SAB2223 Mechanics of Materials and Structures

TOPIC 8
COLUMN

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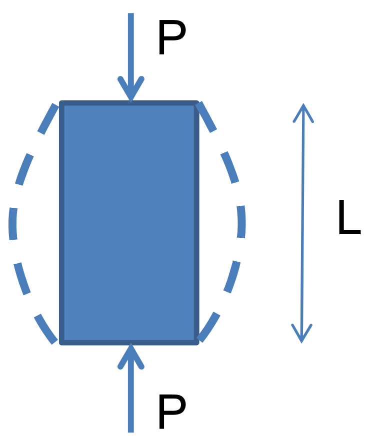
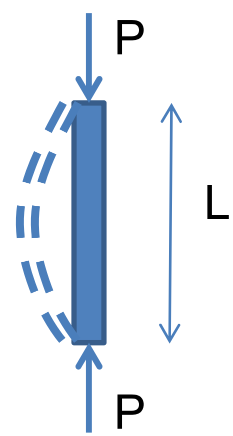
What is Column?

- Normally taking **compression** force
- In **vertical** direction ; if not, is called **strut**
- Divided into 2 categories:

a) Short column (fail in crushing)

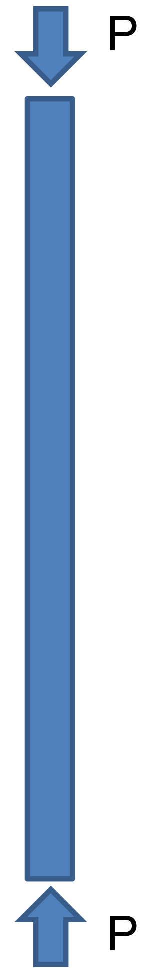


b) Slender column (fail in buckling)



Critical Load

- A slender column pinned at both end subject to load P.
- The maximum axial load that a column can support when it is on the *verge* of buckling is called the critical load, P_{cr}.



Critical Load

- Any additional loading will cause the column to buckle and deform laterally.

Euler Theory (Ideal Column)

- The column is perfectly straight before loading
- The column is made of homogeneous material
- The load is applied through the centroid of the cross section
- The material behaves in a linear-elastic manner
- The column buckles and bends in a single plane

Buckling of Column

P_{cr} = maximum axial load (kN)

σ_{cr} = critical stress (N/mm^2)

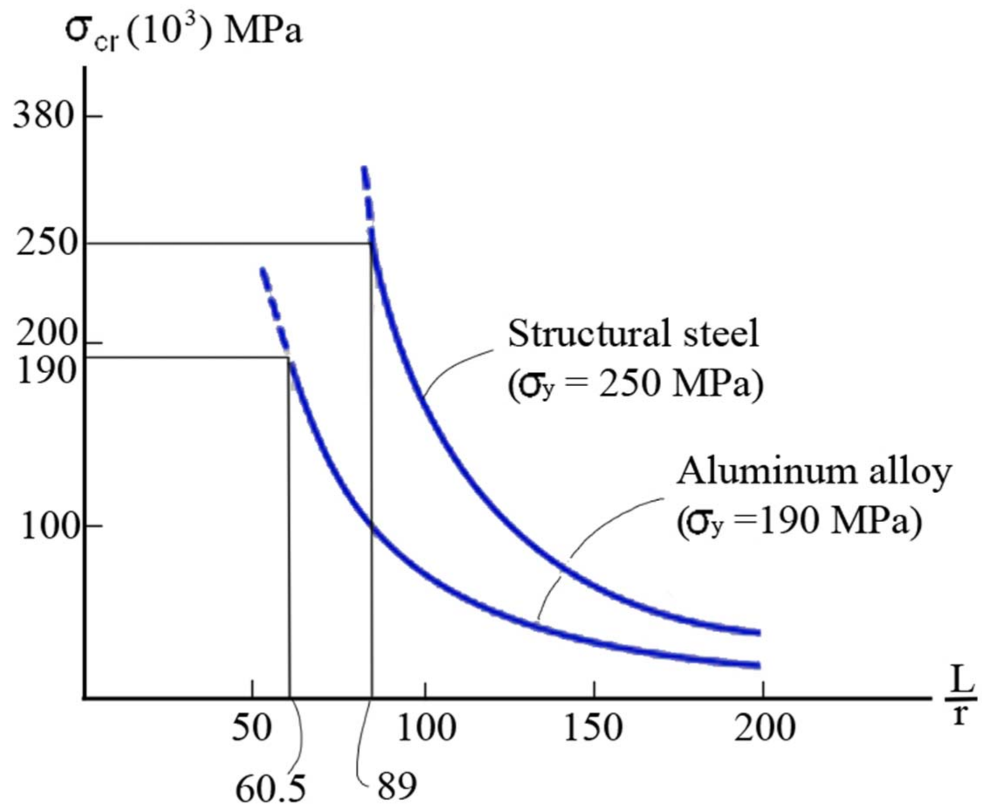
E = modulus of elasticity for the material

I = least moment of inertia for the column's cross-sectional area

L = unsupported length of the column

r = smallest radius of gyration of the column

L/r = slenderness ratio



Important Points

- Columns are **long slender** members that are subjected to **axial loads**
- The **critical load** is the maximum axial load that a column can support when it is on the **verge of buckling**. This loading represents a case of neutral equilibrium
- An **ideal column** is initially **perfectly straight**, made of **homogenous** materials, and the load is applied through the **centroid** of the cross section
- A **pin-connected column** will buckle about the principal axis of the cross section having the **least moment of inertia**
- The **slenderness ratio** is L/r, where r is the smallest radius of gyration of the cross section. Buckling will occur about the axis where this ratio gives the greatest value

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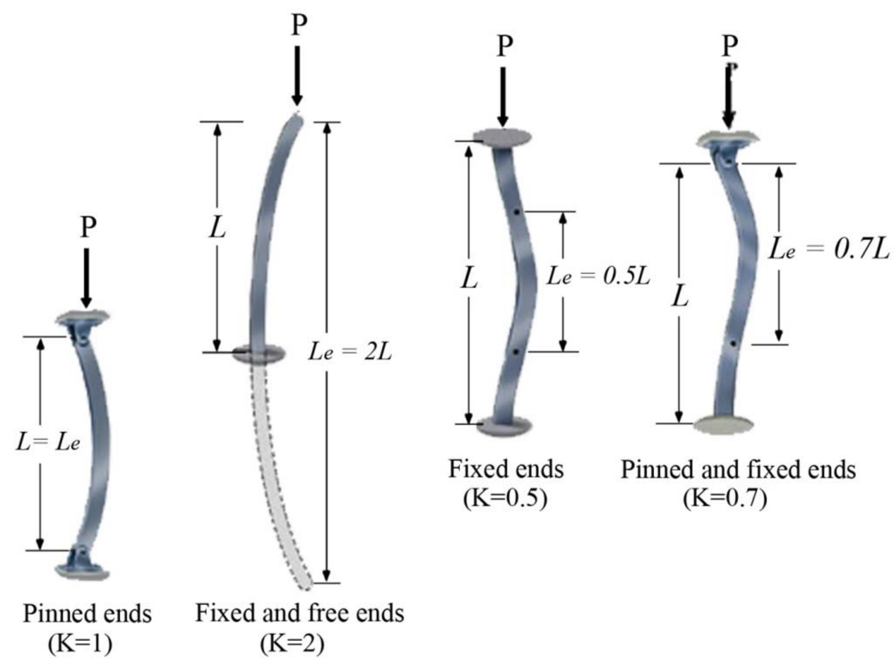
Columns Having Various Types of Supports

- ***Effective length***, L_e – distance between the zero-moment points
- ***Effective-length factor***, K - used to calculate L_e.

$$L_e = KL$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \qquad \sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

KL/r = effective-slenderness ratio



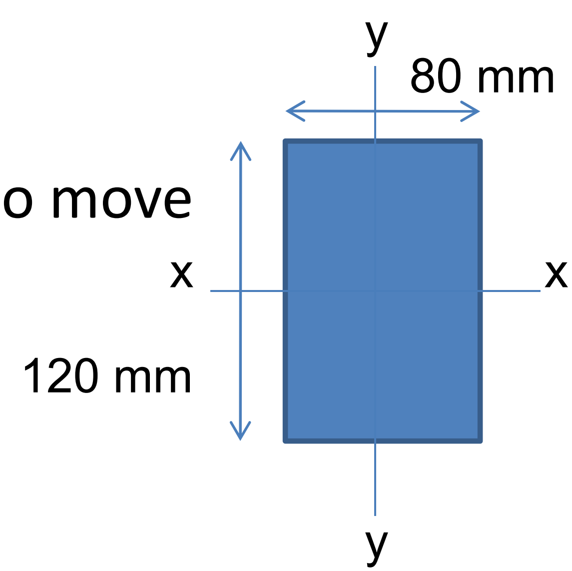
Example 1

A 6-m-long concrete column having the elastic modulus of 200 kN/mm^2 and the cross section shown in the figure is to be used in a building. Determine the maximum allowable axial load (P_{allow}) the column can support so that it does not buckle. The safety factor is taken as 2. Given that

a) Both end of the column are pinned ends

b) Both end of the column are fixed ends

c) One end is fixed and the other end is free to move



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Example 1 (cont.)

Solution

$$I_x = \frac{bh^3}{12} = \frac{80 \times 120^3}{12} = 11.52 \times 10^6 mm^4$$

$$I_y = \frac{hb^3}{12} = \frac{120 \times 80^3}{12} = 5.12 \times 10^6 mm^4$$

$\therefore$ The column will buckle in y-y axis (smallest I)

Example 1 (cont.)

a) Both end of the column are pinned ends

$$L_e = L$$

$$P_{cr} = \frac{\pi^2 EI}{(L)^2} = \frac{\pi^2 \times 200 \times 5.12 \times 10^6}{6000^2} = 280.7kN$$

Maximum allowable axial load (P_{allow}):

$$P_{allow} = \frac{P_{cr}}{Safety \quad factor} = \frac{280.7}{2} = 140.35kN$$

Example 1 (cont.)

b) Both end of the column are fixed ends

$$L_e = 0.5L$$

$$P_{cr} = \frac{\pi^2 EI}{(L)^2} = \frac{\pi^2 \times 200 \times 5.12 \times 10^6}{(0.5 \times 6000)^2} = 1122.94kN$$

Maximum allowable axial load (P_{allow}):

$$P_{allow} = \frac{P_{cr}}{Safety \quad factor} = \frac{1122.94}{2} = 561.47kN$$

Example 1 (cont.)

c) One end is fixed and the other end is free to move

$$L_e = 2L$$

$$P_{cr} = \frac{\pi^2 EI}{(L)^2} = \frac{\pi^2 \times 200 \times 5.12 \times 10^6}{(2 \times 6000)^2} = 70.20kN$$

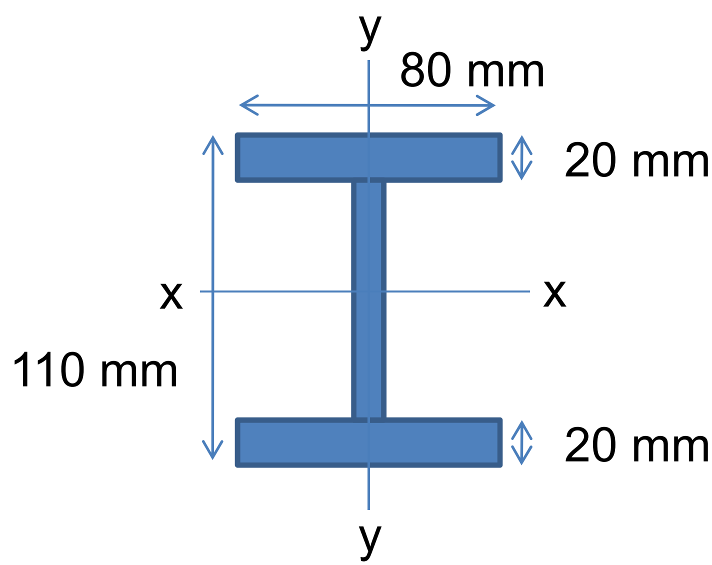
Maximum allowable axial load (P_{allow}):

$$P_{allow} = \frac{P_{cr}}{Safety \quad factor} = \frac{70.20}{2} = 35.10kN$$

Example 2

A 6-m-long steel column having the elastic modulus of 200 kN/mm^2 and the cross section shown in the figure is to be used in a building. The safety factor is taken as 2.

a) Determine the slenderness ratio of the column
b) Determine the critical load (P_{cr})
c) Determine the maximum allowable axial load (P_{allow})



Example 2 (cont.)

Solution

$$I = \frac{bh^3}{12} + A\bar{y}$$

$$I_x = [\frac{80 \times 20^3}{12} + (80 \times 20 \times 55^2)] \times 2 + \frac{10 \times 90^3}{12} = 10.394 \times 10^6 mm^4$$

$$I_y = (\frac{20 \times 80^3}{12}) \times 2 + \frac{90 \times 10^3}{12} = 1.714 \times 10^6 mm^4$$

$\therefore$ The column will buckle in y-y axis (smallest I)

Example 2 (cont.)

a) Determine the slenderness ratio

$$A = (80\times 20)\times 2 + (90\times 10) = 4100mm^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.714\times 10^6}{4100}} = 20.45mm$$

$$Slenderness \quad ratio = \frac{L}{r} = \frac{6000}{20.45} = 293.45$$

Example 2 (cont.)

b) Determine the critical load (P_{cr})

$$P_{cr} = \frac{\pi^2 EI}{(L)^2} = \frac{\pi^2 \times 200 \times 1.714 \times 10^6}{6000^2} = 94kN$$

c) Determine the maximum allowable axial load (P_{allow})

$$P_{allow} = \frac{P_{cr}}{Safety \quad factor} = \frac{94}{2} = 47kN$$

References

1. Hibbeler, R.C., Mechanics Of Materials, 8th Edition in SI units, Prentice Hall, 2011.
2. Gere dan Timoshenko, Mechanics of Materials, 3rd Edition, Chapman & Hall.
3. Yusof Ahmad, ‘Mekanik Bahan dan Struktur’ Penerbit UTM 2001