Statics SKMM1203

Centroid

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Examples:

EXAMPLE 1

a. **Figure 1** shows a 2m x 2m x 0.4 m concrete slab with a 1 m diameter hole in the centre. The plug for the hole which is also concrete and has the same diameter and thickness as the hole is placed at the top left corner as shown. Determine the centroid ($\overline{X}$, $\overline{Y}$ and $\overline{Z}$) of the slab and plug together as one body.

b. Explain the effect on the centroid (C) and centre of gravity (G) if the plug is made of a different material.

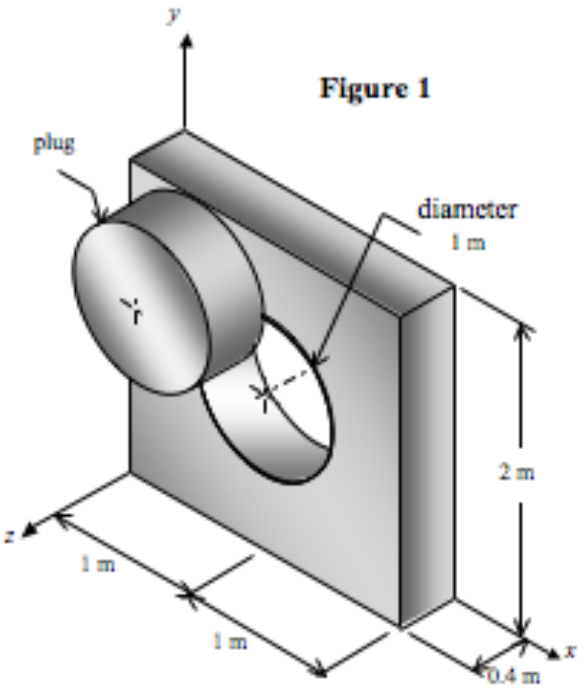
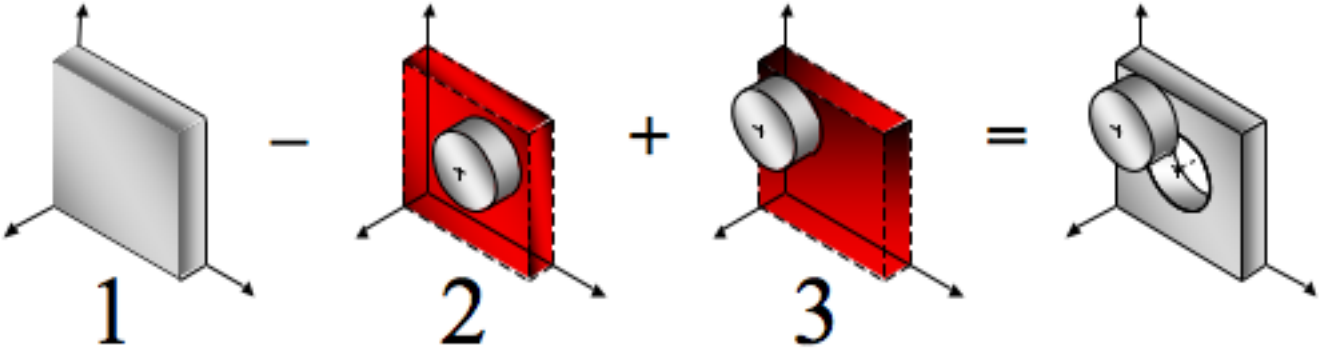


Figure 1

Examples:



Component	V (m^3)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{z}$ (m)	$\bar{x}\,V$ (m^4)	$\bar{y}\,V$ (m^4)	$\bar{z}\,V$ (m^4)
1	2 x 2 x 0.4 = 1.6	1	1	0.2	1.6	1.6	0.32
2	$-\pi(0.5)^2(0.4)$ $= -0.314$	1	1	0.2	-0.314	-0.314	-0.0628
3	$\pi(0.5)^2(0.4)$ $= 0.314$	0.5	1.5	0.6	0.157	0.471	0.1884
	1.6				1.443	1.757	0.4456
	ΣV				$\Sigma \bar{x}\,V$	$\Sigma \bar{y}\,V$	$\Sigma \bar{z}\,V$

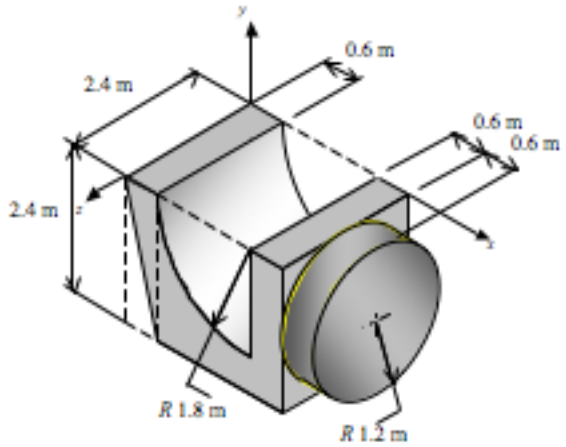
$$\bar{X} = \frac{1.443}{1.6} = 0.902 \text{ m} \qquad \bar{Y} = \frac{1.757}{1.6} = 1.098 \text{ m} \qquad \bar{Z} = \frac{0.4456}{1.6} = 0.279 \text{ m}$$

b. Centroid (C) will maintain and Centre of Gravity (G) will change.

Examples:

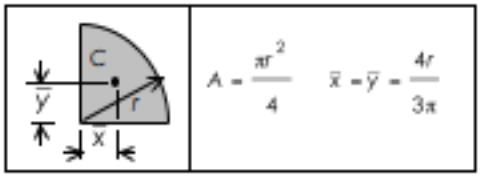
EXAMPLE 2

Determine centroid ($\bar{x}$, $\bar{y}$ and $\bar{z}$) of the composite body shown in **Figure 2**. Separation of the composite body is restricted to four (4) components only.



SOLUTION

Show all four (4) separated components :



Examples:

	V	$\bar{x}$	$\bar{y}$		$\bar{x}\,V$	$\bar{y}\,V$	$\bar{z}\,V$
1	$\frac{1}{2}(0.6)(2.4)(2.4)$ $= 1.728$	$\frac{3}{8}(0.6)$ $= 0.4$	$-\frac{1}{3}(2.4)$ $= -0.8$	1.2	0.6912	-1.3824	2.0736
2	$(2.4)(2.4)(2.4)$ $= 13.824$	$0.6 + 2.4/2$ $= 1.8$	-1.2	1.2	24.8832	-16.59	16.59
3	$-\frac{1}{4}(\pi 1.8^2)(2.4)$ $= -6.107$	$2.4 - \frac{3}{8}(1.8/\pi)$ $= 1.636$	$-\frac{4}{3\pi}(1.8/\pi)$ $= 0.764$	1.2	-9.99	4.666	-7.328
4	$\pi 1.2^2(0.6)$ $= 2.714$	$0.6 + 2.4 + 0.6/2$ $= 3.3$	-1.2	1.2	8.956	-3.257	3.257
	12.159				24.54	-16.56	14.59

$\bar{x} = 24.54/12.159 = 2.02$ m

$\bar{y} = -16.56/12.159 = -1.362$ m

$\bar{z} = 14.59/12.159 = 1.2$ m (this is a check, actually no need to do the calculations

Examples:

EXAMPLE 3

Determine the volume and surface area of the composite body shown in **Figure 3** using the Theorm of Pappus.

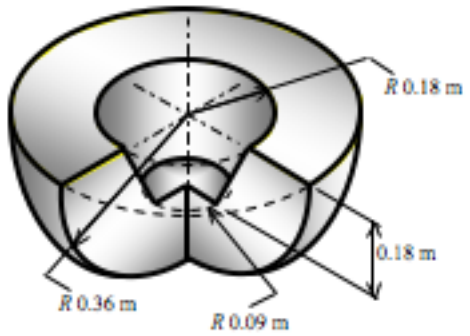
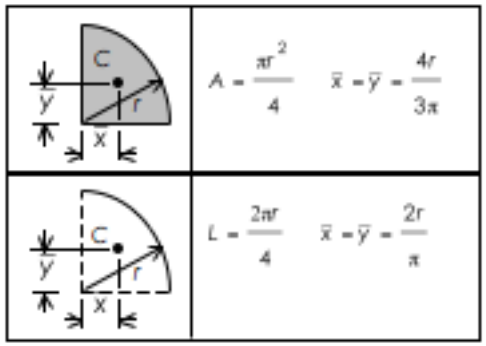
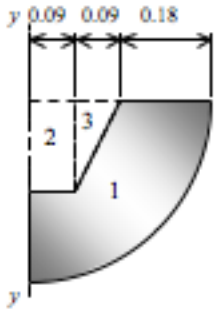


Figure 3



Volume



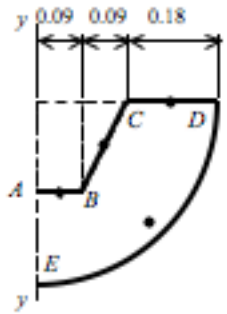
component/	A (m^2)	$\bar{X}$ (m)	$A\bar{X}$ (m^3)
1	$\frac{1}{4}\pi(0.36^2)$	$4(0.36)/3\pi$	0.015555
	$= 0.1018$	$= 0.1528$	
2	$-0.18(0.09)$	$0.09/2$	-0.000729
	$= -0.0162$	$= 0.045$	
3	$-\frac{1}{2}(0.18)(0.09)$	$0.09 + \frac{1}{3}(0.09)$	-0.000972
	$= -0.0081$	$= 0.12$	
	0.0775		0.013854

$$V = \tfrac{3}{4}[2\pi(A\bar{X})] = \tfrac{3}{4}[2\pi(0.013854)]$$

$$V = 0.0653\ m^3$$

Examples:

Surface Area



Segment	L (m)	$\bar{x}$ (m)	$L\bar{x}$ (m^2)
AB	0.09	0.045/2 = 0.045	0.00405
BC	$\sqrt{(0.09^2 + = 0.18^2)}$ = 0.2012	0.09 + 0.09/2 = 0.135	0.02716
CD	0.18	0.18 + 0.18/2 = 0.27	0.0486
DE	$2\pi(0.36)/4$ = 0.5655	$2(0.36)/\pi$ 0.2292	0.1296
	1.0367		0.2094

Surface Area of Revolution

$$A_{rev} = \tfrac{3}{4}[2\pi(L\bar{x})] = \tfrac{3}{4}[2\pi(0.2094)]$$

$$A_{rev} = 0.9868 \text{ m}^2$$

The two end areas

$$A_{ends} = 2(\pi(0.36^2)/4) - (0.09 \times 0.18) - (\tfrac{1}{2}(0.09 \times 0.18))$$

$$= 2(0.1018 - 0.0162 - 0.0081) = 2(0.0775)$$

$$= 0.155 \text{ m}^2$$

Total Surface Area

$$A = A_{rev} + A_{ends} = 0.9868 + 0.155$$

$$A = 1.1418 \text{ m}^2$$

Examples:

EXAMPLE 4

Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the composite sheet metal bracket in **Figure 4** if the material is homogeneous and has a constant thickness.

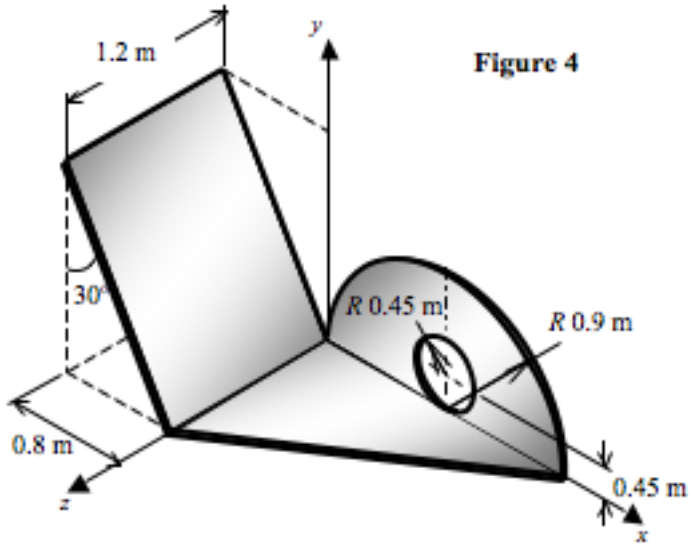


Figure 4

	Area (m^2)	$\bar{x}$	$\bar{y}$	$\bar{z}$	$A\bar{x}$	$A\bar{y}$	$A\bar{z}$
1	1.2 x 1.6 = 1.92	– 0.4	(1.6 cos 30°)/2 = 0.693	0.6	– 0.768	1.331	1.152
2	$\frac{1}{2}(1.2)(1.8)$ = 1.08	$\frac{1}{3}(1.8)$ = 0.6	0	$\frac{1}{3}(1.2)$ = 0.4	0.648	0	0.432
3	$\frac{0.9^2\pi}{2}$ = 1.272	0.9	$\frac{4(0.9)}{3\pi}$ = 0.382	0	1.145	0.486	0
4	$-0.45^2\pi$ = – 0.636	0.9	0.45	0	– 0.572	– 0.286	0
	$\Sigma A = 3.636$				$\Sigma A\bar{x} = 0.453$	$\Sigma A\bar{y} = 1.531$	$\Sigma A\bar{z} = 1.584$

Examples:

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{0.453}{3.636} = 0.1246 \text{ m}$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{1.531}{3.636} = 0.421 \text{ m}$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{1.584}{3.636} = 0.436 \text{ m}$$