SAB2223 Mechanics of Materials and Structures

TOPIC 9
TORSION

Lecturer:
Dr. Shek Poi Ngian

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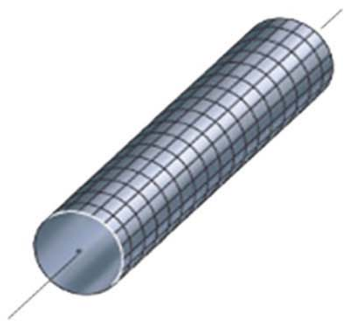
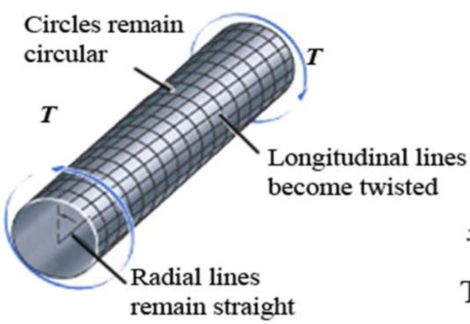
TORSION

Torsion Formula

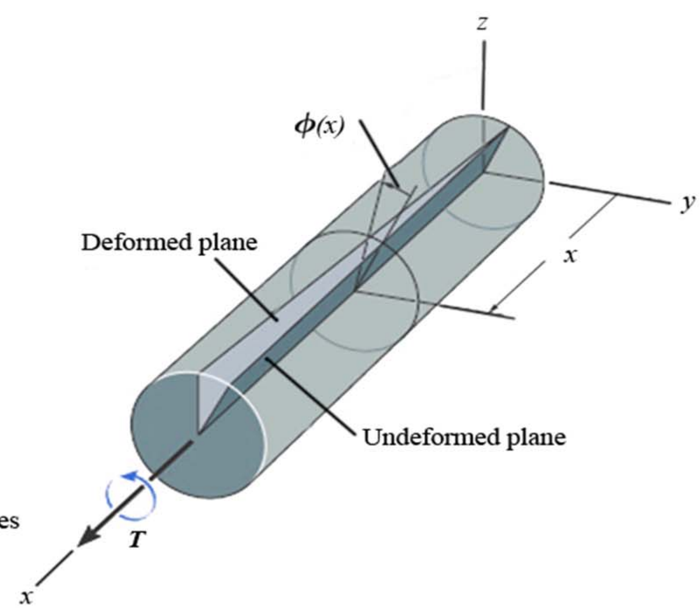
- Two assumptions:
  - Linear and elastic deformation
  - Plane section remains plane and undistorted



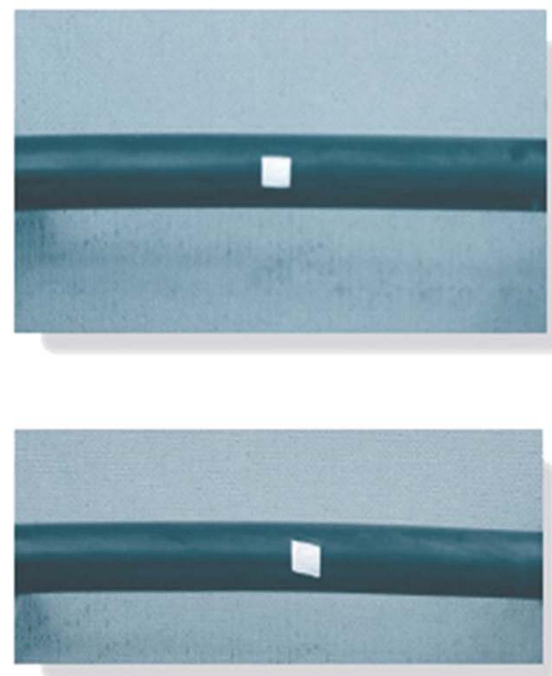
Before deformation
(a)



After deformation
(b)



The angle of twist $\phi(x)$ increases as x increases



Notice the deformation of the rectangular element when this rubber bar is subjected to a torque

Torsion Formula

- Linear distribution of stress: $\tau = \frac{\rho}{c}\tau_{\max}$

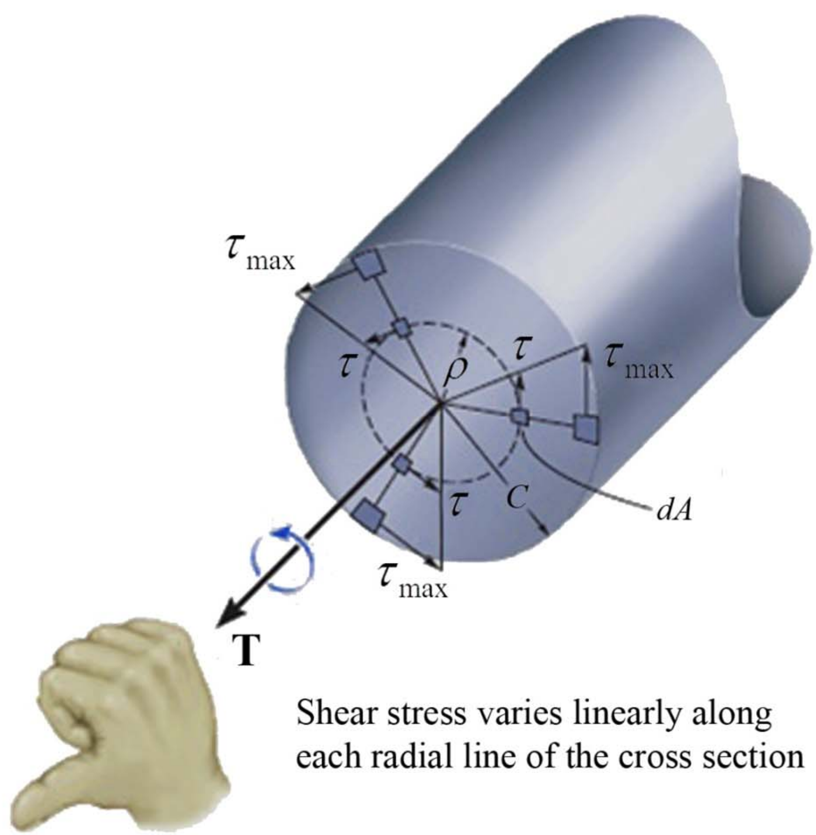
- Torsion – shear relationship:

$$T = \int_A \rho(\tau)dA = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{\max} dA$$

$$T = \frac{\tau_{\max}}{c}\int_A \rho^2 dA$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$\tau = \frac{T\rho}{J}$$



Shear stress varies linearly along each radial line of the cross section

Torsion Formula

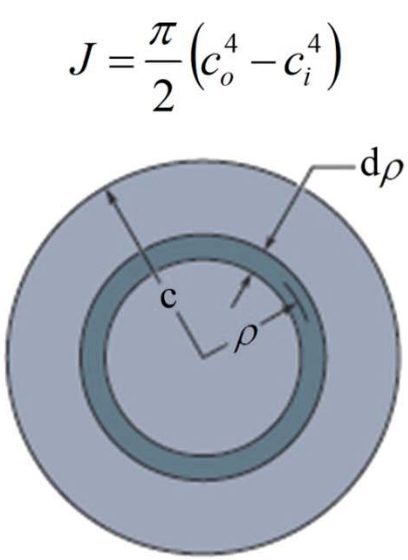
– Solid shaft:

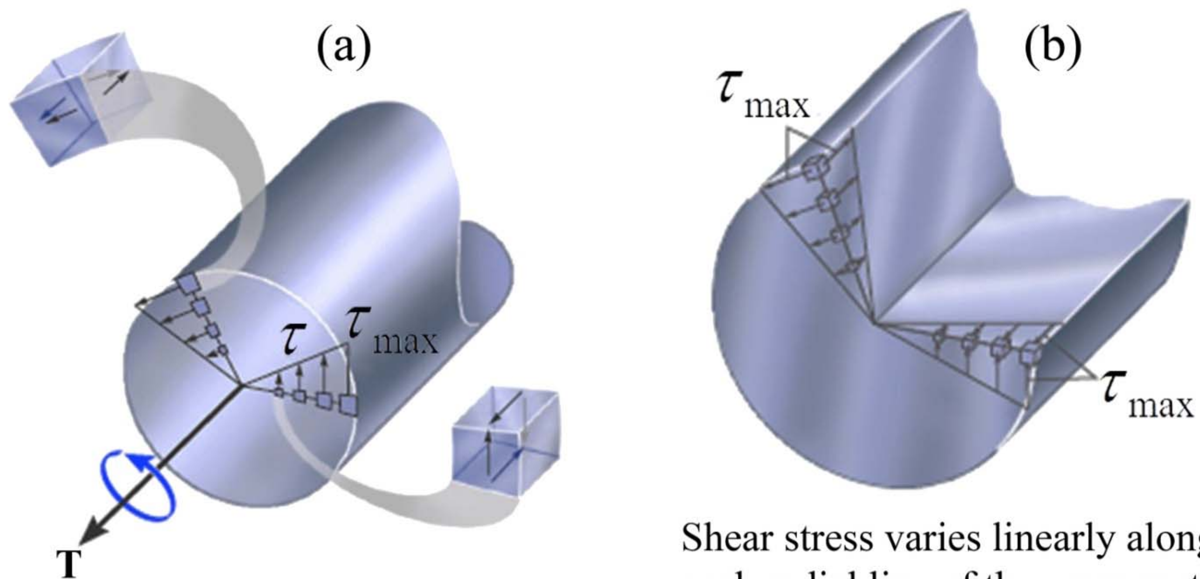
$$J = \int_A \rho^2 dA = \int_0^C \rho^2 (2\pi\rho d\rho) = 2\pi \int_0^C \rho^3 d\rho = 2\pi\left(\frac{1}{4}\right)\rho^4 \Big|_0^C$$

$$J = \frac{\pi}{2} c^4$$

– Tubular shaft:

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$

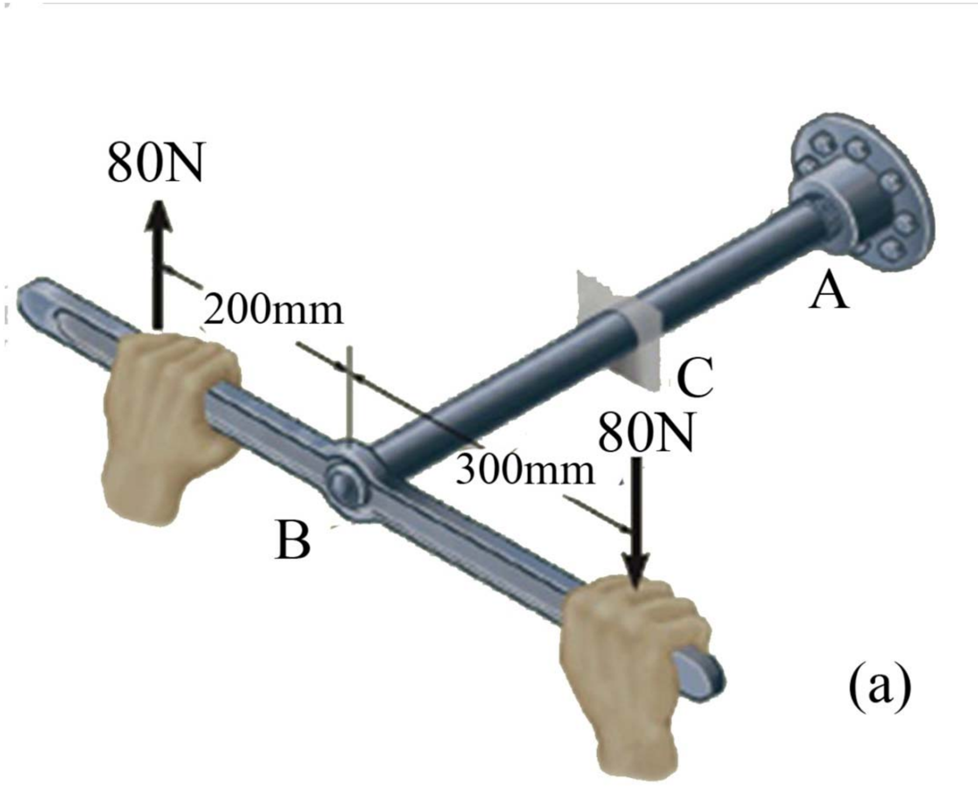




Shear stress varies linearly along each radial line of the cross section

Example 1

The pipe shown has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at *A* using a torque wrench at *B*, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.



Example 1 (cont.)

Solutions

$$\sum M_y = 0;$$

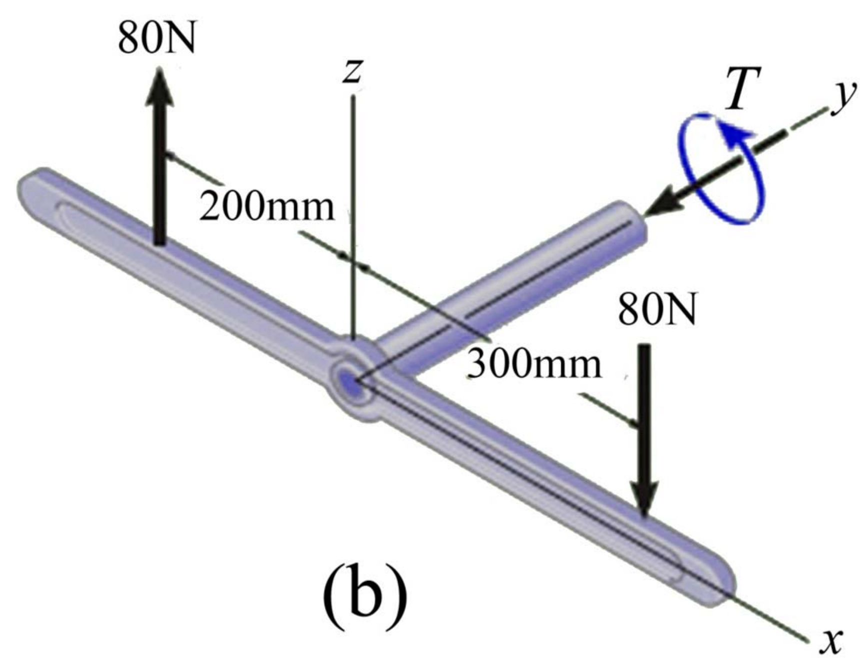
$$80(0.3) + 80(0.2 - T) = 0$$

$$T = 40\ \text{N}\cdot\text{m}$$

$$J = \frac{\pi}{2}\left[(0.05)^4 - (0.04)^4\right] = 5.796\left(10^{-6}\right)\text{m}^4$$

$$\rho = c_0 = 0.05\ \text{m}$$

$$\tau_0 = \frac{Tc_0}{J} = \frac{40(0.05)}{5.796\left(10^{-6}\right)} = 0.345\ \text{MPa}$$

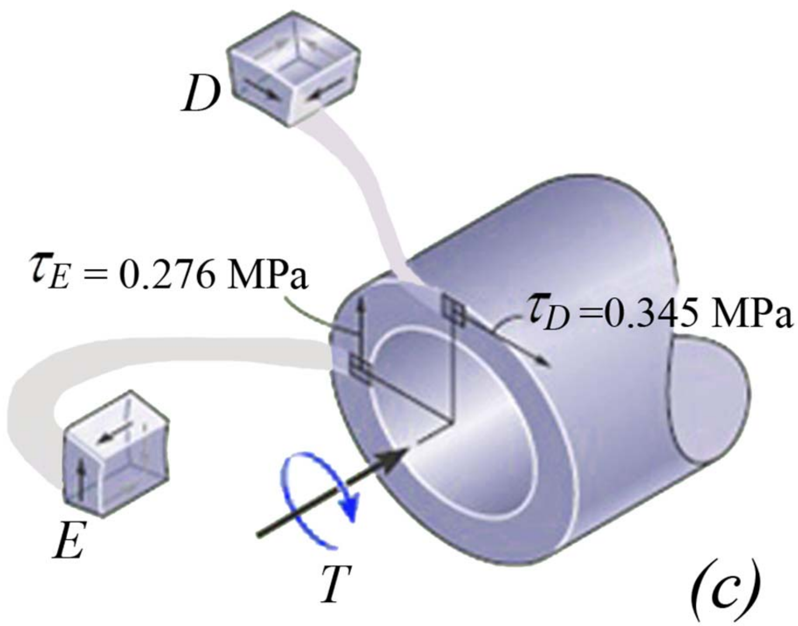


Example 1 (cont.)

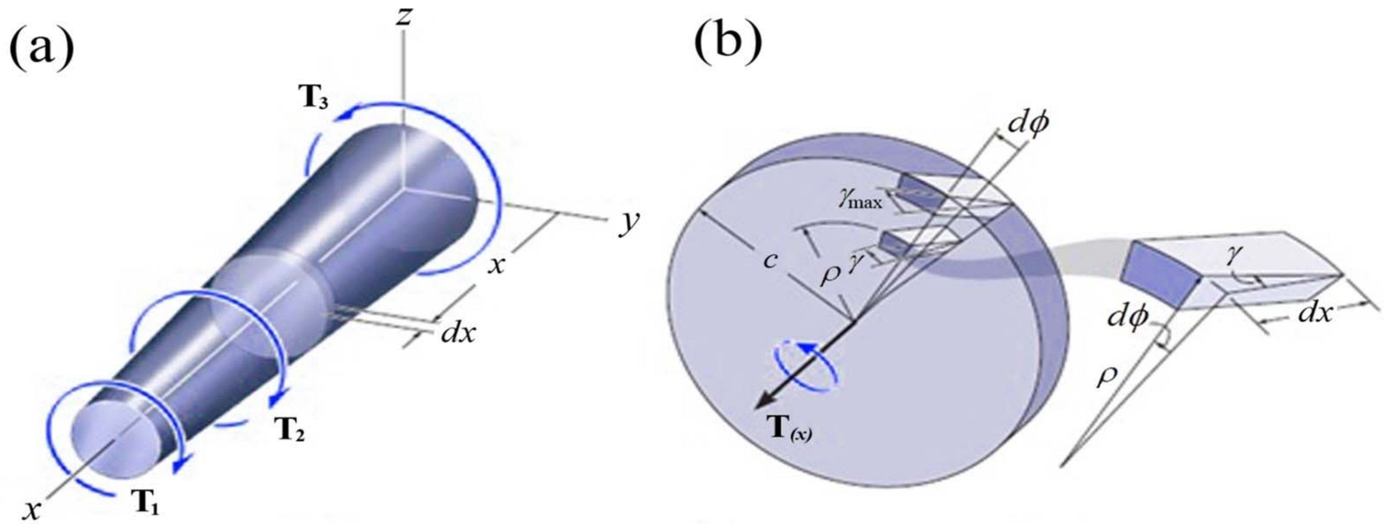
Solutions

$$\rho = c_i = 0.04\,\text{m}$$

$$\tau_i = \frac{Tc_i}{J} = \frac{40(0.04)}{5.796(10^{-6})} = 0.276\ \text{MPa}$$



Angle of Twist



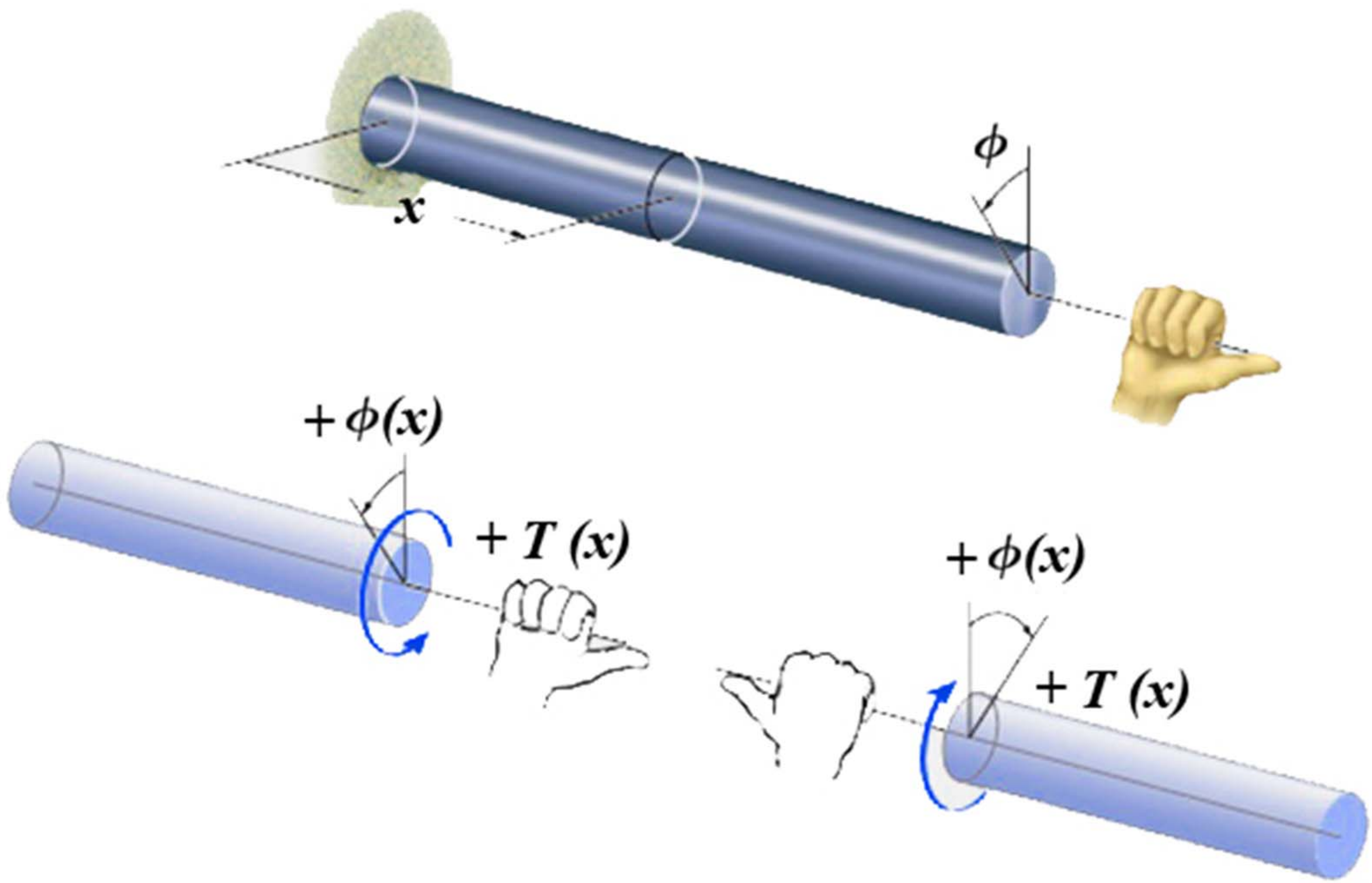
$$d\phi = \gamma \frac{dx}{\rho}$$

$$d\phi = \frac{T(x)}{J(x)G} dx$$

- For constant torque and cross-sectional area: $$\phi = \frac{TL}{JG}$$

Angle of Twist

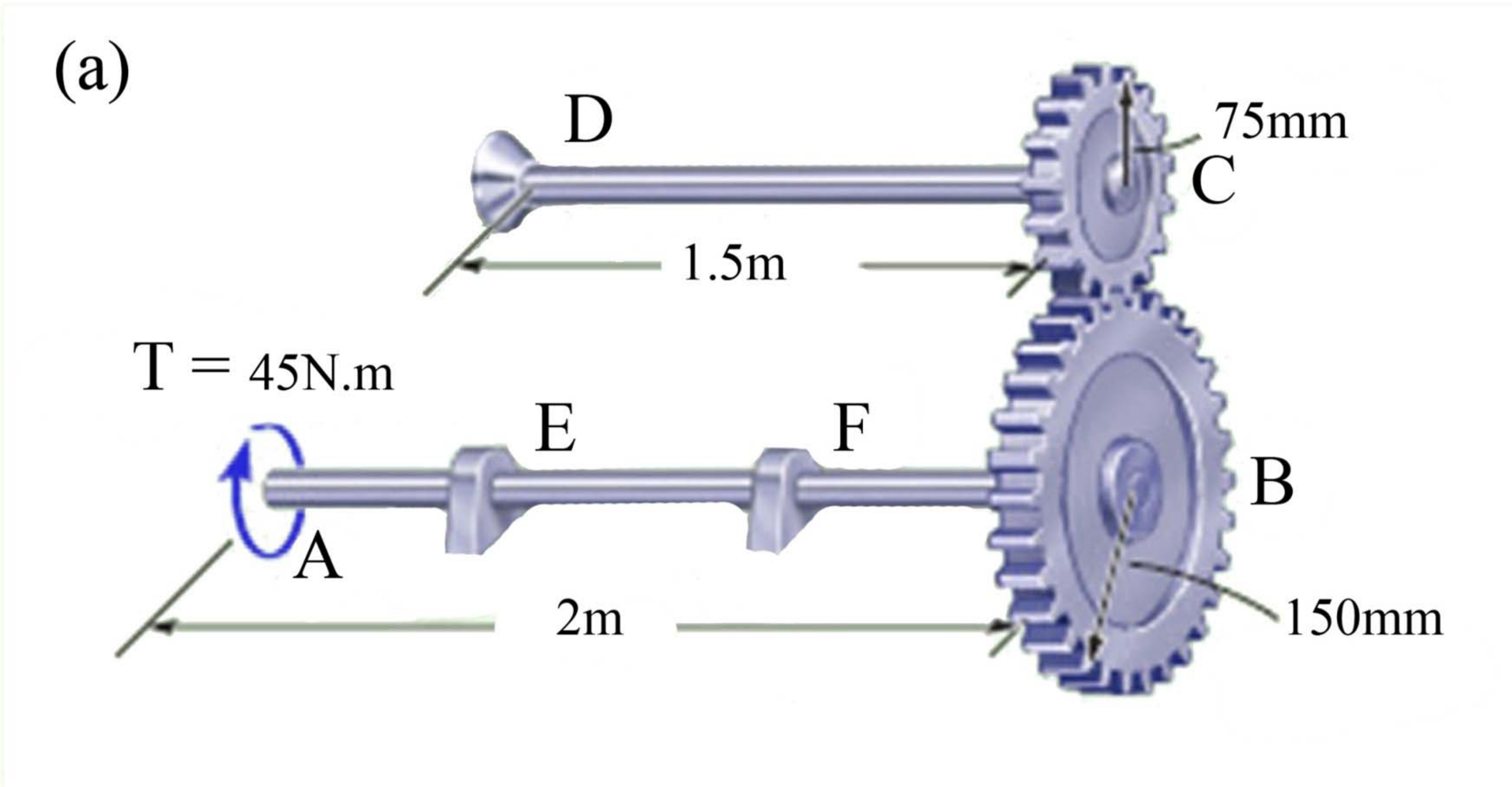
- Sign convention:



Positive sign convention for T and ϕ

Example 2

The two solid steel shafts are coupled together using the meshed gears. Determine the angle of twist of end *A* of shaft *AB* when the torque 45 Nm is applied. Take G to be 80 GPa. Shaft *AB* is free to rotate within bearings *E* and *F*, whereas shaft *DC* is fixed at *D*. Each shaft has a diameter of 20 mm.

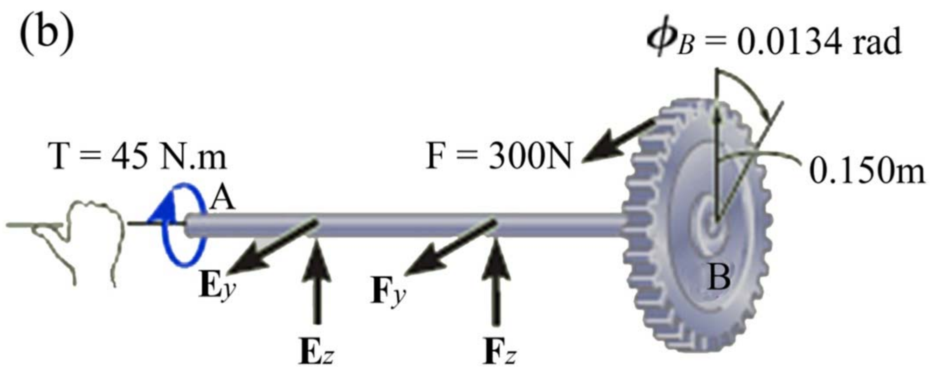


Example 2 (cont.)

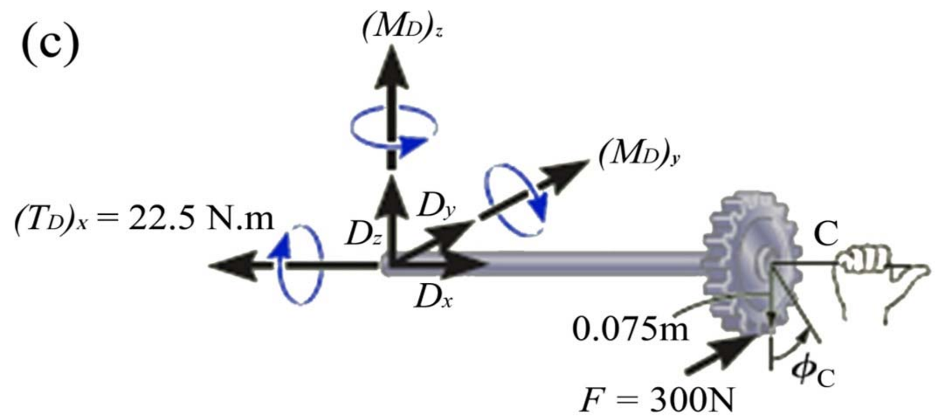
Solutions

$$F = 45/0.15 = 300 \text{ N}$$

$$(T_D)_x = 300(0.075) = 22.5 \text{ Nm}$$



$$\varphi_C = \frac{TL_{DC}}{JG} = \frac{(+22.5)(1.5)}{(\pi/2)(0.001)^4\left[80(10)^9\right]} = +0.0269 \text{ rad}$$



$$\varphi_B(0.15) = (0.0269)(0.075) \Rightarrow 0.0134 \text{ rad}$$

Example 2 (cont.)

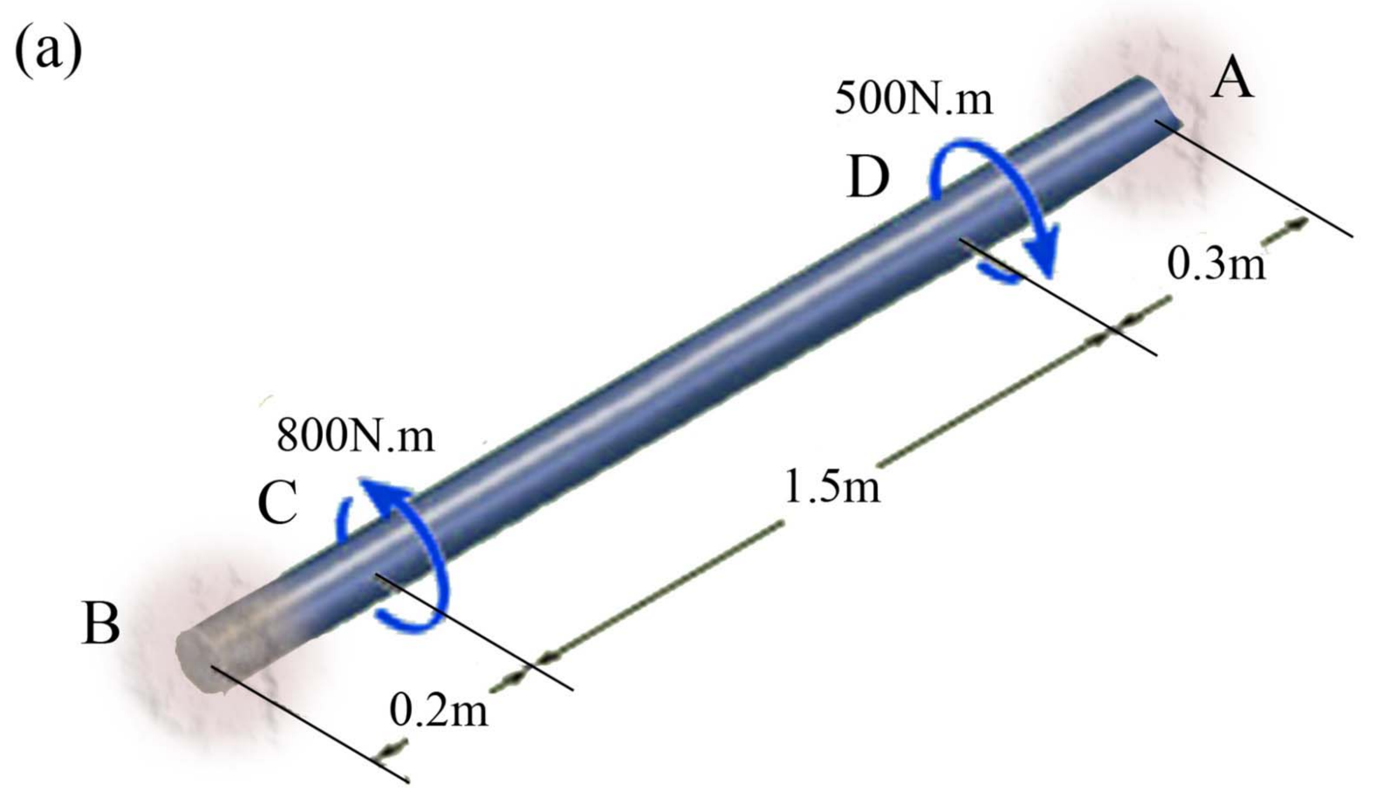
Solutions

$$\varphi_{A/B} = \frac{T_{AB}L_{AB}}{JG} = \frac{(+45)(2)}{(\pi/2)(0.010)^4\left[80\left(10^9\right)\right]} = +0.0716 \text{ rad}$$

$$\varphi_A = \varphi_B + \varphi_{A/B} = 0.0134 + 0.0716 = +0.0850 \text{ rad}$$

Example 3

The solid steel shaft shown below has a diameter of 20 mm. If it is subjected to the two torques, determine the reactions at the fixed supports *A* and *B*.



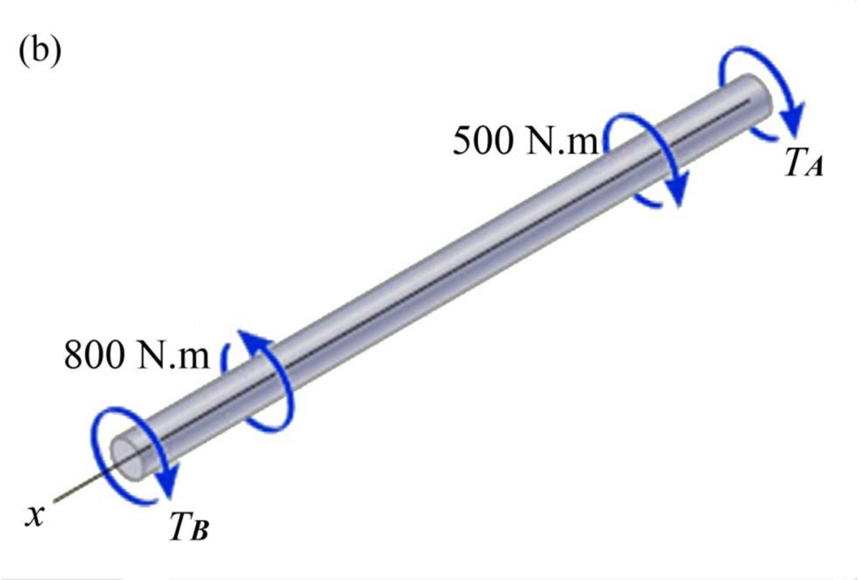
Example 3 (cont.)

Solutions

$$\sum M_x = 0$$

$$-T_b + 800 - 500 - T_A = 0 \qquad (1)$$

$$\phi_{A/B} = 0$$



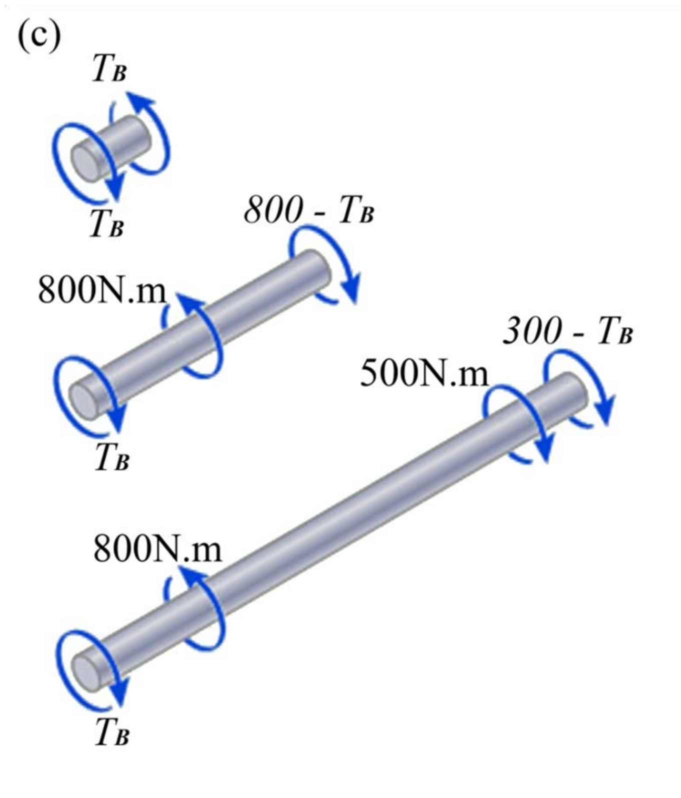
Example 3 (cont.)

Solutions

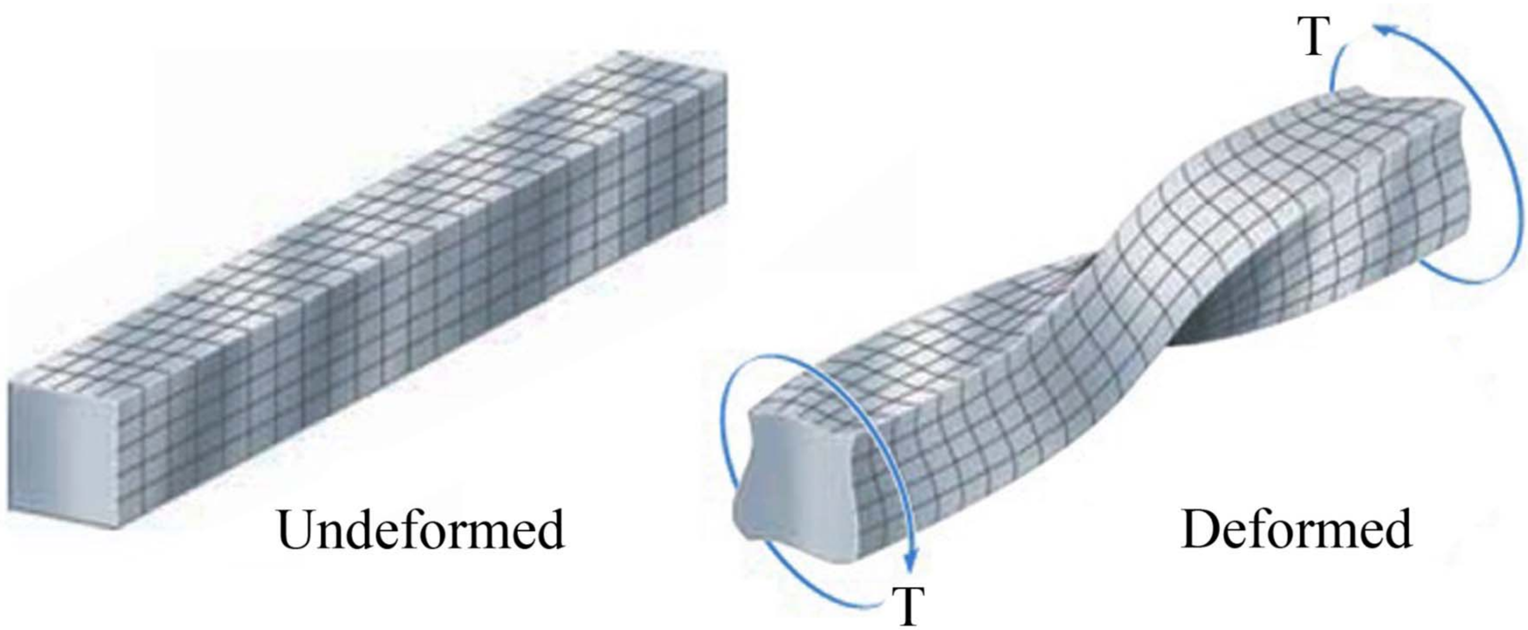
$$\frac{-T_B(0.2)}{JG}+\frac{(800-T_B)(1.5)}{JG}+\frac{(300-T_B)(0.3)}{JG}=0$$

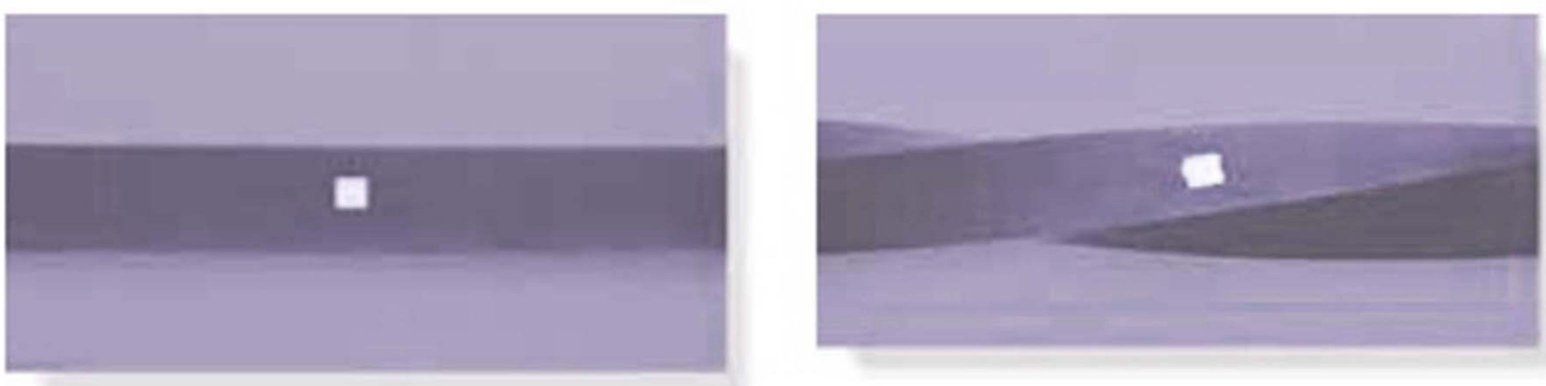
$$T_B = 645\ \text{N}\cdot\text{m}$$

$$T_A = -345\ \text{N}\cdot\text{m}$$

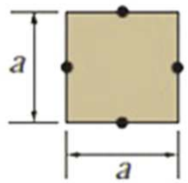
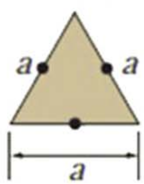
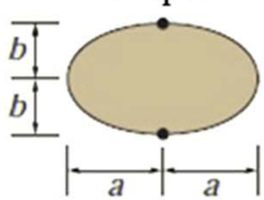


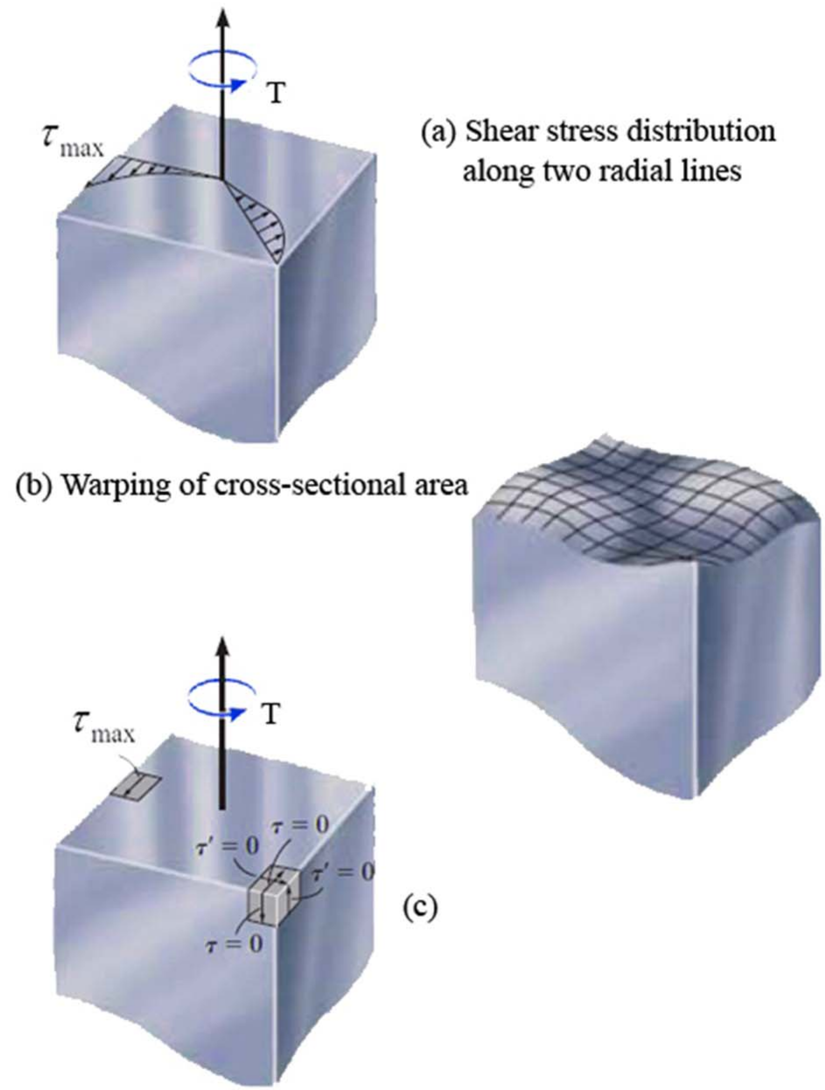
Solid Non-Circular Shafts





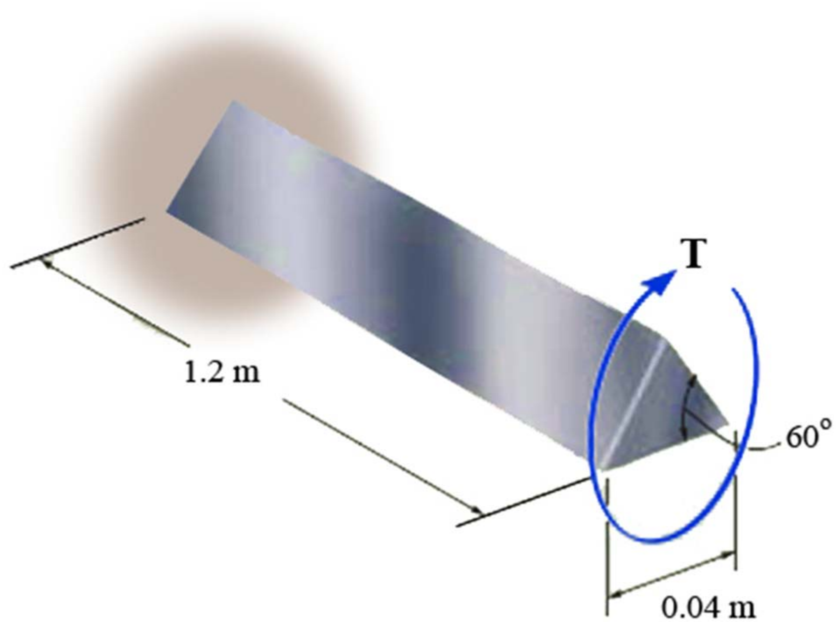
Solid Non-Circular Shafts

Shape of cross section	τ_{max}	ϕ
Square 	$\frac{4.81\,T}{a^3}$	$\frac{7.10\,TL}{a^4 G}$
Equilateral triangle 	$\frac{20\,T}{a^3}$	$\frac{46\,TL}{a^4 G}$
Ellipse 	$\frac{2\,T}{\pi a b^2}$	$\frac{(a^2 + b^2)\,TL}{\pi a^3 b^3 G}$



Example 4

The 6061-T6 aluminum shaft shown below has a cross-sectional area in the shape of an equilateral triangle. Determine the largest torque **T** that can be applied to the end of the shaft if the allowable shear stress is t_{allow} = 56 MPa. and the angle of twist at its end is restricted to Φ_{allow} = 0.02 rad. How much torque can be applied to a shaft of circular cross section made from the same amount of material?

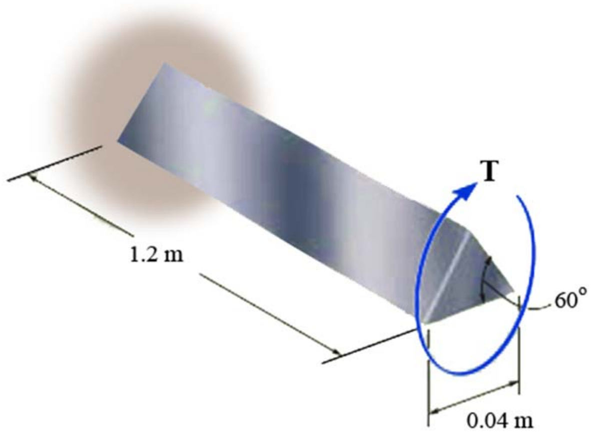


Example 4 (cont.)

Solutions

$$\tau_{allow} = \frac{20T}{a^3}; \qquad 56 = \frac{20T}{40^3} \Rightarrow T = 1779.2 \text{ Nm}$$

$$\sigma_{allow} = \frac{46T}{a^4 G_{al}}; \qquad 0.02 = \frac{46T(1.2)(10^3)}{40^4\left[26(10^3)\right]} \Rightarrow T = 24.12 \text{ Nm}$$



Example 4 (cont.)

Solutions

$$A_{circle} = A_{triangle}; \qquad \pi c^2 = \frac{1}{2}(40)(40\sin 60^\circ) \Rightarrow c = 14.85 \text{ mm}$$

$$\tau_{allow} = \frac{Tc}{J}; \qquad 56 = \frac{T(14.85)}{(\pi/2)(14.85)^4} \Rightarrow T = 288.06 \text{ Nm}$$

$$\varphi_{allow} = \frac{TL}{JG_{al}}; \qquad 0.02 = \frac{T(1.2)(10^3)}{(\pi/2)(14.85)^4\left[26(10^3)\right]} \Rightarrow T = 33.10 \text{ Nm}$$

Thin Wall Tubes Having Closed Section

- Average shear stress

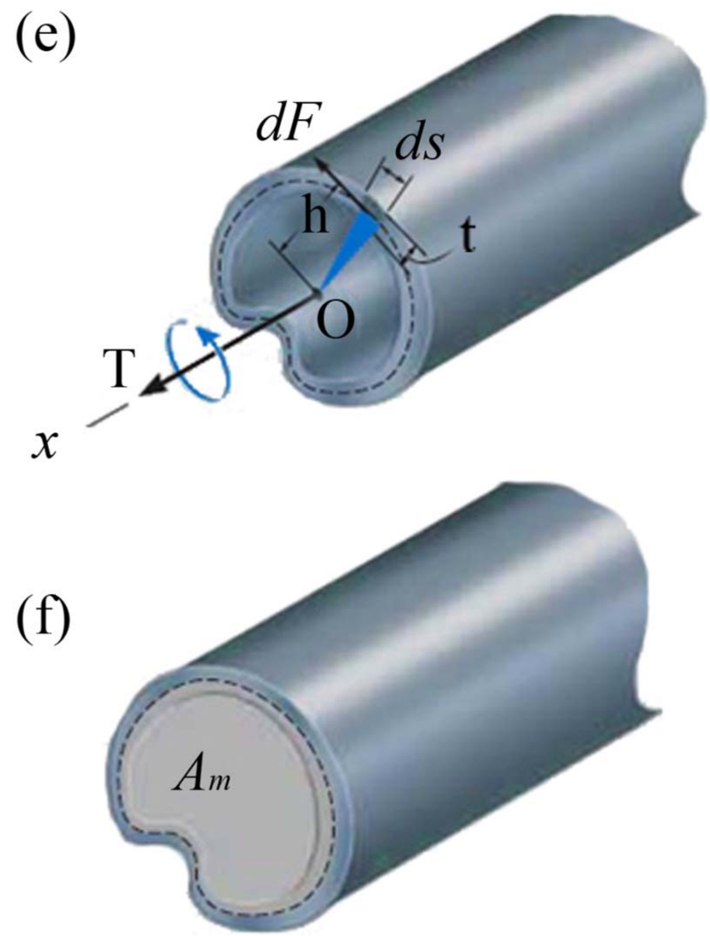
$$\tau_{avg} = \frac{T}{2tA_m}$$

- Shear flow

$$q = \frac{T}{2A_m}$$

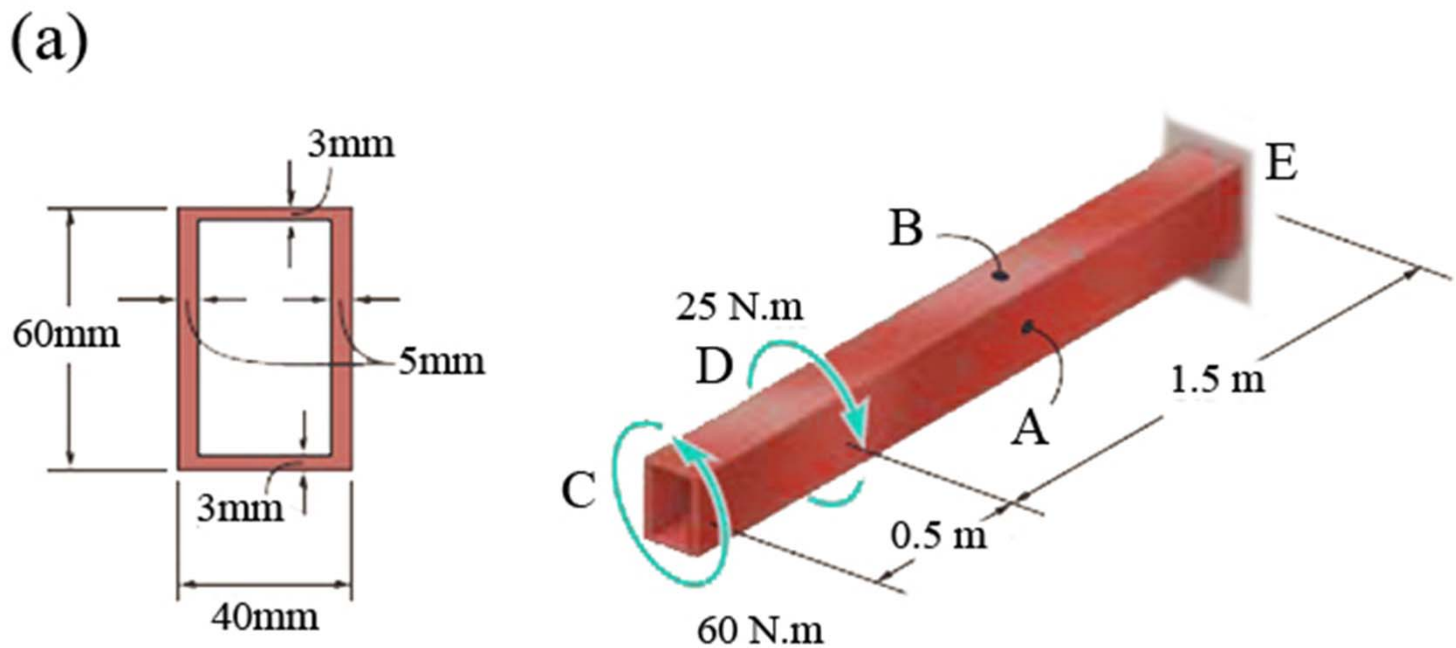
- Angle of twist

$$\varphi = \frac{TL}{4A_m^2 G}\oint \frac{ds}{t}$$



Example 5

The tube is made of C86100 bronze and has a rectangular cross section as shown below. If it is subjected to the two torques, determine the average shear stress in the tube at points *A* and *B*. Also, what is the angle of twist of end *C*? The tube is fixed at *E*.



Example 5 (cont.)

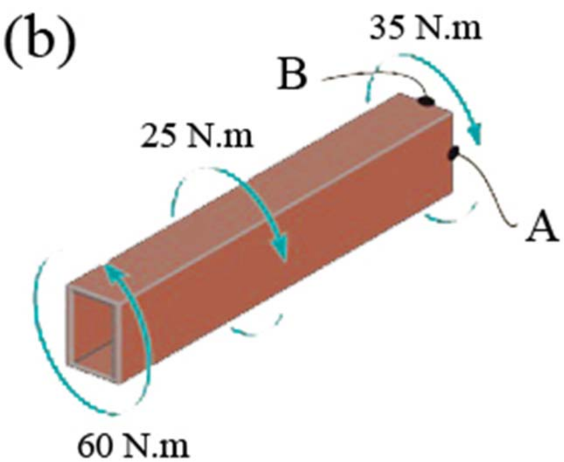
Solutions

$$A_m = (0.035)(0.057) = 0.002\ \text{m}^2$$

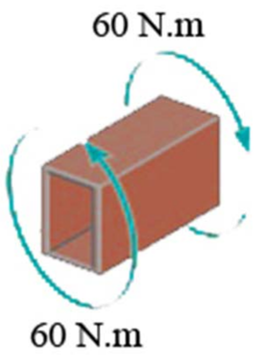
$$\tau_A = \frac{T}{2tA_m} = \frac{35}{2(0.005)(0.002)} = 1.75\ \text{MPa}$$

$$\tau_B = \frac{T}{2tA_m} = \frac{35}{2(0.003)(0.002)} = 2.92\ \text{MPa}$$

(b)



(c)



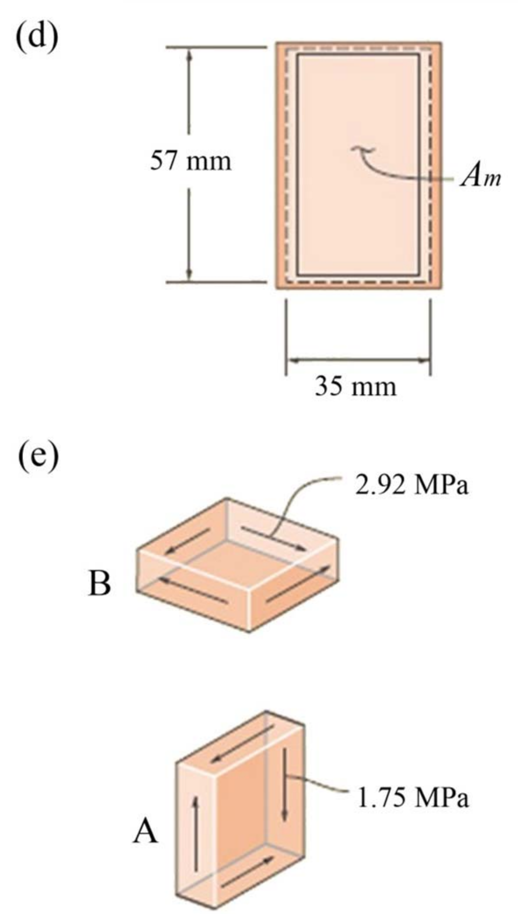
Example 5 (cont.)

Solutions

$$\varphi = \frac{TL}{4A_m^2 G}\oint \frac{ds}{t} = \frac{60(0.5)}{4(0.002)^2\left[38(10^9)\right]}\left[2\left(\frac{57}{5}\right)+2\left(\frac{35}{3}\right)\right]$$

$$+\frac{35(1.5)}{4(0.002)^2\left[38(10^9)\right]}\left[2\left(\frac{57}{5}\right)+2\left(\frac{35}{3}\right)\right]$$

$$= 6.29\left(10^{-3}\right)\ \text{rad}$$



References

1. Hibbeler, R.C., Mechanics Of Materials, 8th Edition in SI units, Prentice Hall, 2011.
2. Gere dan Timoshenko, Mechanics of Materials, 3rd Edition, Chapman & Hall.
3. Yusof Ahmad, 'Mekanik Bahan dan Struktur' Penerbit UTM 2001