SEE 2523 Theory Electromagnetic

Chapter 5
Electrodynamics Fields

You Kok Yeow

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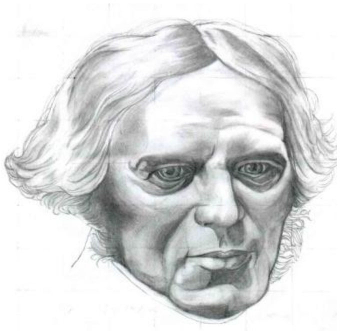

Michael Faraday (1791-1867)

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Reviews

1. Electrostatic is study of static electric charges.

Static charges produce electric field (Capacitor)

2. Magnetostatic is a study of motion electric charges with uniform velocity.

Motion charges produce current

Steady current-carrying conductor produce magnetic field (Inductor)

Steady current-carrying conductor in magnetic field produce motion force (Galvanometer, Electric motor)

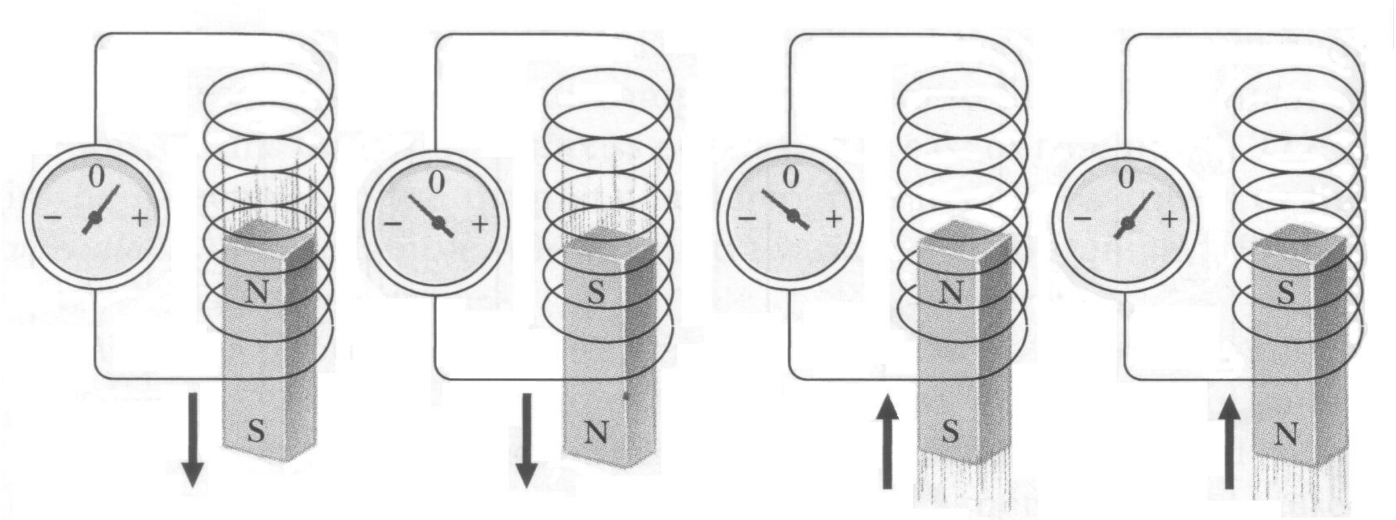
3. Electromagnetic is a study of motion electric charges with acceleration.

Relative motion between conductors and a magnetic field produce current. (Electric generator or Dynamo, Transformer)

Electromagnetic Field

1. **Electromagnet** is a temporary magnet, which its magnetic fields is produced by electric current.

2. **Electromagnetic induction** is the process of producing electromotive force or current in conductor due to relative motion between conductors and a magnetic field.



3. **Electromotive force** (EMF) is a potential difference given to the changes by a battery (in volts).

Electromagnetic Induction (1)

1. There are two laws of electromagnetic induction.

   Faraday's law states the relationship between induced current and the change of flux.

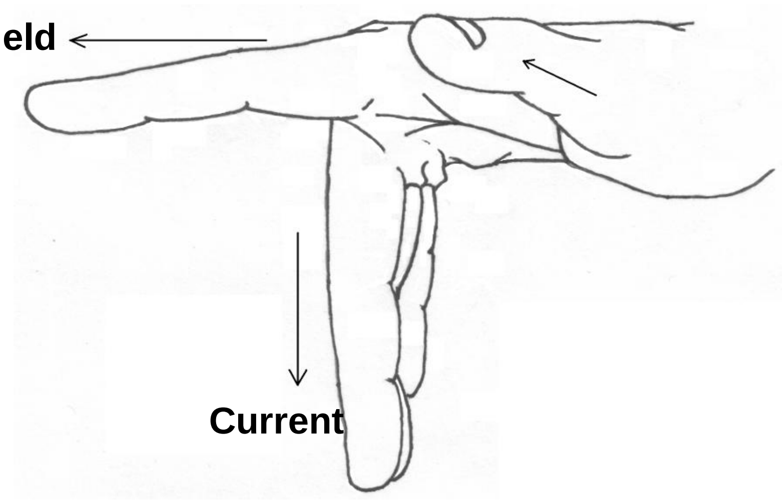
   Lenz's law states the direction of induced current.

Faraday's law states that the magnitude of the induced electromotive force (EMF) in a closed circuit or conductor is proportional to the rate of change of the number of lines of magnetic force linking it.

Lenz's law states that the direction of the induced current is such as to oppose the change causing it

$$Electromotive\ force(\mathrm{EMF}) = -\frac{d\Phi}{dt}$$

Electromagnetic Induction (2)



Fleming's right-hand rule

Electromagnetic Induction (Derivation using Maxwell's Equation)

By using Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Integrating both sides respect to surface

$$\int_S \left(\vec{\nabla} \times \vec{E}\right) \cdot d\vec{S} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{S}$$

Reduce the double integral to single integral using Stoke's Theorem

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$

Thus,

$$Electromotive\ force\left(EMF\right) = -\frac{d\Phi}{dt}$$

$$V\left(EMF\right) = \oint \vec{E} \cdot d\vec{l}$$

Question (1)

A circular loop of N turns of conducting wire lies in the *xy*-plane with its center at the origin of a magnetic field specified by $\vec{B} = \hat{z}B_o \cos(\pi r/2b)\sin\omega t$ where *b* is the radius of the loop and ω is the angular frequency. Determine the emf, V induced in the loop.

Solution

Using $\int x\cos(ax)dx = \frac{1}{a^2}[\cos(ax) + ax\sin(ax)] + C$

The magnetic flux linking each turn of the circular loop is

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$

$$= \int_0^b \left[\hat{z}B_o \cos\left(\frac{\pi r}{2b}\right)\sin\omega t\right] \cdot (\hat{z}2\pi r dr)$$

$$= \frac{8b^2}{\pi}\left(\frac{\pi}{2} - 1\right)B_o \sin\omega t$$

Since there are *N* turn, the total flux linkage is

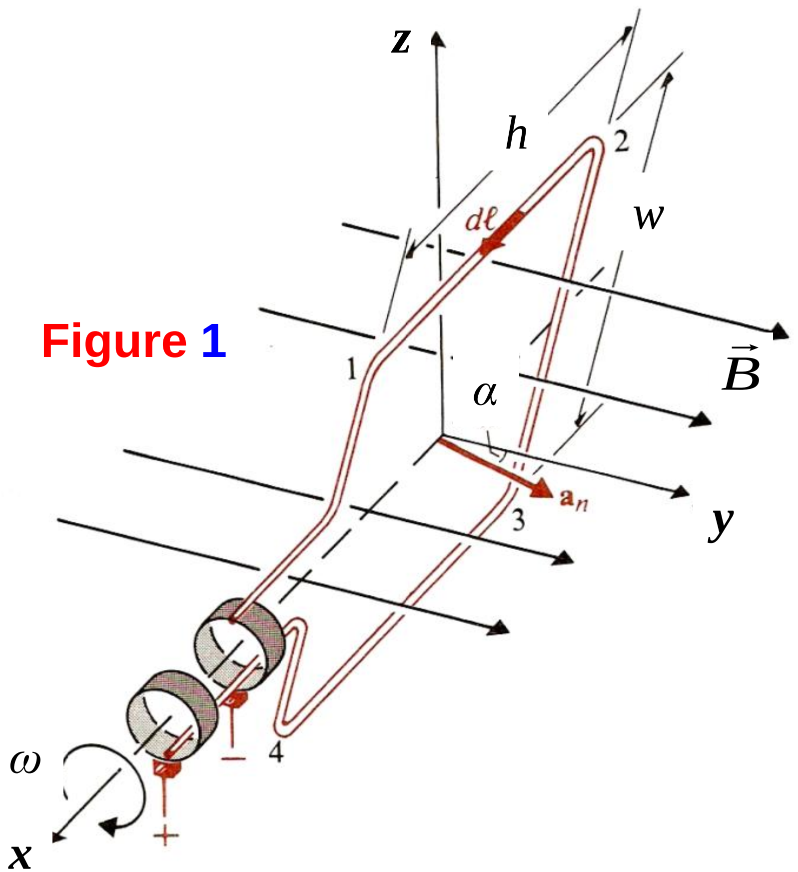
$$V\,(emf) = -N\frac{d\Phi}{dt}$$

$$= -\frac{8N}{\pi}b^2\left(\frac{\pi}{2} - 1\right)B_o\omega\cos\omega t \qquad \text{V}$$

Question (2)

An h by w rectangular conducting loop is situated in a changing magnetic field $\vec{B} = \hat{y}B_o \sin\omega t$. The normal of the loop initially makes an angle α with ω as shown in Figure 1. Determine the induced emf , V in the loop when the loop is at rest.

Figure 1



Solution

When the loop is at rest.

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$

$$= (\hat{y}B_o \sin\omega t) \cdot (\hat{a}_n hw)$$

$$= B_o hw \sin\omega t \cos\alpha$$

Therefore

$$V(emf) = -\frac{d\Phi}{dt}$$

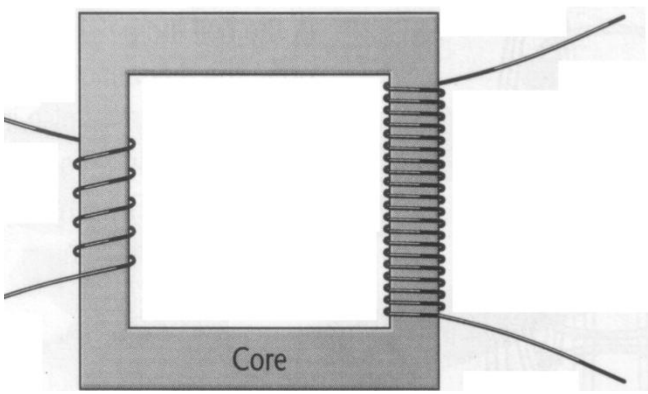
$$= -B_o hw\omega \cos\omega t \cos\alpha$$

Mutual Coupling Induction (Transformer) 1

1. For generator, a moving loop with a time-varying area in a static magnetic field.

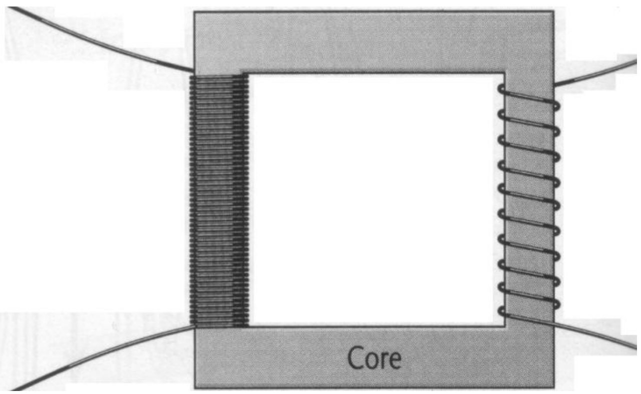
2. For transformer, a time-varying magnetic field linking a stationary loop.

primary secondary



Step-up Transformer

primary secondary



Step-down Transformer

Mutual Coupling Induction (Transformer) 2

Principle

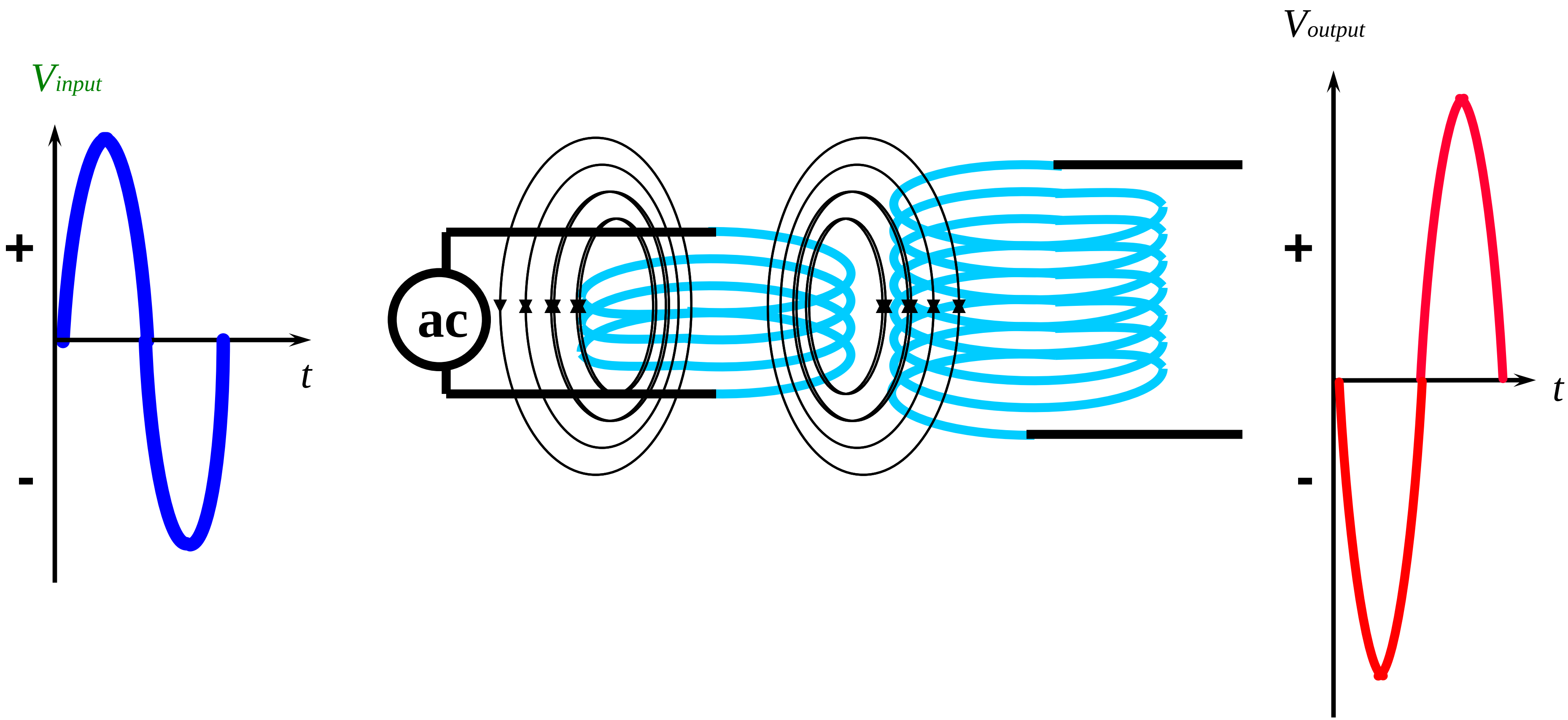
a) When the primary coil is connected to source of a.c voltage, the changing current creates a varying magnetic field.

b) The varying magnetic field is carried through the core to the secondary coil.

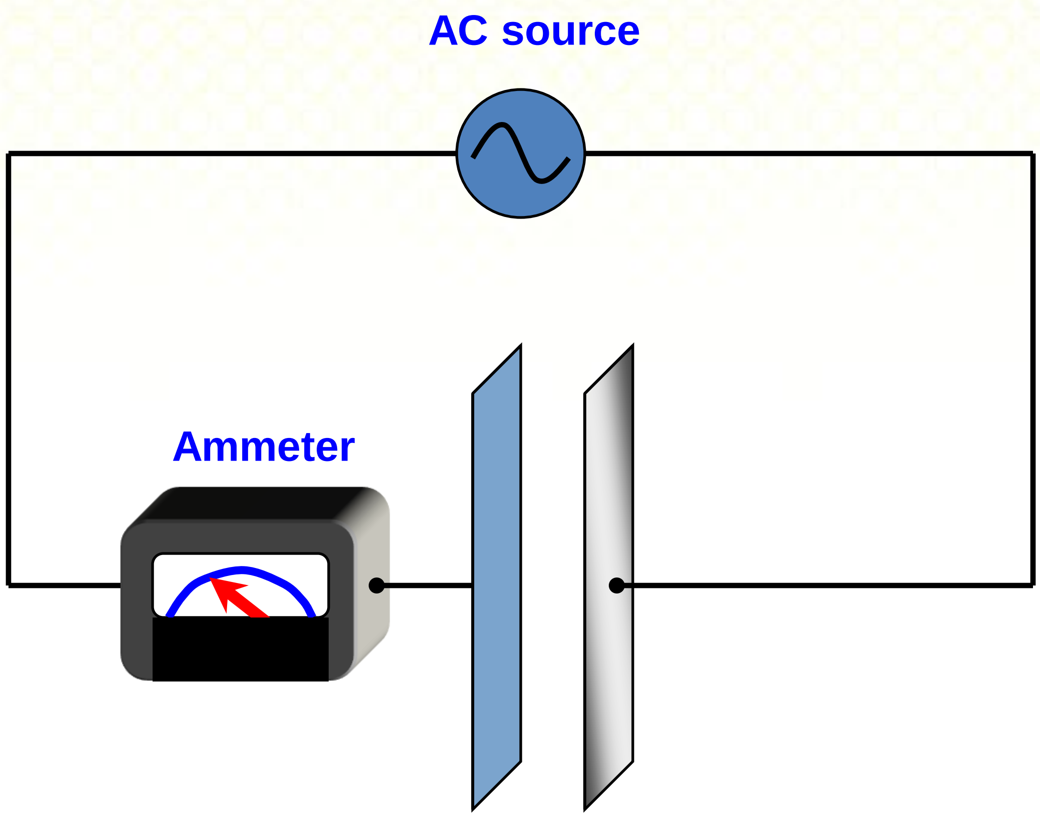
c) In the secondary coil, the varying field induces a varying electromotive force (EMF).

d) This effect is called **mutual inductance**

Mutual Coupling Induction (Transformer) 3



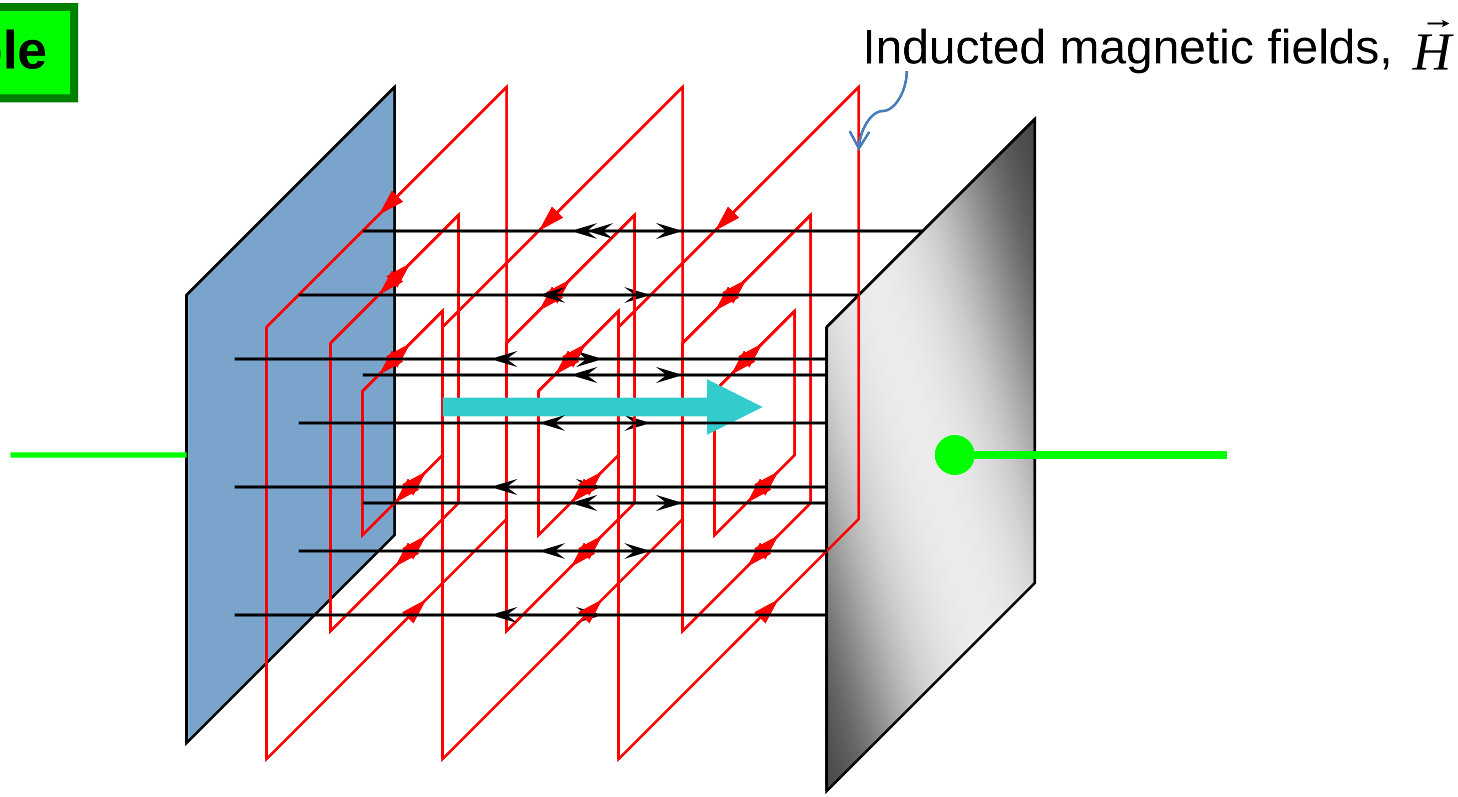
Displacement Current Density (1)



How can the ammeter read any value of current since the capacitor is an open circuit ?

Displacement Current Density (2)

Principle



This displacement current does not exist in a time-independent system

$$\vec{\nabla} \times \vec{H} = \vec{J} + \boxed{\frac{\partial \vec{D}}{\partial t}}$$

Displacement Current Density (3)

Question

Verify that the conduction current in the wire equals the displacement current between the plates of the parallel plate capacitor in the circuit. The voltage source has $V_c = V_o \sin\omega t$

Answer

The conduction current in the wire is given by

$$I_c = C\frac{dV_c}{dt}$$

$$= CV_o\omega\cos\omega t$$

The electric field between the plates

$$\vec{E} = V_c / d$$

The displacement flux density

$$\vec{D} = \varepsilon\vec{E}$$

The displacement current is computed from

$$I_d = \int_A \frac{\partial\vec{D}}{\partial t}\cdot d\vec{S}$$

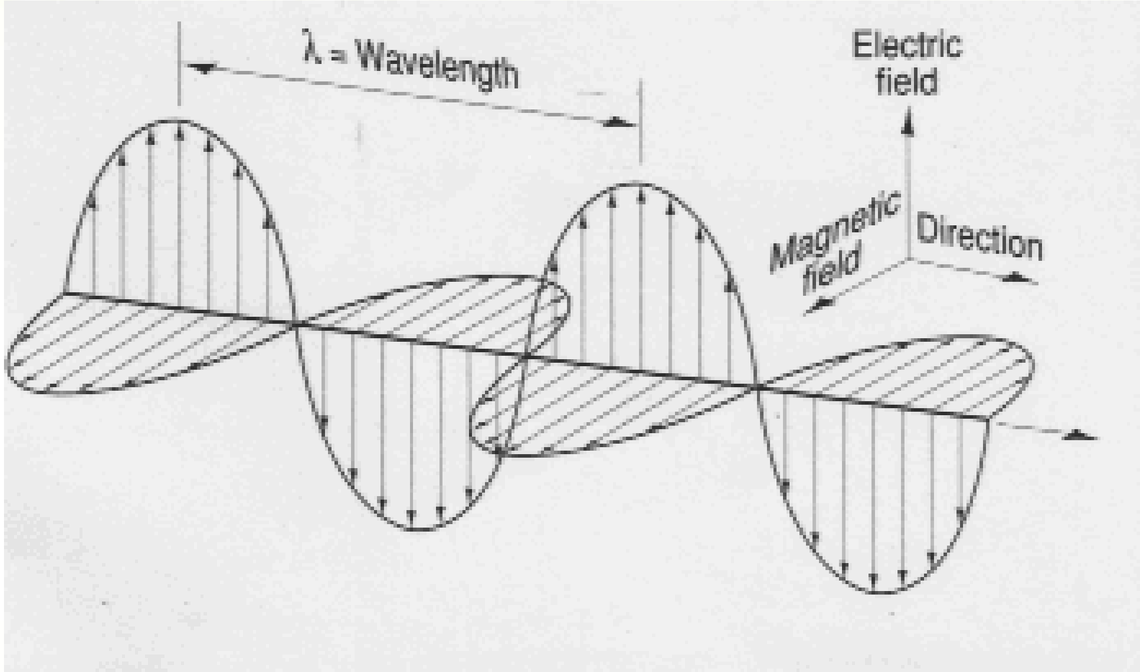
$$= \left(\frac{\varepsilon A}{d}\right)V_o\omega\cos\omega t$$

$$= CV_o\omega\cos\omega t$$

$$= I_c$$

Plane Wave (1)

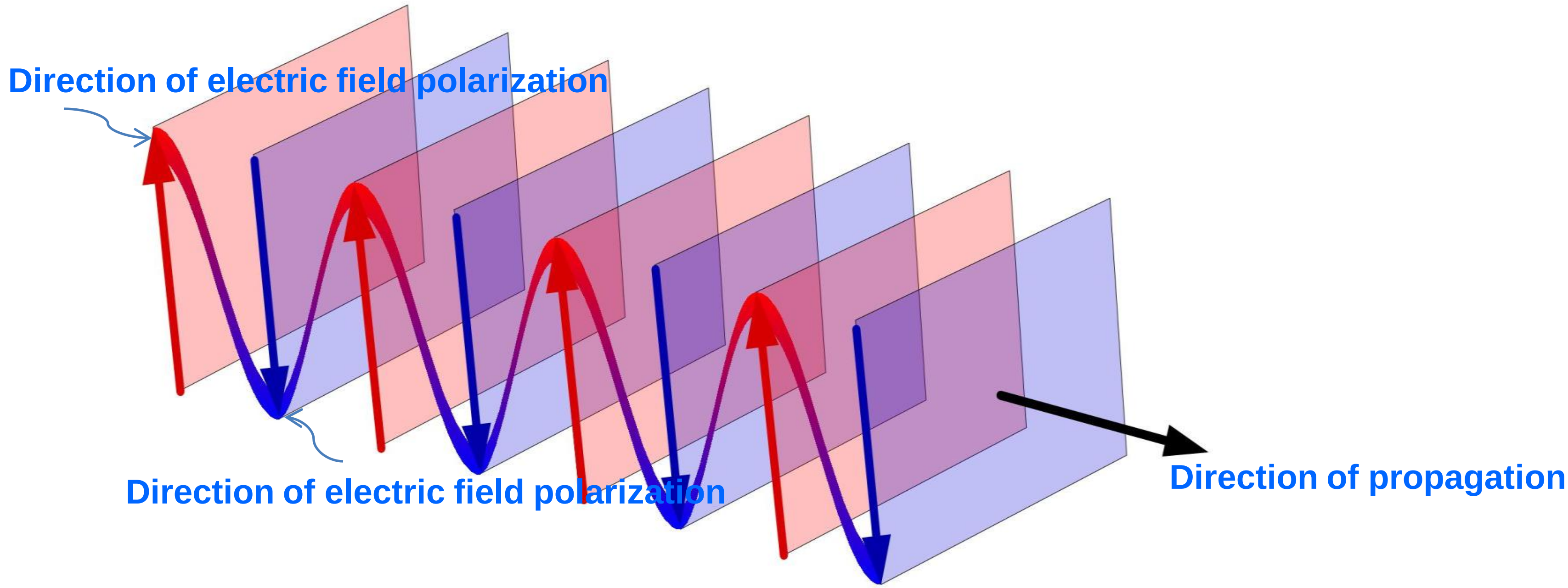
1. A **uniform plane wave** is the wave that the electric field, $\vec{E}$ or magnetic field, $\vec{H}$ in same direction, same magnitude and same phase in infinite planes perpendicular to the direction of propagation.



2. A **plane wave** has no electric field, $\vec{E}$ and magnetic field, $\vec{H}$ components along its direction of propagation

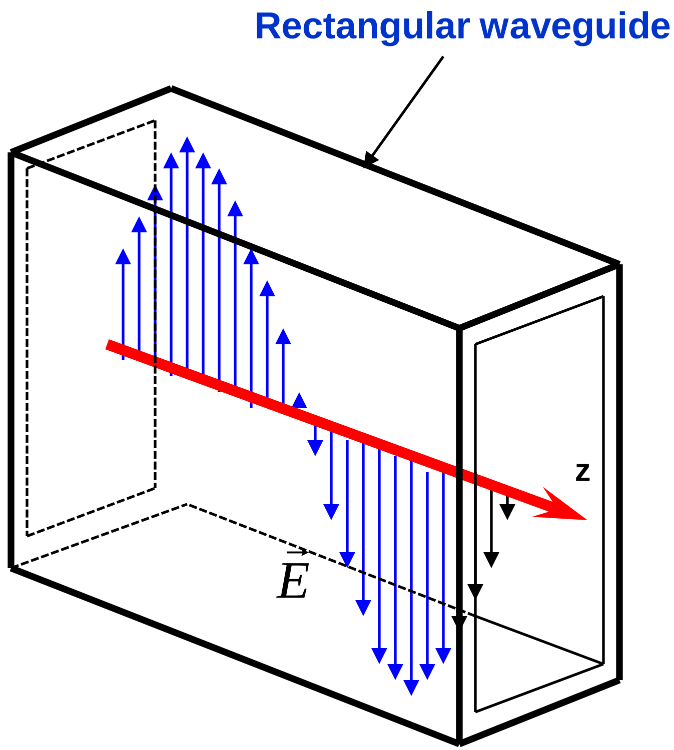
Plane Wave (2)

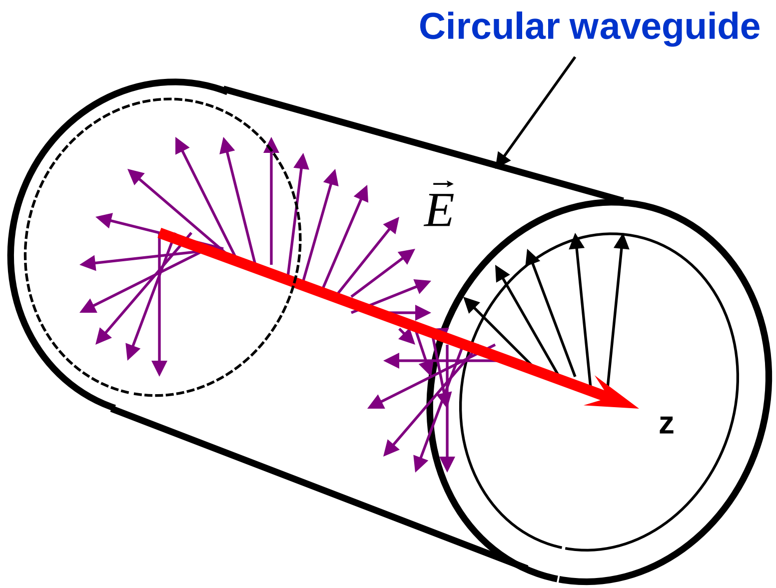
Example



At a particular location z and at the particular time, t, the electric field $\hat{y}E_y$ have the same phase at all points in the transverse plane.

Plane Wave and Polar Wave

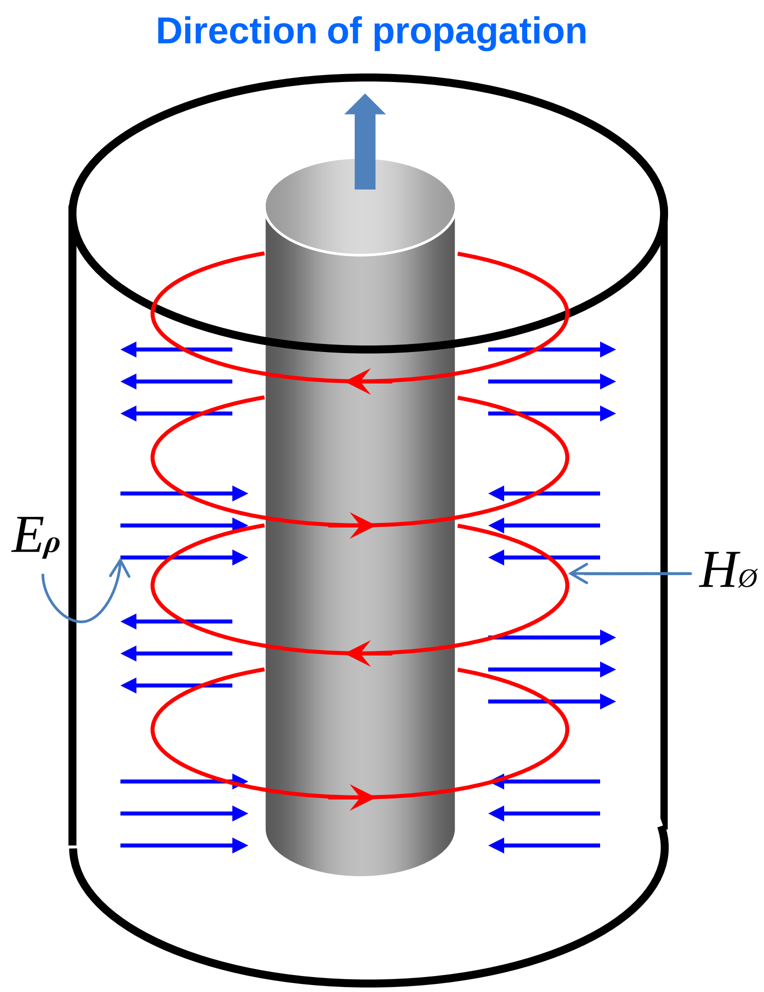




Plane wave

Polar wave

Wave Propagation (Example)



Wave Propagate in Coaxial Line

Wave Equations (1)

1. If the wave is in simple (linear, isotropic and homogeneous) nonconducting medium ($\sigma = 0$), Maxwell's equation reduce to

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

2. The first-order differential equations in the two variables $\vec{E}$ and $\vec{H}$.

3. They can combine to give $\vec{E}$ or $\vec{H}$ alone using second-order equation.

Wave Equations (2)

Example

Using Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (2)$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (3)$$

The curl of equation of (1)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{H}\right)$$

Replace equation (2)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

We know that $\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{E}\right) - \vec{\nabla}^2 \vec{E}$ **because of equation (3), thus**

$$\vec{\nabla}^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Homogeneous vector wave equation

Wave Equations (3)

1. The wave equation also can written as

$$\vec{\nabla}^2\vec{E} - k^2\vec{E} = 0 \qquad (1)$$

Assuming an implicit time dependence $e^{j\omega t}$ in the field vectors.

2. Equation (1) also called **Helmholtz equation.**

3. The k is called the wave number or propagation constant.

$$k = k_o\sqrt{\varepsilon_r} = \frac{2\pi f}{c}\sqrt{\varepsilon_r}$$

and

$$c = \frac{1}{\sqrt{\varepsilon\mu}}$$

where c is the velocity of light in free space.

Wave Equations (Example)

4. For magnetic intensity domain, $\vec{H}$

$$\vec{\nabla}^2\vec{H}-\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}=0 \quad \text{or} \quad \vec{\nabla}^2\vec{H}-\mu_r\varepsilon_r k_o^2\vec{H}=0$$

Example:

For coaxial line, $\vec{H}$ field is a function of ρ and z, but independent of ϕ.

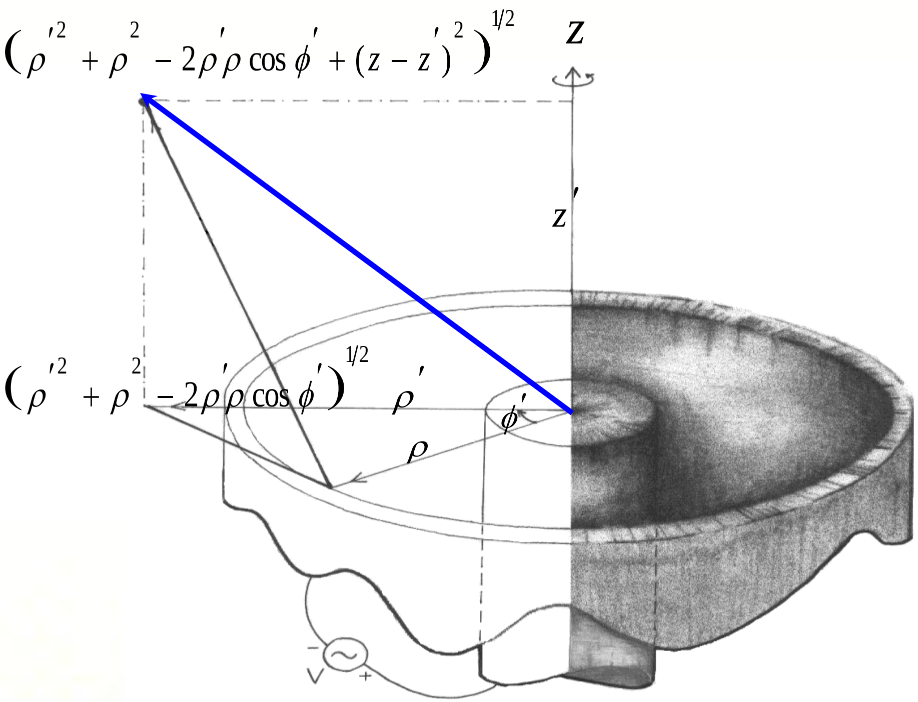
Therefore, vector wave equation can be simplified to a scalar equation for H_ϕ

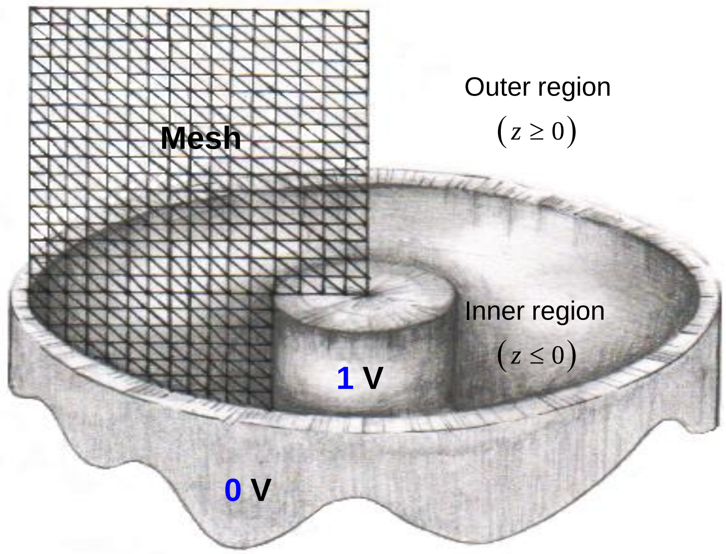
$$-\frac{\partial}{\partial\rho}\left[\frac{1}{\varepsilon_r\rho}\frac{\partial}{\partial\rho}(\rho H_\phi)\right]-\frac{1}{\varepsilon_r}\frac{\partial^2}{\partial z^2}H_\phi-\mu_r k^2 H_\phi=0$$

Wave Equations (Example)

$$H_\phi = \frac{j\omega\varepsilon_o\varepsilon_r}{2\pi\ln(b/a)}\int_a^b\int_0^\pi \frac{\exp(-jkr)}{r}\cos\phi'\rho' d\phi' d\rho'$$

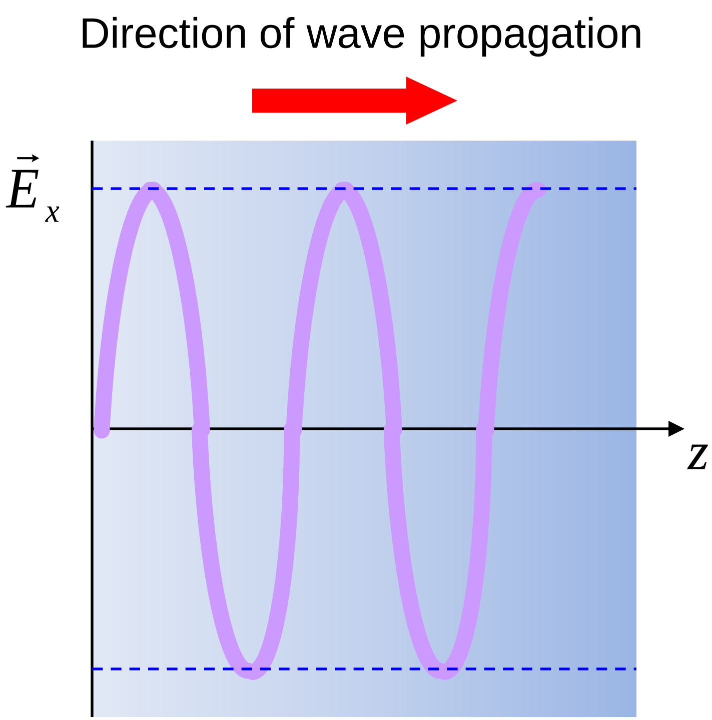
$$-\frac{\partial}{\partial\rho}\left[\frac{1}{\varepsilon_r\rho}\frac{\partial}{\partial\rho}(\rho H_\phi)\right]-\frac{1}{\varepsilon_r}\frac{\partial^2}{\partial z^2}H_\phi-\mu_r k^2 H_\phi=0$$



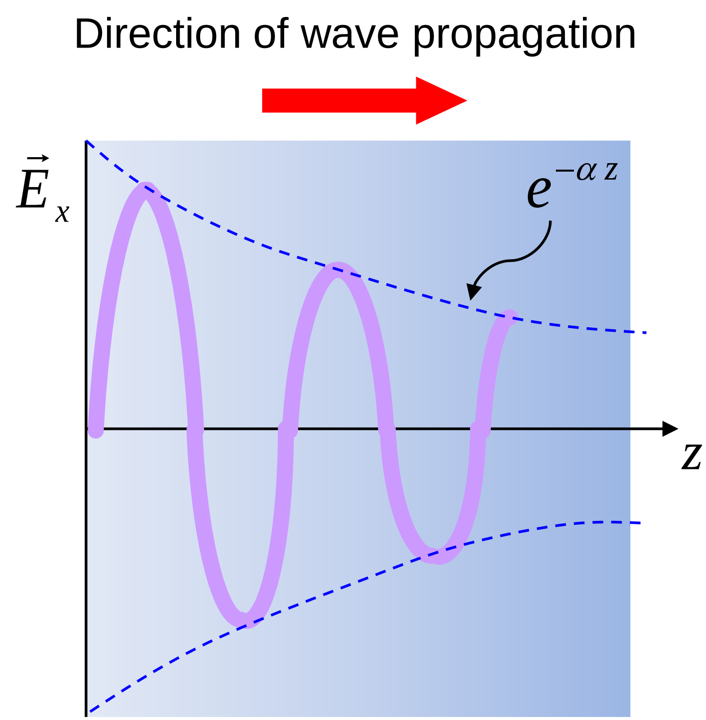


Integral and **Differential S**olutions for **C**oaxial **W**aveguides

Plane Wave Propagation in Medium (1)

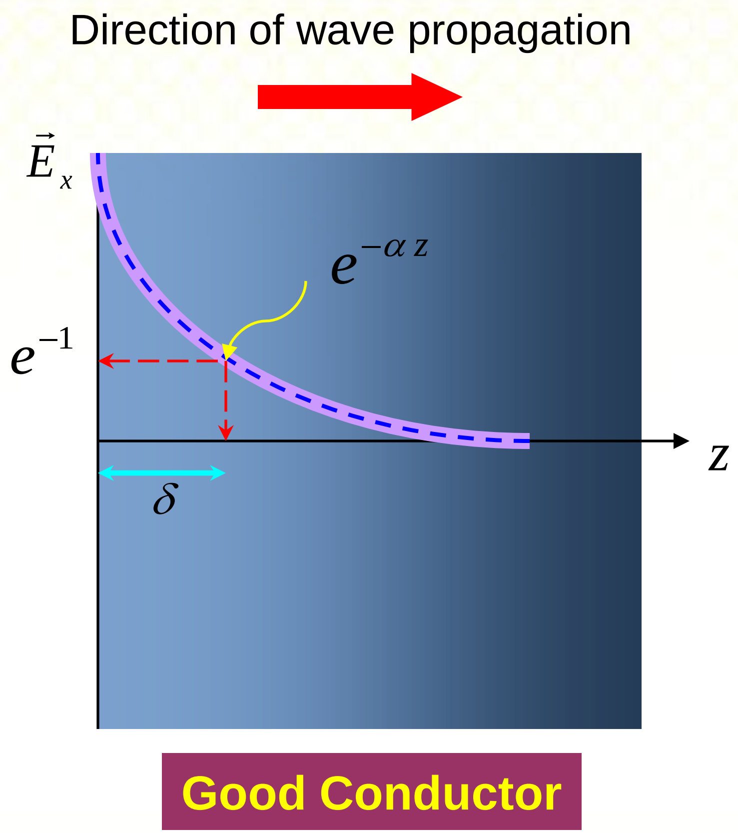


Lossless Medium



Lossy Medium

Plane Wave Propagation in Medium (2)



The fields decrease with penetration, falling to $1/e$ of their surface values in a distance equal **skin depth**, δ .

Plane Wave Propagation in Medium (3)

1. For a uniform plane wave with an electric field $\vec{E} = \hat{x} E_x$ traveling in the z-direction, the wave equation can be reduced as

$$\frac{\partial^2 \vec{E}_x(z)}{\partial z^2} - k^2 \vec{E}_x(z) = 0$$

2. The solution of this wave equation

$$\begin{aligned} \vec{E}(z) &= \hat{x} E_x \\ &= \hat{x} E_o e^{-kz} \\ &= \hat{x} E_o e^{-\alpha z} e^{-j\beta z} \end{aligned}$$

where α is the attenuation constant of the medium and β is its phase constant

Plane Wave Propagation in Medium (4)

3. The k is called the wave number or propagation constant.

$$k^2 = k_o^2 \varepsilon_r \mu_r$$

or

$$k^2 = k_o^2 \mu_r \left(\varepsilon_r' - j\varepsilon_r'' \right)$$

4. The wave number can also be written in terms of α and β.

$$\begin{aligned} k^2 &= (\alpha + j\beta)^2 \\ &= \left(\alpha^2 - \beta^2\right) + j2\alpha\beta \end{aligned}$$

5. Thus,

$$\alpha^2 - \beta^2 = k_o^2 \mu_r \varepsilon_r' \quad (1)$$

$$2\alpha\beta = -k_o^2 \mu_r \varepsilon_r'' \quad (2)$$

Plane Wave Propagation in Medium (5)

6. By solving the (1) and (2),

$$\alpha = \sqrt{\frac{k_o^2 \mu_r \varepsilon_r'}{2}\left(\sqrt{1+\left(\frac{\varepsilon_r''}{\varepsilon_r'}\right)^2}-1\right)}$$

$$\beta = \sqrt{\frac{k_o^2 \mu_r \varepsilon_r'}{2}\left(\sqrt{1+\left(\frac{\varepsilon_r''}{\varepsilon_r'}\right)^2}+1\right)}$$

Plane Wave Propagation in Medium (6)

Lossless Medium $(\sigma = 0)$	Low-loss Medium $(\varepsilon''/\varepsilon' \neq 0)$	Conductor $(\varepsilon''/\varepsilon' \to \infty)$
$\alpha = 0$	$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\alpha = \sqrt{\pi f \mu \sigma}$
$\beta = \omega\sqrt{\mu\varepsilon}$	$\beta = \omega\sqrt{\mu\varepsilon}$	$\beta = \sqrt{\pi f \mu \sigma}$

Plane Wave Propagation in Medium (7)

7. **The associated magnetic field, $\vec{H}$**

$$\vec{H}(z) = \hat{y} H_y$$

$$= \hat{y} \frac{\vec{E}_x}{\eta}$$

$$= \hat{y} \frac{E_o}{\eta} e^{-\alpha z} e^{-j\beta z}$$

where η is the intrinsic impedance of the medium.

Electromagnetic Phenomena are described by using four Maxwell's equations

	Maxwell's equation		
	Integral form:	**Description**	**Information**
Gauss's Law (Electric fields)	$\underbrace{\varepsilon_o \oint \vec{E} \cdot d\vec{S}}_{Left} = \underbrace{q}_{Right}$	**Left side:** The number of electric field lines – perpendicularly passing through to a closed surface, $\vec{S}$ **Right side:** Total amount of charge, q contained within that surface, .	Electric charge produces an electric field, $\vec{E}$ and the flux of that field passing through any closed surface is proportional to the total charge, q contained within that surface. Charge on an insulated conductor moves outward surface. The electric field, $\vec{E}$ produced by electric charge diverges from positive charge and converges upon negative charge. The electric field, $\vec{E}$ is tendency to propagate perpendicularly away from a surface charge.
	Differential form: $\underbrace{\varepsilon_o \vec{\nabla} \cdot \vec{E}}_{Left} = \underbrace{\rho}_{Right}$	**Left side:** Divergence of the electric field, $\vec{E}$ – the tendency of the field to "flow" away from a specified location. **Right side:** Electric charge density, ρ	

Gauss's Law **(Magnetic fields)**	**Integral form:** $\underbrace{\mu_o \oiint \vec{H} \cdot d\vec{S}}_{Left} = \underbrace{0}_{Right}$	**Left side:** The number of magnetic field lines – perpendicularly passing through a closed surface. **Right side:** Identically zero.	The total magnetic flux passing through any closed surface is zero. Flux enter the closed surface is same with the flux come out from the surface. The divergence of the magnetic field at any point is zero.
	Differential form: $\underbrace{\mu_o \vec{\nabla} \cdot \vec{H}}_{Left} = \underbrace{0}_{Right}$	**Left side:** Divergence of the magnetic field – the tendency of the field to "flow" away from a point than toward it. **Right side:** Identically zero.	

Faraday's Law	**Integral form:** $$\underbrace{\oint_C \vec{E}\cdot d\vec{l}}_{Left} = \underbrace{-\mu_o \int_S \frac{\partial \vec{H}}{\partial t}\cdot d\vec{S}}_{Right}$$	**Left side:** The circulation of the vector electric field, $\vec{E}$ around a closed path, *C*. **Right side:** The rate of change with time (*d*/*dt*) of magnetic field, through any surface, $\vec{S}$.	Changing magnetic flux through a surface induces an emf in any boundary path, *C* of that surface, and a changing magnetic field, $\vec{H}$ induces a circulating electric field.
	Differential form: $$\underbrace{\vec{\nabla}\times\vec{E}}_{Left} = \underbrace{-\mu_o \frac{\partial \vec{H}}{\partial t}}_{Right}$$	**Left side:** Curl of the electric field, – the tendency of the field lines to circulate around a point. **Right side:** The rate of change of the magnetic field, $\vec{H}$ over time (*d*/*dt*)	A circulating electric field, is produced by a magnetic field, $\vec{H}$ that changes with time.

Ampere's Law	**Integral form:** $$\underbrace{\oint_C \vec{H}\cdot d\vec{l}}_{Left} = \underbrace{\int_S \left(\vec{J}_c + \varepsilon_o \frac{\partial \vec{E}}{\partial t}\right)\cdot d\vec{S}}_{Right}$$	**Left side:** The circulation of the magnetic field, $\vec{H}$ around a closed path, *C*. **Right side:** Two sources for the magnetic field, $\vec{H}$; a steady conduction current, $\vec{J}_c$ and a changing electric field, $\vec{E}$ through any surface, bounded by closed path, *C*.	An electric current or a changing electric flux through a surface produces a circulating magnetic field around any path, *C* that bounds that surface.
	Differential form: $$\underbrace{\vec{\nabla}\times\vec{H}}_{Left} = \underbrace{\vec{J}_c + \varepsilon_o \frac{\partial \vec{E}}{\partial t}}_{Right}$$	**Left side:** Curl of the magnetic field, – the tendency of the field lines to circulate around a point. **Right side:** Two terms represent the electric current density, $\vec{J}_c$ and the time rate of change of the electric field, $\vec{E}$.	A circulating electric field, is produced by a magnetic field, $\vec{H}$ that changes with time. An electric current, or a changing electric field, through a surface produces a circulating magnetic field, $\vec{H}$ around any path that bounds that surface.

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