}{AE} = \frac{\left[-80\left(10^3\right)\right](0.4)}{\left[400\left(10^{-6}\right)\right]\left[70\left(10^9\right)\right]} = -0.001143 = 0.001143\text{ m} \rightarrow$$

- Since both displacements are to the right,

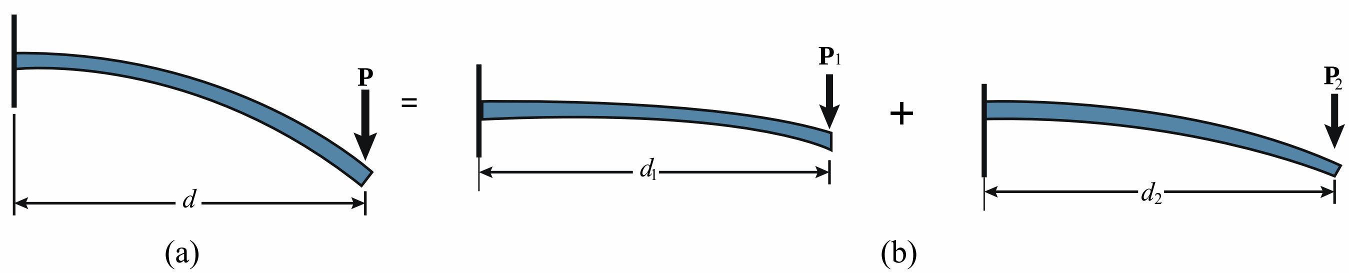
$$\delta_C = \delta_C + \delta_{C/B} = 0.0042\text{ m} = 4.20\text{ mm} \rightarrow$$



(b)

Principle of Superposition

- It can be used to simply problems having complicated loadings. This is done by dividing the loading into components, then algebraically adding the results.
- It is applicable provided the material obeys Hooke's Law and the deformation is small.
- If P = P1 + P2 and d ≈ d1 ≈ d2, then the deflection at location x is sum of two cases, $\delta_x = \delta_{x1} + \delta_{x2}$



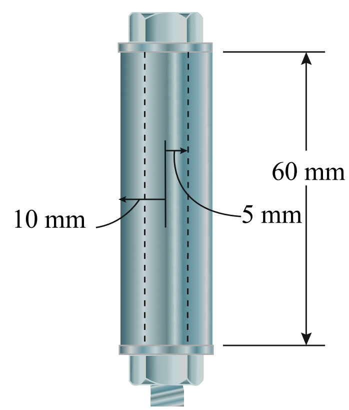
Compatibility Conditions

- When the force equilibrium condition alone cannot determine the solution, the structural member is called **statically indeterminate.**

- In this case, compatibility conditions at the constraint locations shall be used to obtain the solution. For example, the stresses and elongations in the 3 steel wires are different, but their displacement at the common joint A must be the same.

Example 24

The bolt is made of 2014-T6 aluminum alloy and is tightened so it compresses a cylindrical tube made of Am 1004-T61 magnesium alloy. The tube has an outer radius of 10 mm, and both the inner radius of the tube and the radius of the bolt are 5 mm. The washers at the top and bottom of the tube are considered to be rigid and have a negligible thickness. Initially the nut is hand-tightened slightly; then, using a wrench, the nut is further tightened one-half turn. If the bolt has 20 threads per inch, determine the stress in the bolt.



Example 24 (cont.)

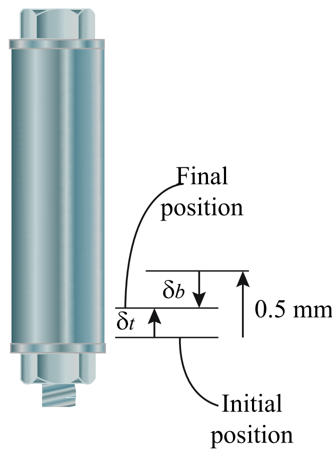
Solutions

- Equilibrium requires

$$+\uparrow \sum F_y = 0; \qquad F_b - F_t = 0 \qquad (1)$$

- When the nut is tightened on the bolt, the tube will shorten.

$$(+\uparrow) \qquad \delta_t = 0.5 - \delta_b$$



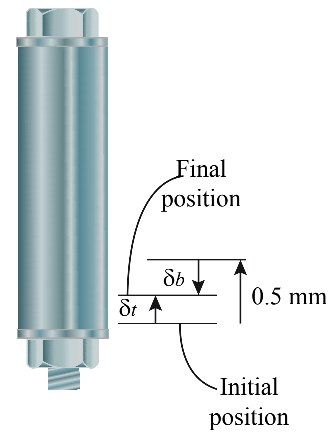
Example 24 (cont.)

Solutions

- Taking the 2 modulus of elasticity,

$$\frac{F_t(60)}{\pi\left[10^2-5^2\right]\left[45\left(10^3\right)\right]}=0.5-\frac{F_b(60)}{\pi\left[5^2\right]\left[75\left(10^3\right)\right]}$$

$$5F_t=125\pi(1125)-9F_b \qquad (2)$$



- Solving Eqs. 1 and 2 simultaneously, we get

$$F_b=F_t=31556=31.56\text{ kN}$$

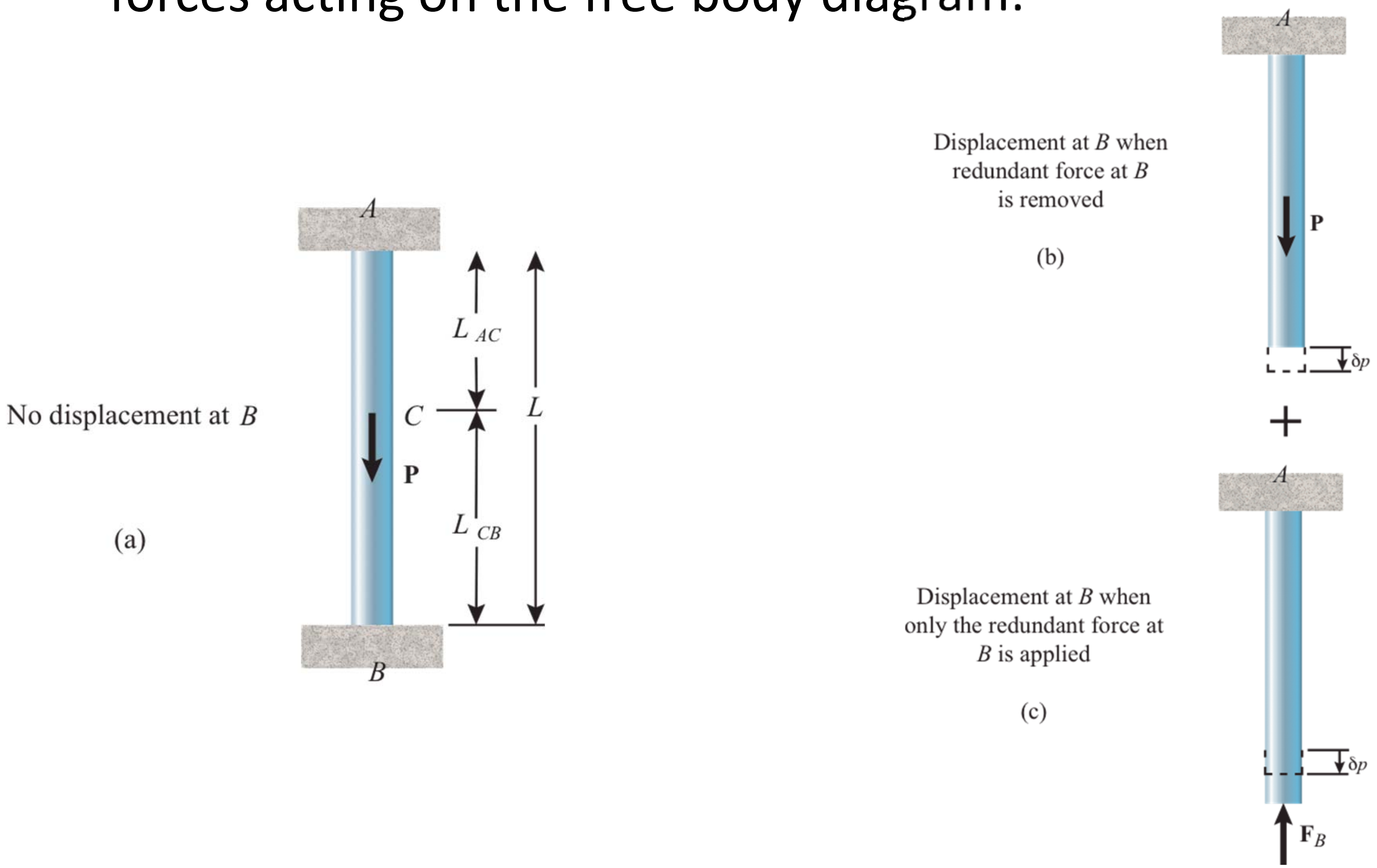
- The stresses in the bolt and tube are therefore

$$\sigma_b=\frac{F_b}{A_b}=\frac{31556}{\pi(5)}=401.8\text{ N/mm}^2=401.8\text{ MPa (Ans)}$$

$$\sigma_s=\frac{F_t}{A_t}=\frac{31556}{\pi\left(10^2-5^2\right)}=133.9\text{ N/mm}^2=133.9\text{ MPa (Ans)}$$

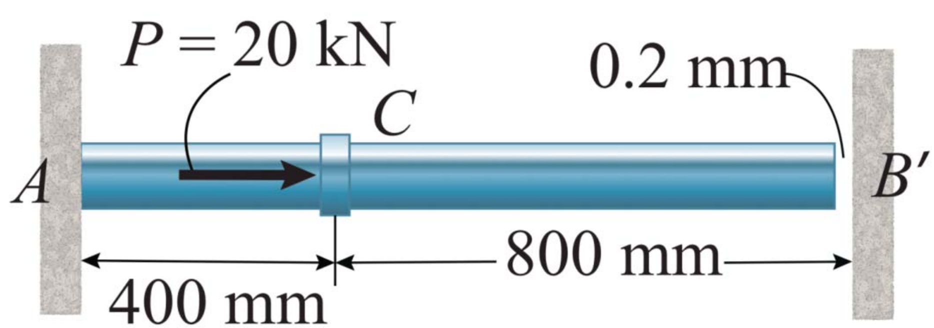
Force Method of Analysis

- It is also possible to solve statically indeterminate problem by writing the compatibility equation using the superposition of the forces acting on the free body diagram.



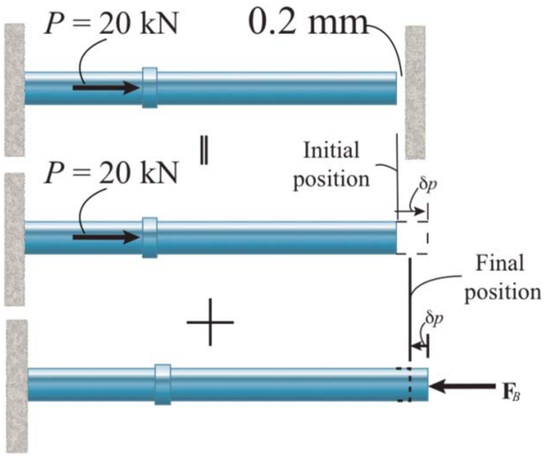
Example 25

The A-36 steel rod shown below has a diameter of 10 mm. It is fixed to the wall at *A*, and before it is loaded there is a gap between the wall at B' and the rod of 0.2 mm. Determine the reactions at *A* and Neglect the size of the collar at *C*. Take Est = 200GPa.



(a)

Example 25 (cont.)



Solutions

- Using the principle of superposition,

$$(\xrightarrow{+}) \quad 0.0002 = \delta_P - \delta_B \qquad (1)$$

- From Eq. 4-2,

$$\delta_P = \frac{PL_{AC}}{AE} = \frac{\left[20\left(10^3\right)\right](0.4)}{\pi(0.005)^2\left[200\left(10^9\right)\right]} = 0.5093\left(10^{-3}\right)$$

$$\delta_B = \frac{F_B L_{AB}}{AE} = \frac{F_B(1.2)}{\pi(0.005)^2\left[200\left(10^9\right)\right]} = 76.3944\left(10^{-9}\right)F_B$$

- Substituting into Eq. 1, we get

$$0.0002 = 0.5093\left(10^{-3}\right) - 76.3944\left(10^{-9}\right)F_B$$

$$F_B = 4.05\left(10^3\right) = 4.05 \text{ kN} \quad \text{(Ans)}$$

Example 25 (cont.)

Solutions

- From the free-body diagram,

$$(+\rightarrow)\ \sum F_x = 0$$

$$-F_A + 20 - 4.05 = 0$$

$$F_A = 16.0\ \text{kN} \quad \text{(Ans)}$$

(c)

Thermal Stress

- Ordinarily, the expansion or contraction δ_T is linearly related to the temperature increase or decrease ΔT that occurs.

$$\delta_T = -\alpha \Delta T L$$

α = ***linear coefficient of thermal expansion***, property of the material (/°C)
ΔT = algebraic change in temperature of the member (°C)
L = original length of the member
δ_T = algebraic change in length of the member

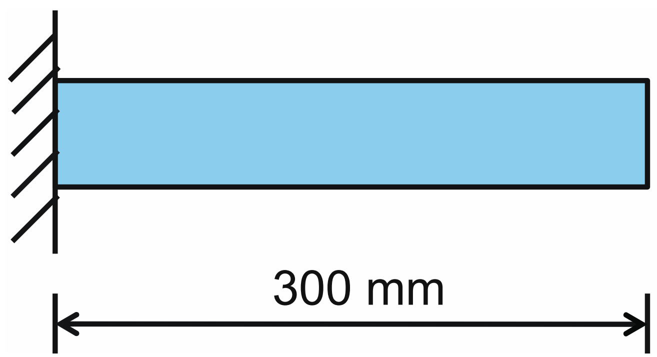
Strain,

$$\varepsilon_t = \frac{\delta_t}{L} = \frac{\alpha(\Delta T)L}{L}$$

$$\therefore \varepsilon_t = \alpha(\Delta T)$$

Example 26

One steel bar with 300 mm length tied on one end and free on the other end exposed to a temperature of 30 ˚C. If the temperature increased to 75 ˚C, calculate the total elongation, δ_t and strain, ε_t. ($\alpha = 13.1 \times 10^{-6}$ / ^0C)



Example 26 (cont.)

Solution

Variation in temperature, $\Delta T = 75 - 30 = 45°C$

$$Elongation, \delta_t = \alpha(\Delta T)L$$

$= (13.1 \times 10^{-6})(45)(300)$

$= 0.177$ mm

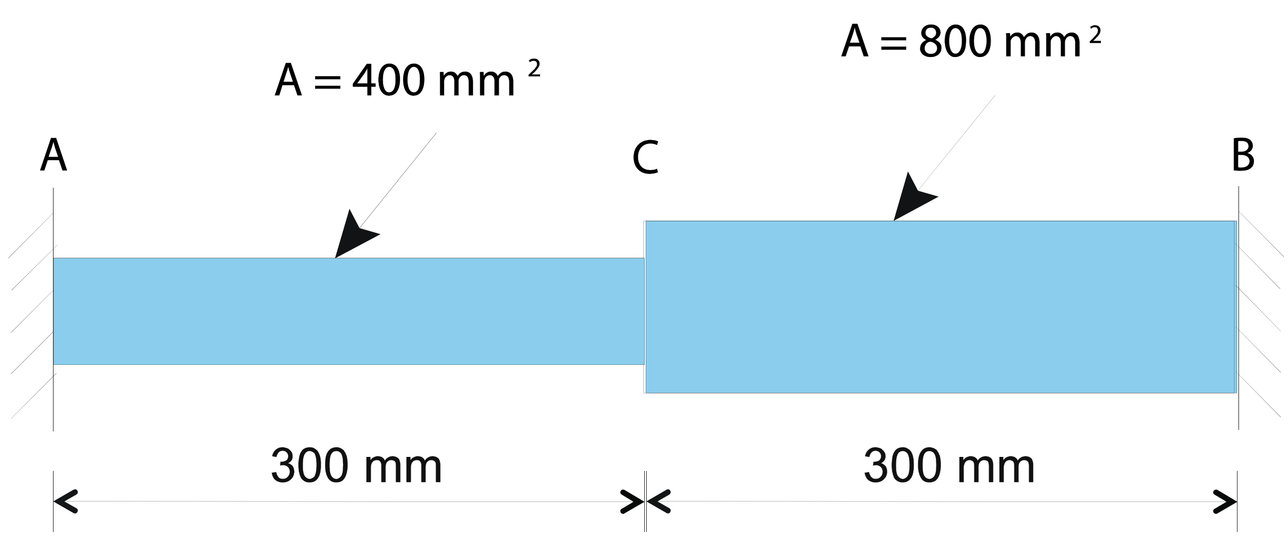
$$Strain, \varepsilon_t = \alpha(\Delta T)$$

$= (13.1 \times 10^{-6})(45)$

$= 5.895 \times 10^{-4}$

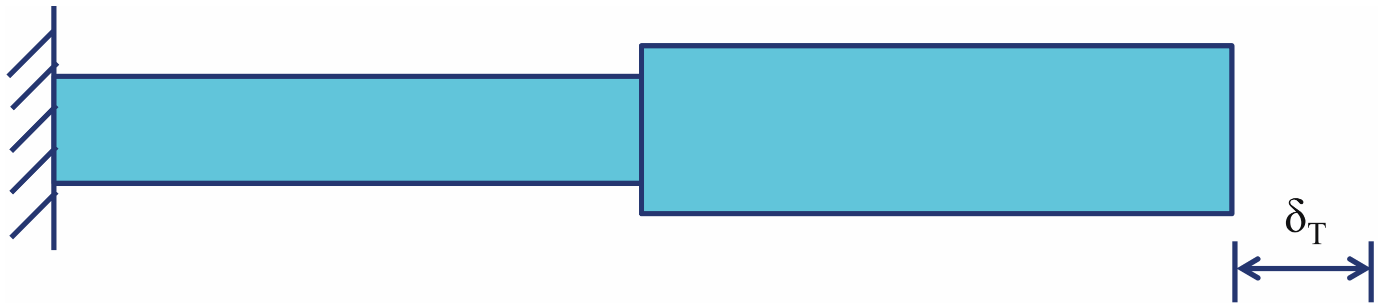
Example 27

Calculate the stress in segment AC and CB for the steel bar when temperature decreased from +25°C (original temperature) to -50°C.



Example 27 (cont.)

Solution



Variation in temperature, $\Delta T = -50 - 25 = -75°C$

Total elongation in the bar can be calculated by taking out the support at B:

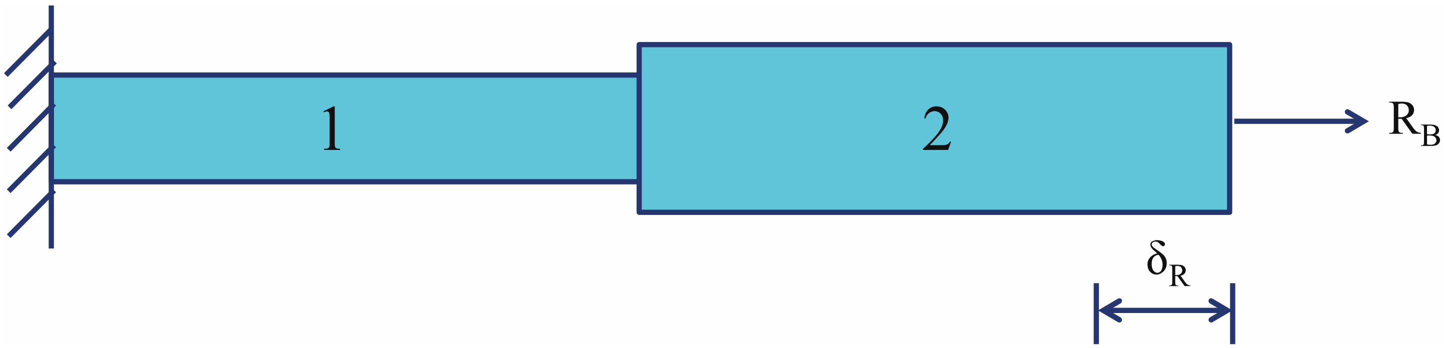
$$Elongation, \delta_t = \alpha(\Delta T)L$$

$$= (12 \times 10^{-6})(75)(600)$$

$$= 0.54 \text{ mm}$$

Example 27 (cont.)

Solution (cont.)



Reaction force R_B is required to return the bar to its original length. The elongation due to R_B is taken as δ_R.

$$P_1 = P_2 = R_B$$

$$Elongation, \delta_R = \sum \frac{P_i L_i}{A_i E} = \left(\frac{P_1 \times 300}{400 \times 200}\right) + \left(\frac{P_2 \times 300}{800 \times 200}\right) = \frac{R_B}{200}\left(\frac{300}{400} + \frac{300}{800}\right)$$

$$= 5.625 \times 10^{-3} R_B$$

Example 27 (cont.)

Solution (cont.)

We know that $\delta_T = \delta_R$

Then, $5.625 \times 10^{-3} R_B = 0.54$

$R_B = 96$ kN

Stress in each segment:

$$\sigma_{AC} = (\frac{96 \times 10^3}{400}) = 240 N / mm^2$$

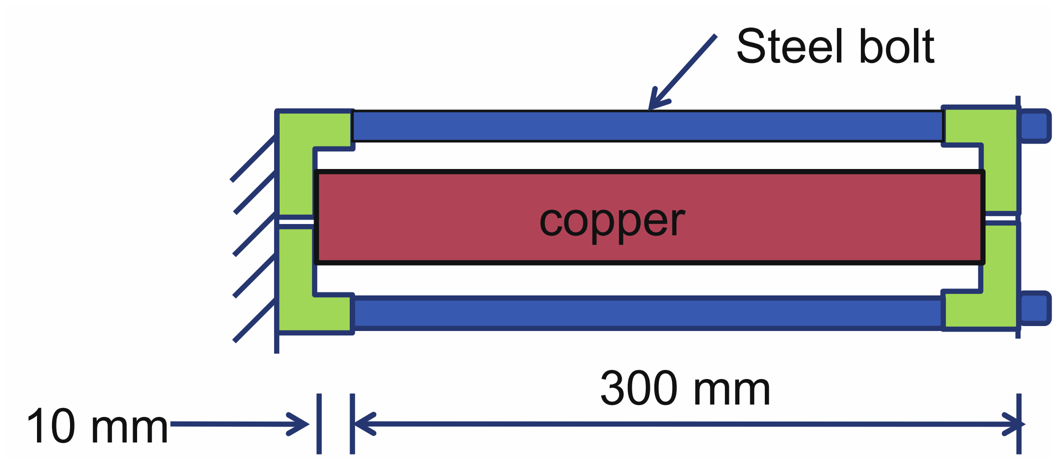
$$\sigma_{CB} = (\frac{96 \times 10^3}{800}) = 120 N / mm^2$$

Example 28

The figure shows one copper rod with 30mm diameter tighten by 2 steel bolts of 20mm diameter. No changes was observed at temperature of 20°C. Calculate the stress in copper and steel at a temperature of 70°C.

E_{steel} = 200 kN/mm^2 α_{steel} = 11.7 × 10^{-6} /°C

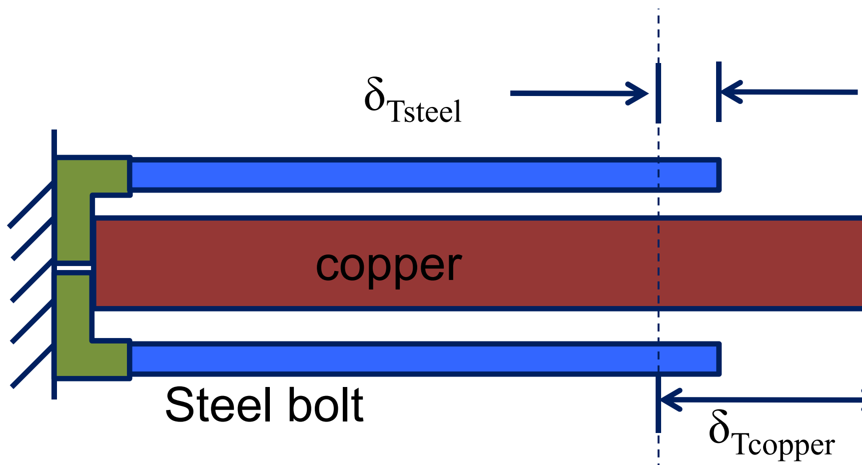
E_{copper} = 83 kN/mm^2 α_{copper} = 18.9 × 10^{-6} /°C



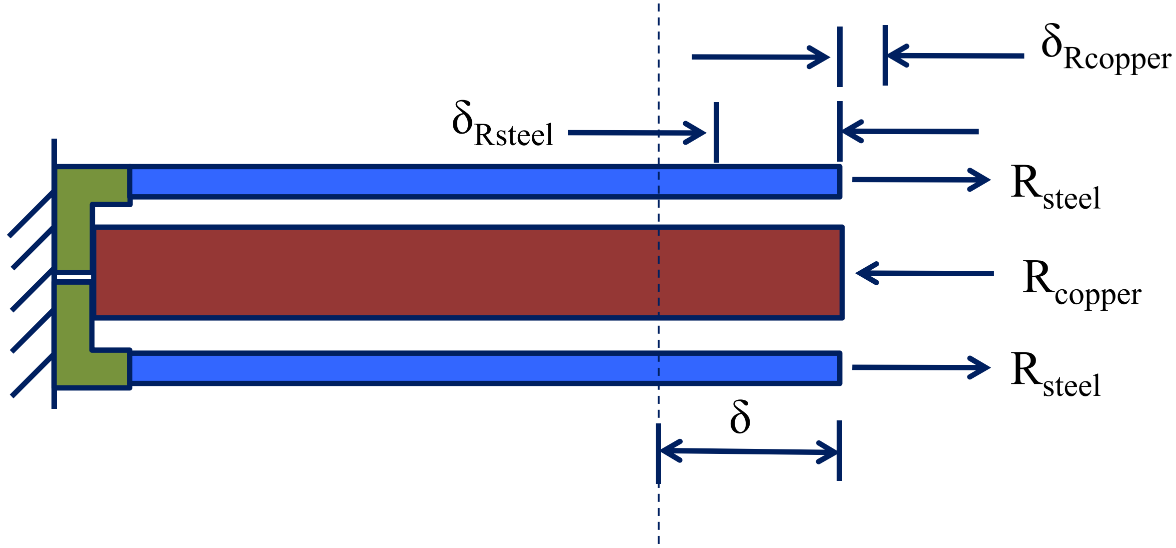
Example 28 (cont.)

Solution

a)



b)



Example 28 (cont.)

Solution (cont.)

Variation in temperature, $\Delta T = 70 - 20 = 50°C$

Elongation due to variation in temperature as shown in (a):

$$\delta_{tsteel} = \alpha_{steel}(\Delta T)L$$

$$= (11.7 \times 10^{-6})(50)(300)$$

$$= 0.1755 \text{ mm}$$

$$\delta_{tcopper} = \alpha_{copper}(\Delta T)L$$

$$= (18.9 \times 10^{-6})(50)(310)$$

$$= 0.2930 \text{ mm}$$

Example 28 (cont.)

Solution (cont.)

Final elongation for both steel and copper should be the same, which the steel bolt will be subjected to a tensile force of R_{steel} and the copper will be subjected to a compression force of R_{copper} (as shown in (b)).

$$Elongation, \delta_{Rsteel} = \frac{R_{steel} L_{steel}}{A_{steel} E_{steel}} = \left(\frac{R_{steel} \times 300}{\frac{\pi \times 20^2}{4} \times 200}\right) = 4.775 \times 10^{-3} R_{steel}$$

$$Elongation, \delta_{Rcopper} = \frac{R_{copper} L_{copper}}{A_{copper} E_{copper}} = \left(\frac{R_{copper} \times 310}{\frac{\pi \times 30^2}{4} \times 83}\right) = 5.284 \times 10^{-3} R_{copper}$$

Example 28 (cont.)

Solution (cont.)

We know that $\delta = \delta_{Tsteel} + \delta_{Rsteel}$ — **(1)**

and $\delta = \delta_{Tcopper} + \delta_{Rcopper}$ — **(2)**

and $R_{copper} = 2R_{steel}$ — **(3)**

(1) = **(2)**, then

$0.1755 + 4.775 \times 10^{-3} R_{steel} = 0.2930 - 5.284 \times 10^{-3} R_{copper}$

$0.1755 + 4.775 \times 10^{-3} R_{steel} = 0.2930 - 5.284 \times 10^{-3} (2R_{steel})$

$15.343 \times 10^{-3} R_{steel} = 0.1175$

R_{steel} = 7.66 kN and R_{copper} = 15.32 kN

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1. Hibbeler, R.C., Mechanics Of Materials, 8th Edition in SI units, Prentice Hall, 2011.
2. Gere dan Timoshenko, Mechanics of Materials, 3rd Edition, Chapman & Hall.
3. Yusof Ahmad, ‘Mekanik Bahan dan Struktur’ Penerbit UTM 2001