SEE 2523 Theory Electromagnetic

Chapter 4
Magnetic Fields

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Magnet (1)

1. Magnet is polarized, since it is dipole and has two ends,

   a) one is the north pole (N),

   b) other is the south pole (S).

2. There are two general types of magnets, which are

   a) Permanent magnet

   b) Temporary magnet

3. Temporary magnet is produced by a constant current flow.

4. Magnetic fields are the space, which magnetic forces (repel or attract) can be detected.

Magnet (2)

5. The **direction** of the field lines are come out at **north poles** (N) and enter at **south poles** (S).

Magnetic fields can be visualized by sprinkling iron filings on a piece of paper suspended over a bar magnet.

6. The field lines are in terms of the magnetic field intensity, $\vec{H}$ in units of $\mathrm{A/m}$.

Magnetostatic Field (1)

1. In electromagnetic study, a magnet field is produced by a current flow.

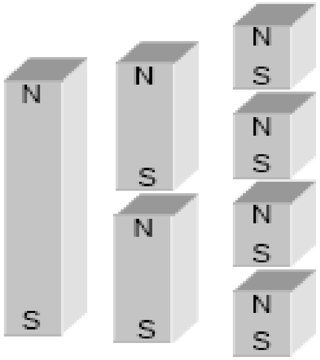
2. The two fundamental properties of magnetostatic field

$$\vec{\nabla} \cdot \vec{B} = 0$$ **Gauss's Law**

$$\vec{\nabla} \times \vec{H} = \bar{J}$$ **Ampere's Law**

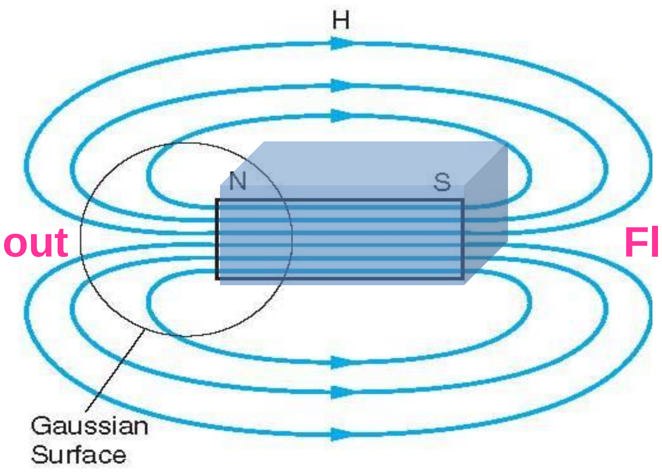
3. The SI units of magnetic flux density, $\vec{B}$ are given in Tesla (T).

4. The first property is the magnetic field lines are continuous and do not originate nor terminate at a point, since the magnetic monopole has not been observed to exist in nature

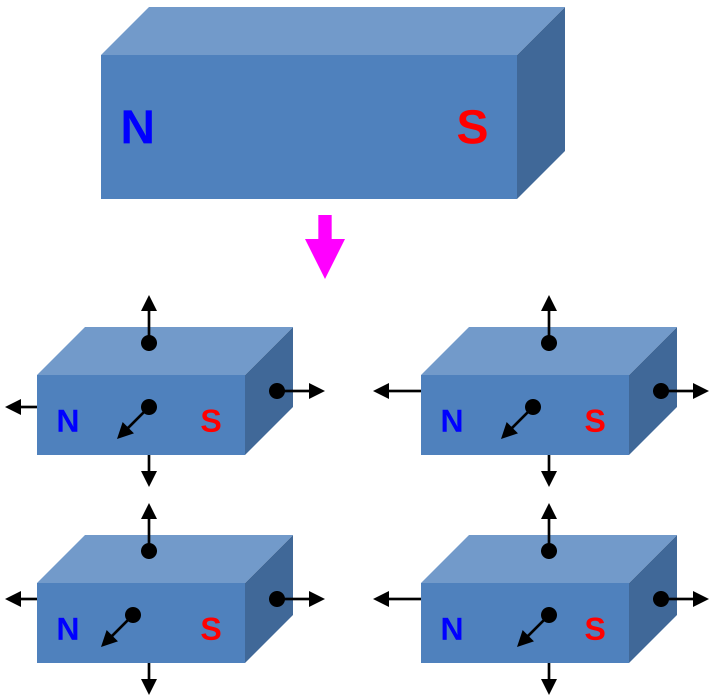


$$\oint \vec{B} \cdot d\vec{S} = \int_v \vec{\nabla} \cdot \vec{B}\, dv = 0$$

Magnetostatic Field (2)



Gauss Theorem



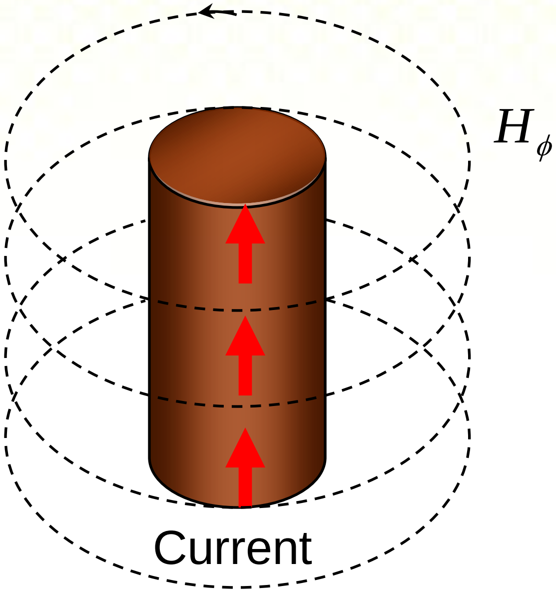
Magnetic flux, $\vec{B}$ is conserved because of

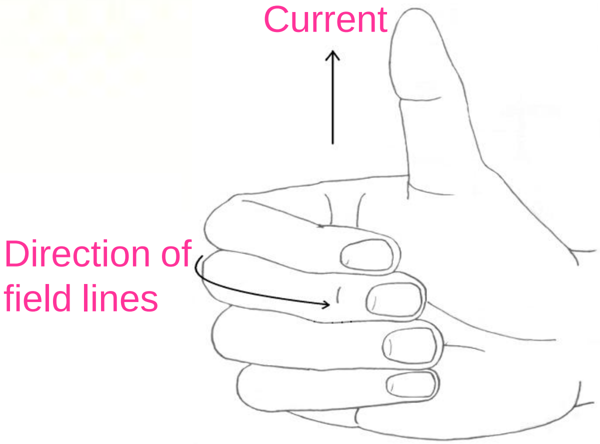
Total flux through surface bar magnet = Flux out - Flux in

= 0 Tesla

Magnetostatic Field (3)

5. The second property is **magnetic field created by electrical current**.





$$\oint \vec{B}\cdot d\vec{l} = \int_S \vec{\nabla}\times\vec{B}\cdot d\vec{S} = \mu_o I$$

6. The are two laws governing magnetostatic fields

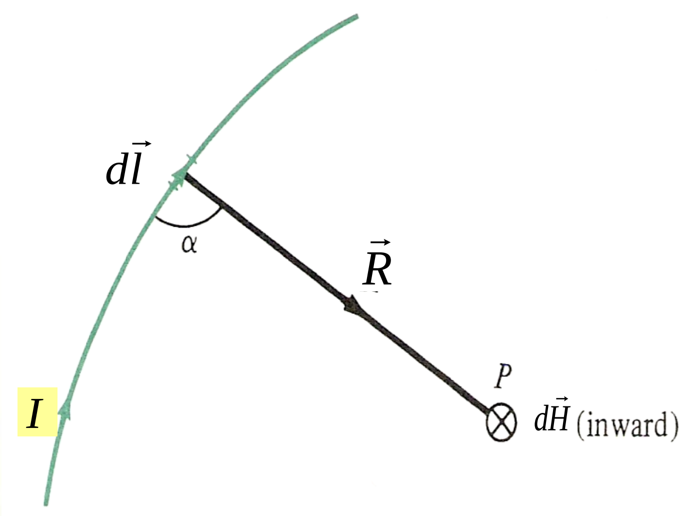
a) Biot-Savart's law

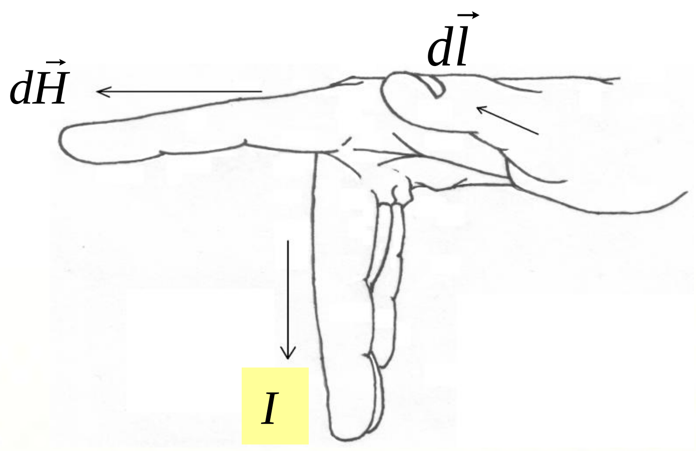
b) Ampere's law

Biot-Savart Law

1. The **Biot-Savart law** states that the differential magnetic field, $d\vec{H}$ generated by a steady current, I flowing through a differential length, $d\vec{l}$ is given by

$$d\vec{H} = \frac{I}{4\pi R^2}\left(d\vec{l} \times \hat{r}\right) = \frac{I}{4\pi R^2}\left(d\vec{l} \times \frac{\vec{R}}{R}\right)$$



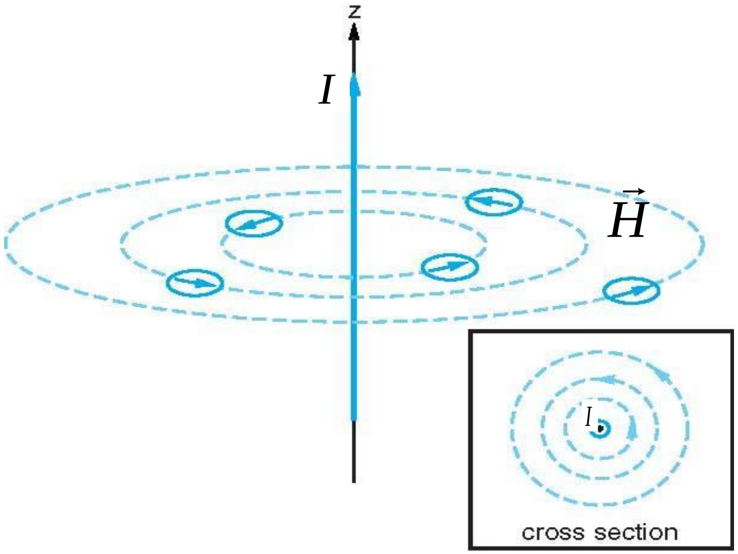


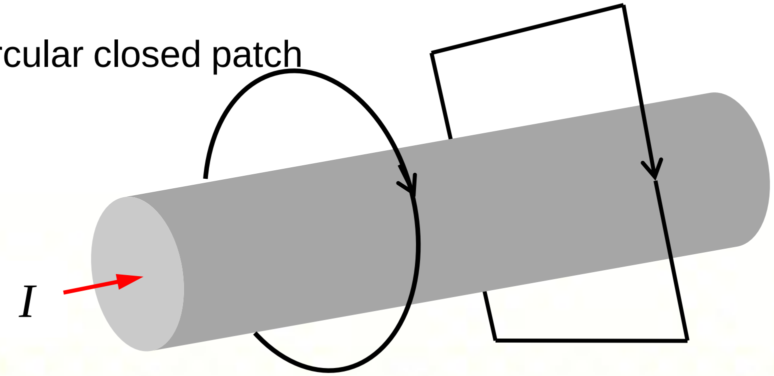
Fleming's right-hand rule

Ampere's Law

2. The **Ampere's law** states that the line integral of magnetic field, $\vec{H}$ for any closed path is equal to the direct current, I enclosed by that path.

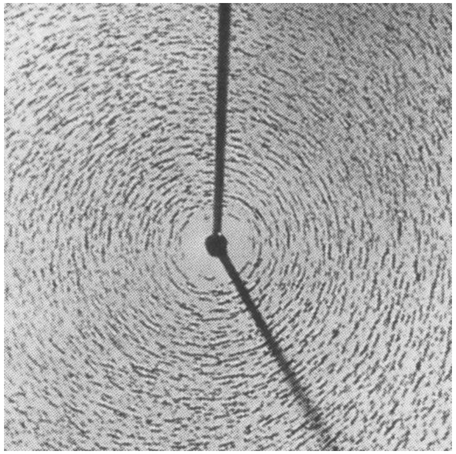
$$\oint \vec{H} \cdot d\vec{l} = I$$

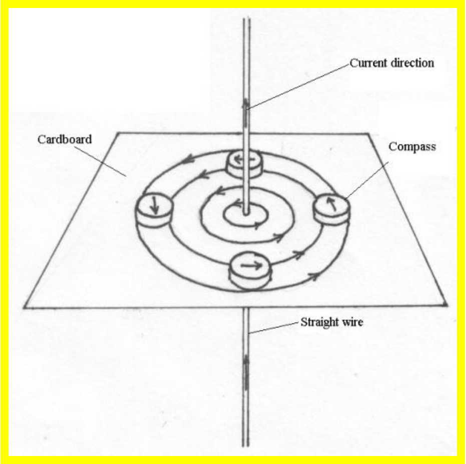


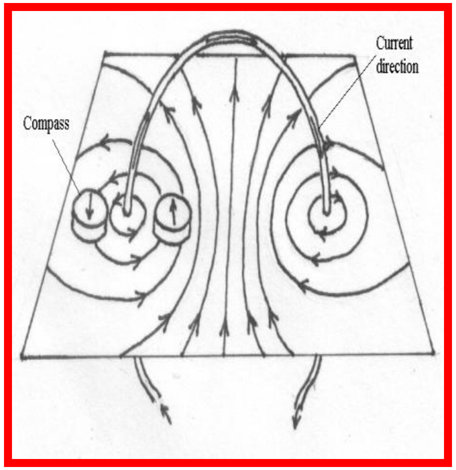


Magnetostatic Field (Example)

Pattern and direction of the magnetic field due to a current in a conductor

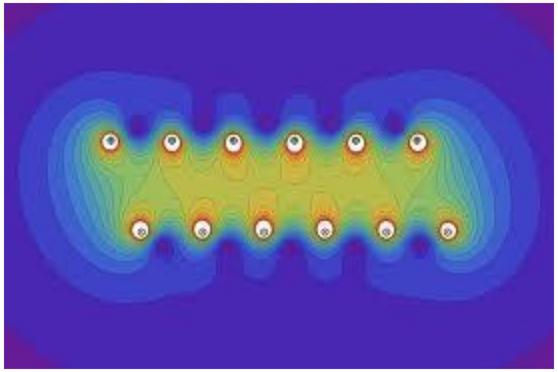




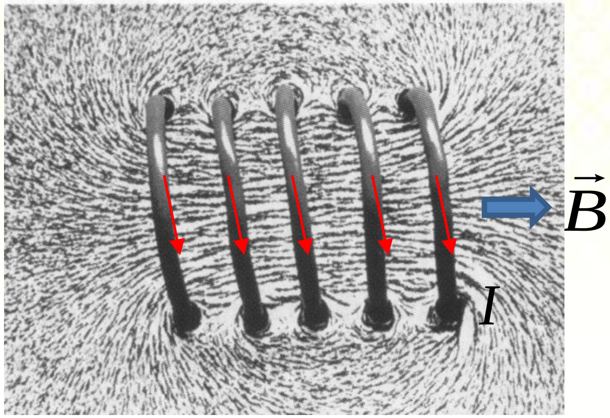


Wire **Coil**

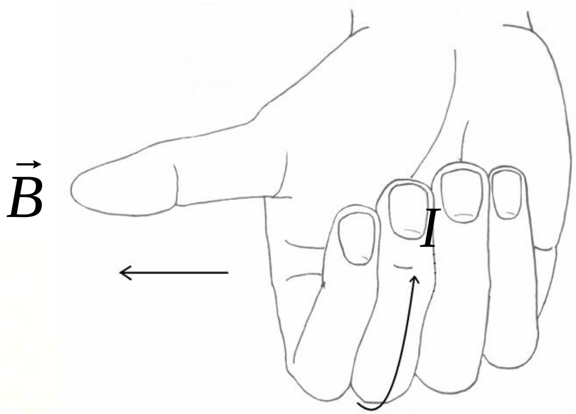
Magnetostatic Field (Example)



2D Contour fields

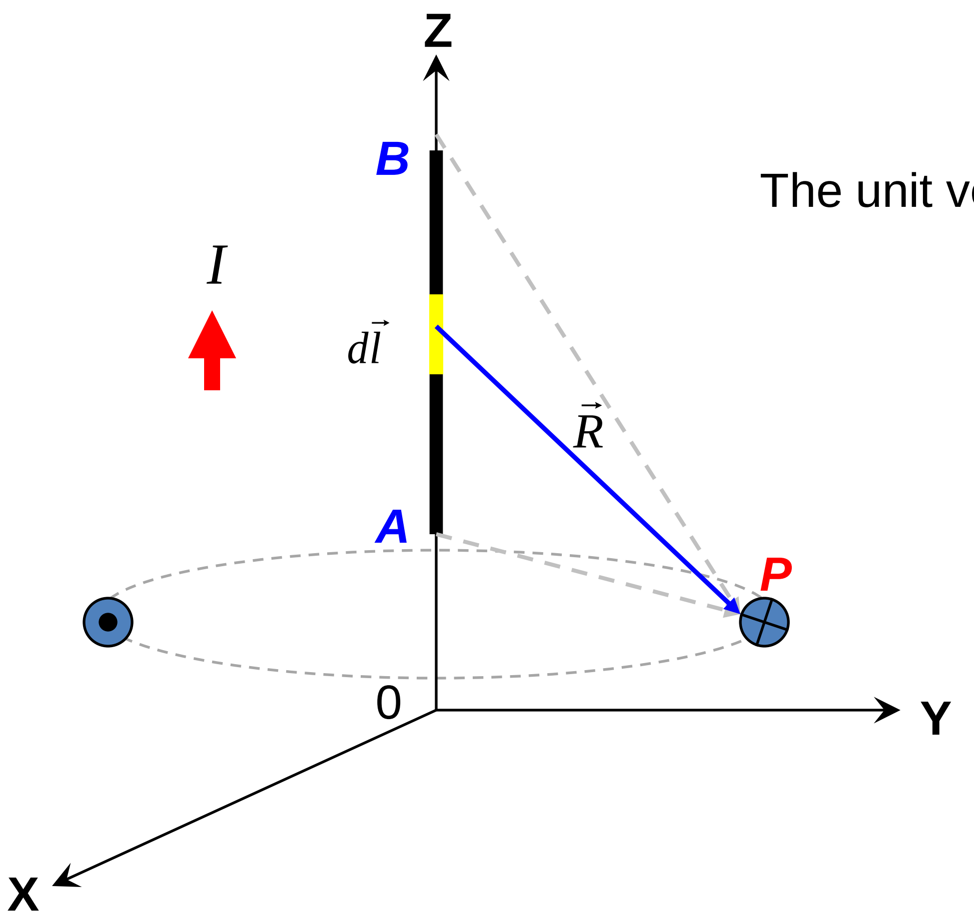


Experiment



Right hand rule

Mathematatic Derivation 1 (Biot-Savart)



The term $d\vec{l} = \hat{z}dz$ and the vector distance, $\vec{R}$ from the source to the test point ***P*** is:

$$\vec{R} = -\hat{z}z + \hat{\rho}\rho$$

The unit vector

$$\hat{a}_R = \frac{\vec{R}}{R} = \frac{-\hat{z}z + \hat{\rho}\rho}{\sqrt{z^2 + \rho^2}}$$

Thus,

$$\vec{H} = \int \frac{Id\vec{l} \times \hat{a}_R}{4\pi R^2}$$

$$= \int \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$

$$= \int_A^B \frac{I\,dz\hat{z} \times \left(-\hat{z}z + \hat{\rho}\rho\right)}{4\pi\left(z^2 + \rho^2\right)^{3/2}}$$

Mathematic Derivation 2 (Biot-Savart)

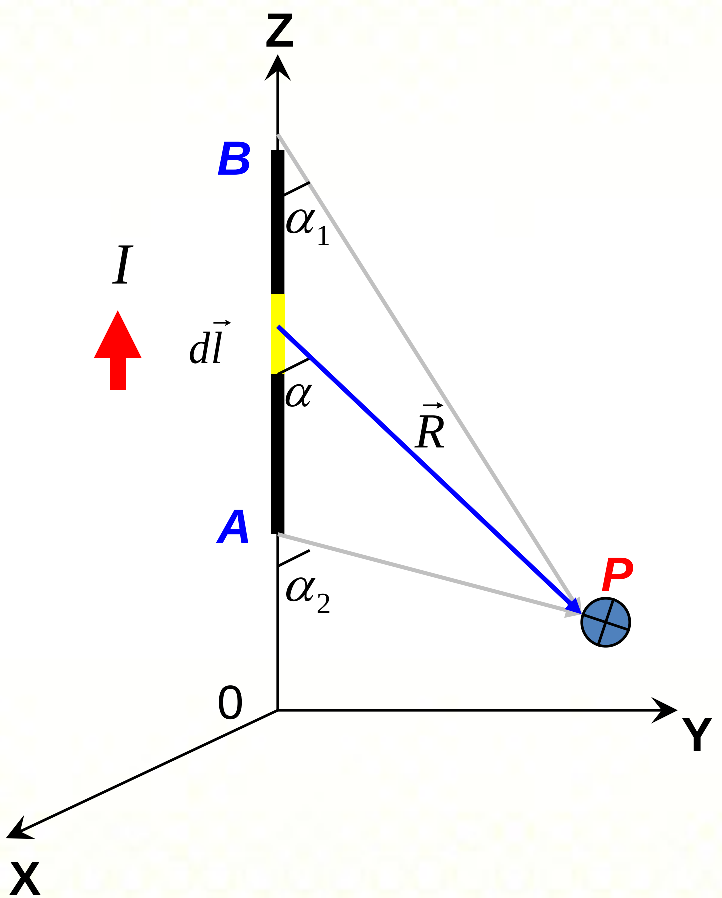
Cross product

$$\vec{dl} \times \vec{R} = \hat{z}\,dz \times \left(-z\hat{z} + \rho\hat{\rho}\right)$$
$$= \begin{vmatrix} \hat{\rho} & \hat{\phi} & \hat{z} \\ 0 & 0 & dz \\ \rho & 0 & -z \end{vmatrix}$$
$$= \rho dz \hat{\phi}$$

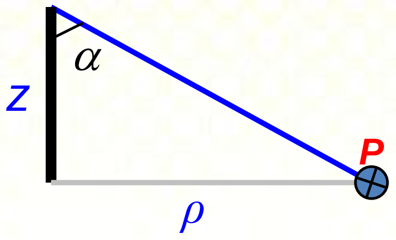
Finally

$$\vec{H} = \int_A^B \frac{I\rho dz}{4\pi\left(z^2 + \rho^2\right)^{3/2}} \hat{\phi}$$

Mathematic Derivation 3 (Biot-Savart)



Trigonometry



$\tan\alpha = \dfrac{\rho}{z}$ and $z = \rho\cot\alpha$

$$\vec{H} = \frac{I}{4\pi}\int_A^B \frac{\rho\, dz}{\left(z^2+\rho^2\right)^{3/2}}\hat{\phi}$$

$$= -\frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2\alpha\, d\alpha}{\left(\rho^2+\rho^2\cot^2\alpha\right)^{3/2}}\hat{\phi} \qquad \leftarrow \quad dz = -\rho\operatorname{cosec}^2\alpha\, d\alpha$$

$$= -\frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2\alpha\, d\alpha}{\rho^3\operatorname{cosec}^3\alpha}\hat{\phi} \qquad \leftarrow \quad 1+\cot^2\alpha = \operatorname{cosec}^2\alpha$$

$$= -\frac{I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha\,\hat{\phi} \qquad \leftarrow \quad \operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$$

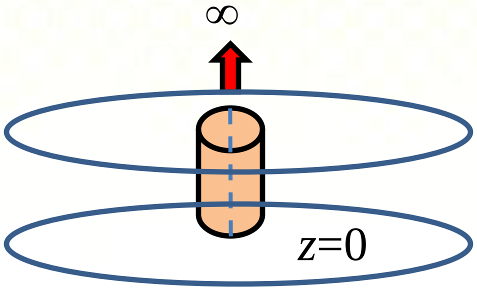
$$= \frac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right)\hat{\phi}$$

Mathematic Derivation 4 (Biot-Savart)

If *A* at origin (0, 0, 0) and *B* at (0, 0, ∞), the angle becomes $\alpha_1 = 90^o$ and $\alpha_2 = 0^o$

Thus,

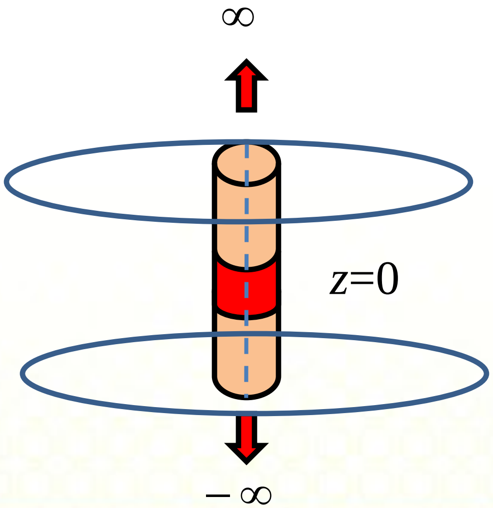
$$\vec{H} = \frac{I}{4\pi\rho}\hat{\phi}$$



If *A* at (0, 0, $-\infty$) and *B* at (0, 0, ∞), the angle becomes $\alpha_1 = 180^o$ and $\alpha_2 = 0^o$

Thus,

$$\vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$$



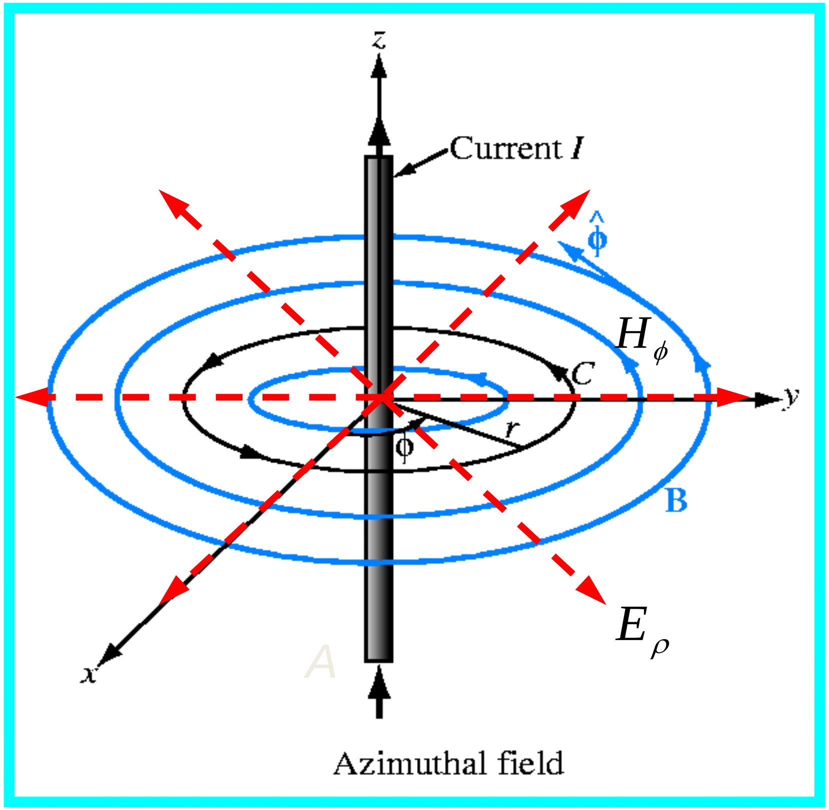
Relationship Electrostatic and Magnetostatic

Electrostatic

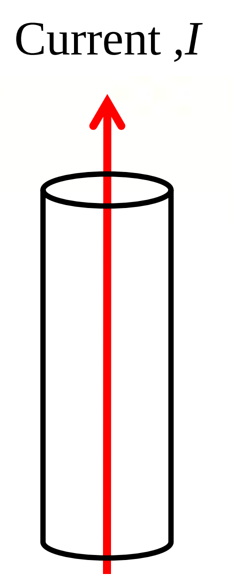
$$\vec{E} = E_\rho \hat{\rho}$$
$$= \int_A^B \frac{\rho_l \, \rho dz}{4\pi\varepsilon_o \left(\rho^2 + z^2\right)^{3/2}} \hat{\rho}$$

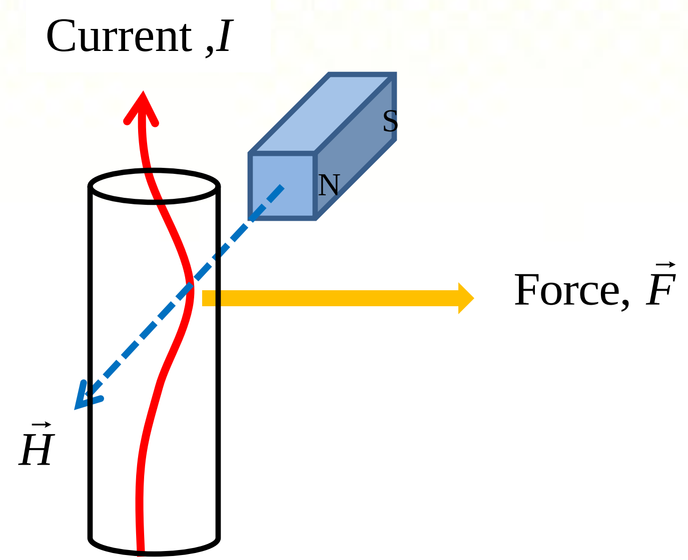
Magnetostatic

$$\vec{H} = H_\phi \hat{\phi}$$
$$= \int_A^B \frac{I \rho dz}{4\pi \left(z^2 + \rho^2\right)^{3/2}} \hat{\phi}$$



Magnetic Force (1)

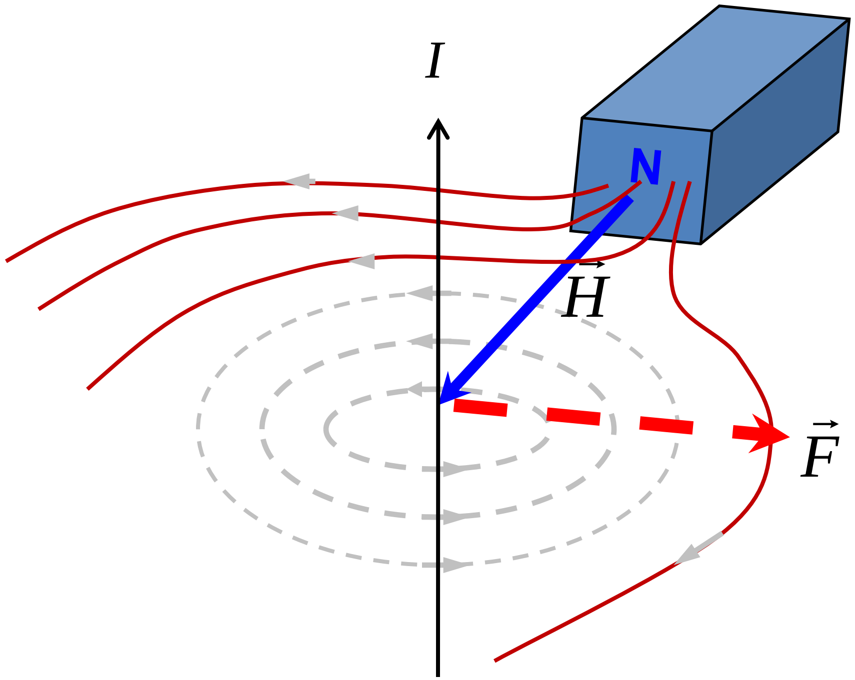


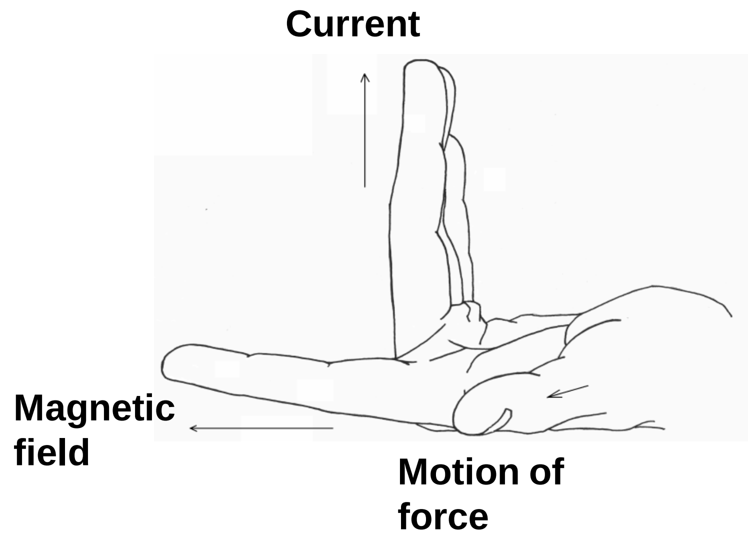


A wire is carrying a current, I.

If an external magnetic field, $\vec{H}$ occurs, wire is deflected in a direction normal to both the field and the direction of current, I.

Magnetic Force (2)





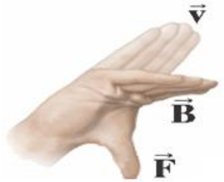
Fleming's left hand rule

One magnetic field is provided by permanent magnet, and another is produced by the current-carrying conductor.

The interaction between the two magnetic fields produces a resultant field which will force on the conductor.

Magnetic Force (3)

1. The magnetic force, $\vec{F}_m$ acting on the individual charges, q moving with constant velocity, $\vec{u}$ in the conductor due to magnetic flux, $\vec{B}$.



$$\vec{F}_m = q\vec{u} \times \vec{B}$$

Magnetic force, magnetic field and velocity in perpendicular direction

2. The electric force, $\vec{F}_e$ acting on a charge q within an electric field, $\vec{E}$.

$$\vec{F}_e = q\vec{E}$$

Electric force and electric field in same direction

3. A total force, $\vec{F}$ on a charge

$$\vec{F} = q\left(\vec{E} + \vec{u} \times \vec{B}\right)$$

Lorentz force equation

Magnetic Force (4)

4. To find a force on a current element, consider a line conducting current in the presence of magnetic field with differential segment dQ of charge moving with velocity, $\vec{u}$

$$d\vec{F} = dQ\vec{u} \times \vec{B}$$

But $\vec{u} = d\vec{l} \big/ dt$ thus

$$d\vec{F} = \frac{dQ}{dt} d\vec{l} \times \vec{B}$$

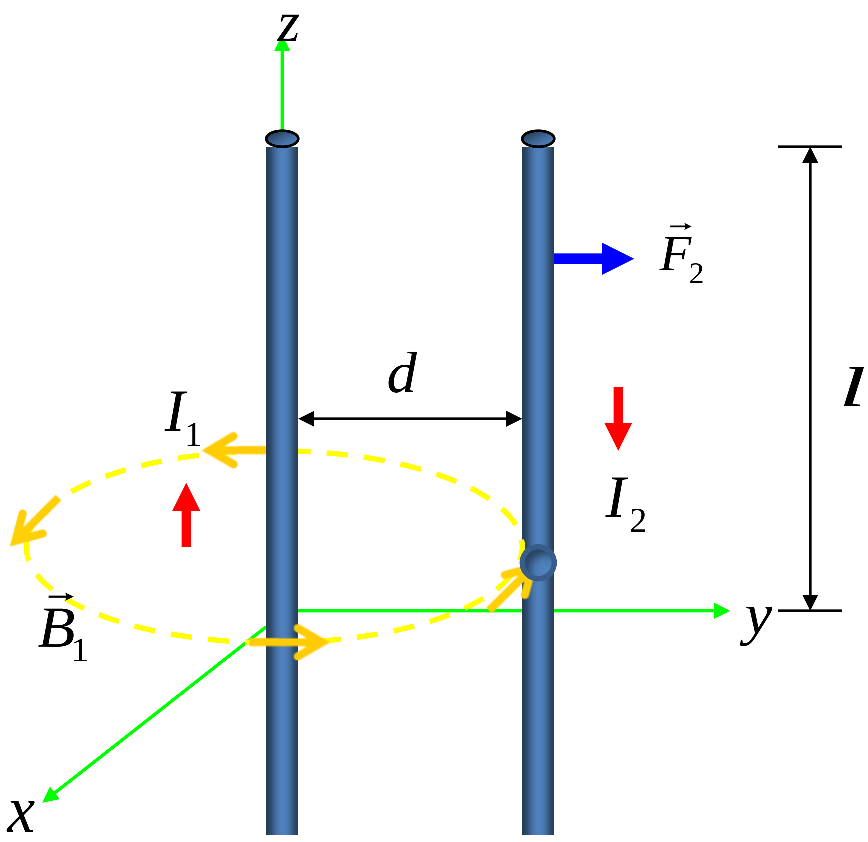
Since $dQ \big/ dt$ is equal to current, I in the line,

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$$\vec{F} = \int I \, d\vec{l} \times \vec{B}$$

Magnetic Force (5)

We can find the force from a collection of current elements



$\vec{B}_1$ is the magnetic field due to current I_1

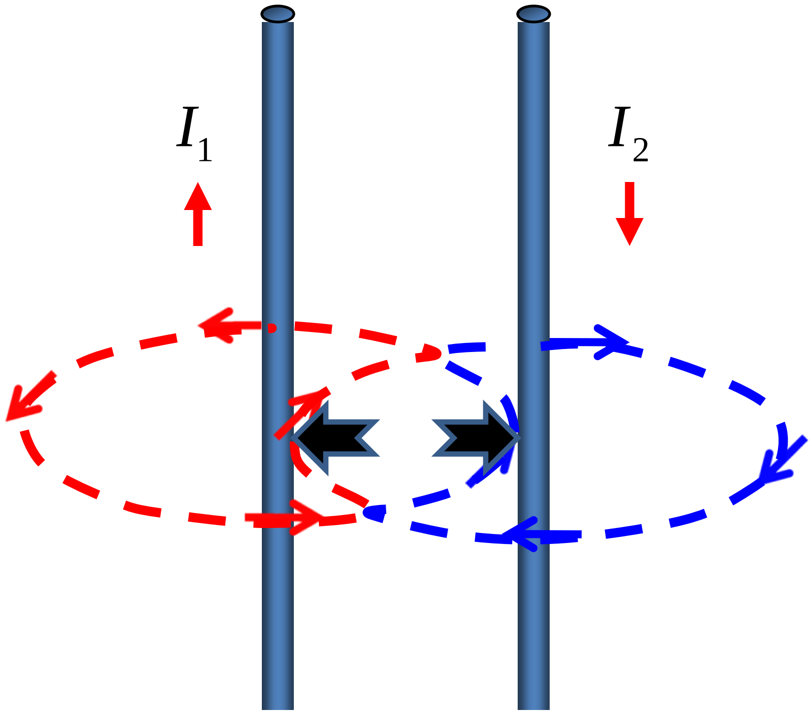
$$\vec{B}_1 = -\hat{x}\frac{\mu_0 I_1}{2\pi d}$$

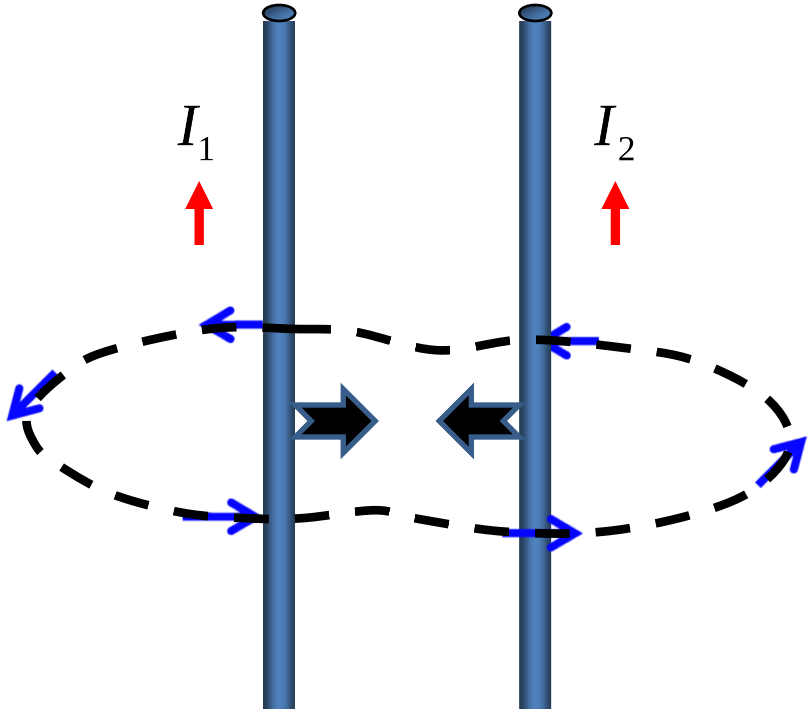
$\vec{F}_2$ is the force exerted on a length *l* of second wire due to $\vec{B}_1$

$$\vec{F}_2 = I_2 l\,\hat{z}\times\vec{B}_1$$
$$= I_2 l\,\hat{z}\times -\hat{x}\frac{\mu_0 I_1}{2\pi d}$$
$$= \hat{y}\frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Magnetic Force (6)

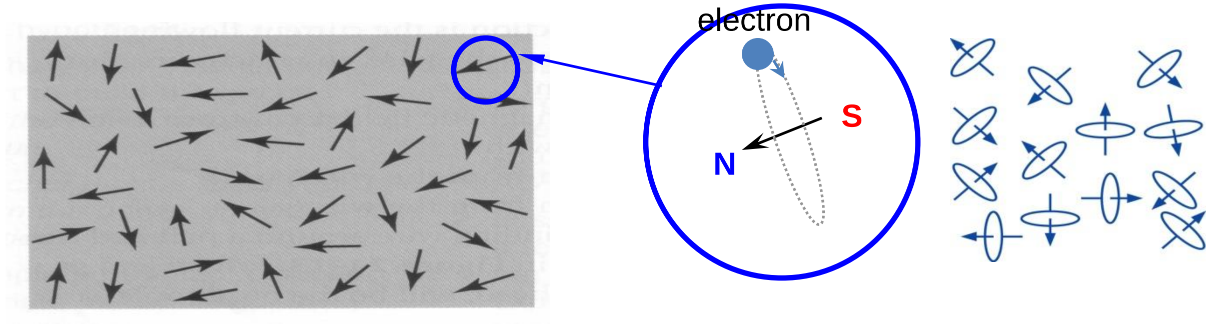
$$\vec{F}_1 = -\vec{F}_2$$



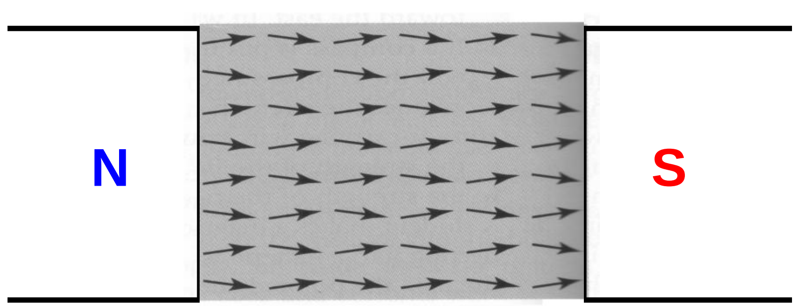


Magnetic Materials (1)

1. We will now investigate the characteristics of a material made of a very large number of atoms and their corresponding randomly magnetic dipoles.



2. What will happen to these magnetic fields from individual atoms if an external magnetic field is applied to the material ?



The magnetic dipoles line up.

UTM

Magnetic Materials (2)

3. Similar with dielectric materials, the magnetic materials can be said linear and isotropic if the magnitude of the magnetization, $\vec{M}$ is directly proportional to the magnitude of magnetic fields, $\vec{H}$.

$$\vec{M} = \chi_m \vec{H}$$

where χ_m is called the magnetic susceptibility of the material.

4. The magnetization, $\vec{M}$ is the magnetic dipole moment per unit volume.

5. Magnetic materials is classified into 3 main groups

   a) Diamagnetic

   b) Paramagnetic

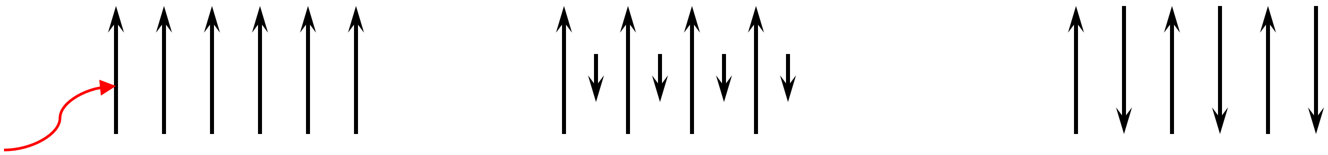
   c) Ferromagnetic

Magnetic Materials (3)

Magnetic Materials

Linear	Linear	Nonlinear
Diamagnetic	**Paramagnetic**	**Ferromagnetic**
$\chi_m < 0$	$\chi_m > 0$	$\chi_m >> 0$
$\mu_r < 1$	$\mu_r \geq 1$	$\mu_r >> 1$
copper silicon diamond	platinum air tungsten	iron nickel alloy

Ferromagnetic < Paramagnetic (Ferrimagnetic) < Diamagnetic (Antiferromagnetic)



magnetic dipole moment

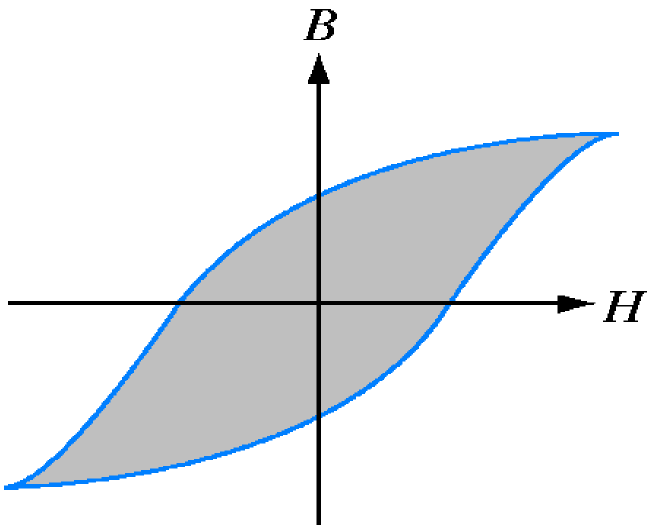
Magnetic Materials (4)

Ferromagnetic is a nonlinear material because magnetization, $\vec{M}$ is not directly proportional to the magnitude of magnetic fields, $\vec{H}$.

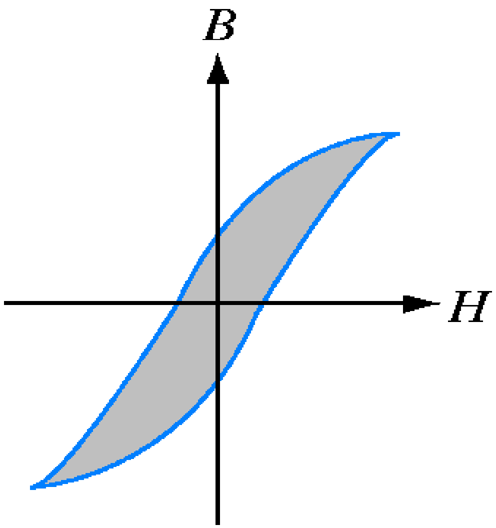
Example

$$\begin{aligned} \vec{B} &= \mu_o \vec{H} + \vec{M} \\ &= \mu_o \vec{H} + \chi_m \vec{H} \\ &= \mu_o (1+\chi_m) \vec{H} \\ &= \mu_o \mu_r \vec{H} \end{aligned}$$

for ferromagnetic materials

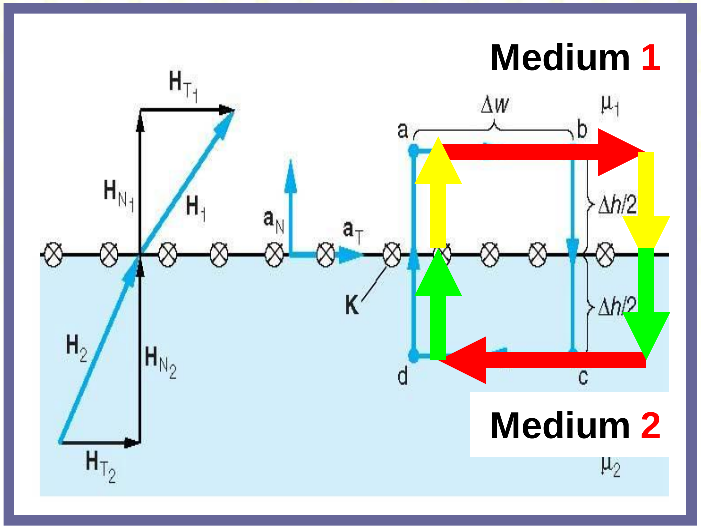


Hard Ferromagnetic Material



Soft Ferromagnetic Material

Boundary Conditions (1)



***K* is a surface current on the boundary**

By using Ampere's law

$$\oint \vec{H} \cdot d\vec{l} = I \qquad (1)$$

Equation (1) can be written as

$$\int_a^b \vec{H} \cdot d\vec{l} + \int_b^c \vec{H} \cdot d\vec{l} + \int_c^d \vec{H} \cdot d\vec{l} + \int_d^a \vec{H} \cdot d\vec{l} = I \qquad (2)$$

Equation (2) can be written in discrete form

$$H_{1t} \cdot \Delta w + H_{1n} \cdot \frac{\Delta h}{2} + H_{2n} \cdot \frac{\Delta h}{2} - H_{2t} \cdot \Delta w - H_{2n} \cdot \frac{\Delta h}{2} - H_{1n} \cdot \frac{\Delta h}{2} = K\Delta w$$

Boundary Conditions (2)

Finally

$$H_{1t} \cdot \Delta w - H_{2t} \cdot \Delta w = K \Delta w$$

$$H_{1t} - H_{2t} = K$$

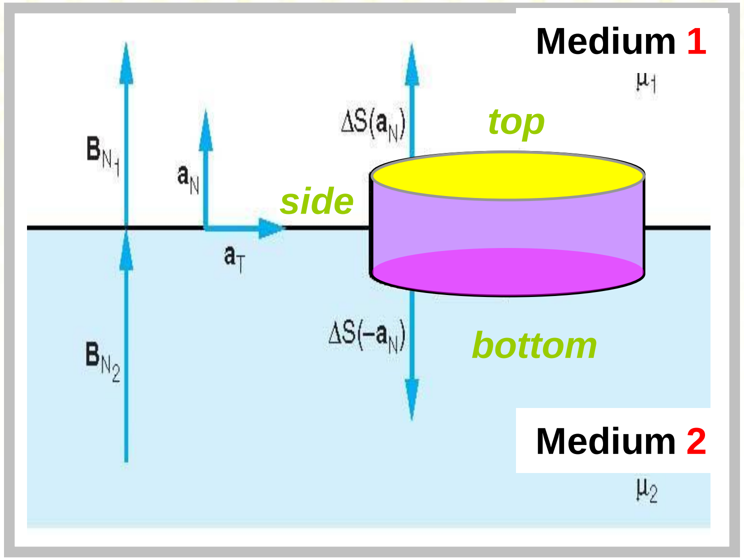
In general case

$$\left(\vec{H}_1 - \vec{H}_2\right) \times \hat{a}_n = \vec{K}$$

If the boundary is free of current or the media are not conductors ($K = 0$)

$$\vec{H}_{1t} = \vec{H}_{2t}$$

Boundary Conditions (3)



By using magnetic **Gauss's law**

$$\oint \vec{B} \cdot d\vec{S} = 0 \qquad (1)$$

Equation (1) can be written as

$$\int_{top} \vec{B} \cdot d\vec{S} + \int_{bottom} \vec{B} \cdot d\vec{S} + \int_{side} \vec{B} \cdot d\vec{S} = 0 \qquad (2)$$

Equation (2) can be written in discrete form

$$B_{1n} \Delta S - B_{2n} \Delta S = 0$$

Boundary Conditions (4)

Finally

$$B_{1n} - B_{2n} = 0$$

$$B_{1n} = B_{2n}$$

Question

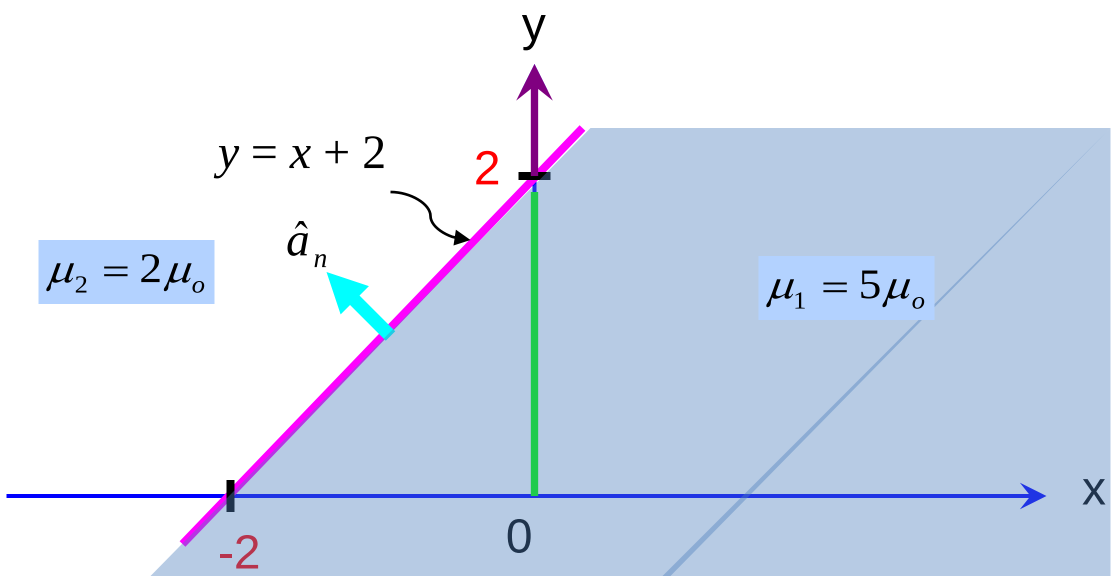
Given that $\vec{H}_1 = -2\hat{x} + 6\hat{y} + 4\hat{z}$ A/m in region $y - x - 2 \le 0$, where $\mu_1 = 5\mu_o$. Determine

a) $\vec{M}_1$ and $\vec{B}_1$

b) $\vec{H}_2$ and $\vec{B}_2$

in region $y - x - 2 \ge 0$, where $\mu_2 = 2\mu_o$

Initial Step of Solution



For region **1**

$$y \leq x + 2$$

If $x = 0$, $y \leq 2$

For region **2**

$$y \geq x + 2$$

If $x = 0$, $y \geq 2$

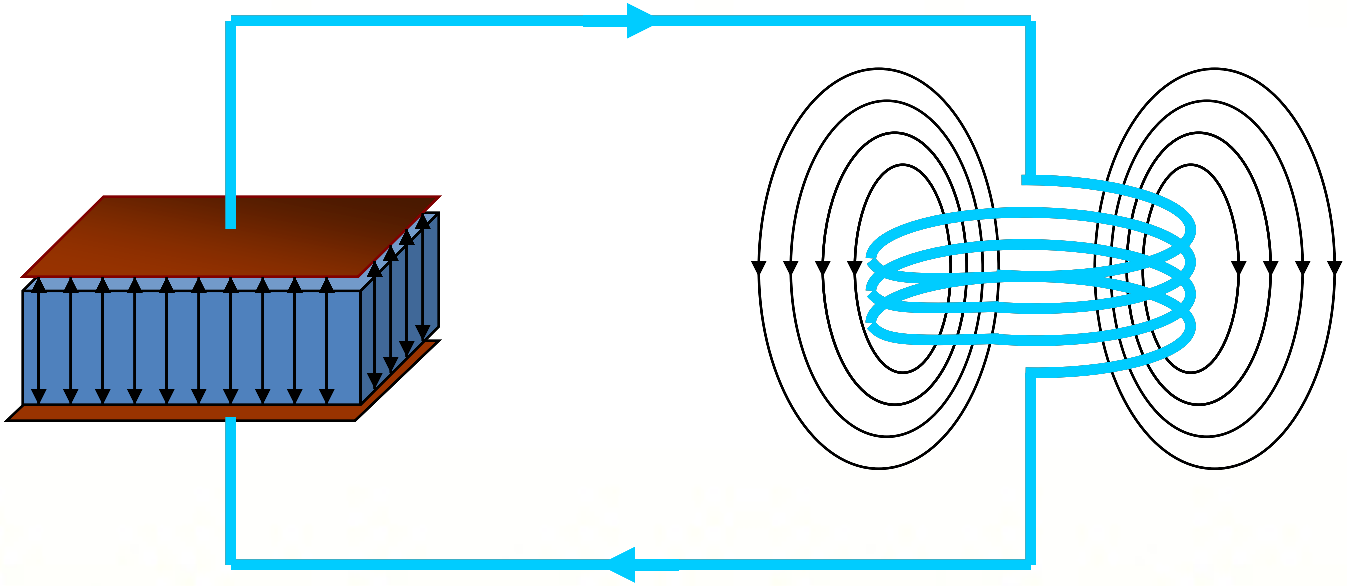
Magnetic Energy (1)

Potential Energy

$$W_e = \frac{1}{2}\int_v \rho V\, dv = \frac{1}{2}\int \varepsilon_o \vec{E}^2\, dv$$

Magnetic Energy

$$W_m = \int I\, V\, dt = \frac{1}{2} L I^2 = \frac{1}{2}\int \mu_o \vec{H}^2\, dv$$



Analogies between four types of oscillations system

	Mechanical System (without friction)		Electromagnetic System (without resistance)	
Accumulated system	Spring Mass	**Spring system** Energy, $U_{Spring} = \frac{1}{2}kx^2$ Energy, $U_{Mass} = \frac{1}{2}mv^2$ Velocity, $v = \frac{\partial x}{\partial t}$ Angular frequency, $\omega = \sqrt{\frac{k}{m}}$	Capacitor Inductor	**Circuit system** Energy, $U_{Capacitor} = \frac{1}{2}\frac{q^2}{C}$ Energy, $U_{Inductor} = \frac{1}{2}L \times I^2$ Current, $I = \frac{\partial q}{\partial t}$ Angular frequency, $\omega = \sqrt{\frac{1}{LC}}$
Distributed system	Viscosity Density	**Accoustic waveguide** Energy, $U_{Viscosity} = \frac{1}{2}\rho_o v_{gases}^2$ Energy, $U_{Density} = \frac{1}{2}C_2\left(\frac{C_1\rho}{\rho_o}\right)^2$ Velocity, $v = \sqrt{\frac{C_2}{\rho_o}}$	Magnetic field Electric field	**Electromagnetic waveguide** Energy, $U_{Electric} = \frac{1}{2}\varepsilon_o E^2$ Energy, $U_{Magnetic} = \frac{1}{2}\frac{B^2}{\mu_o}$ Velocity, $v = \sqrt{\frac{1}{\varepsilon_o \mu_o}}$

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