SAB2223 Mechanics of Materials and Structures

TOPIC 6
STATICALLY DETERMINATE SPACE TRUSSES

Lecturer:
Dr. Shek Poi Ngian

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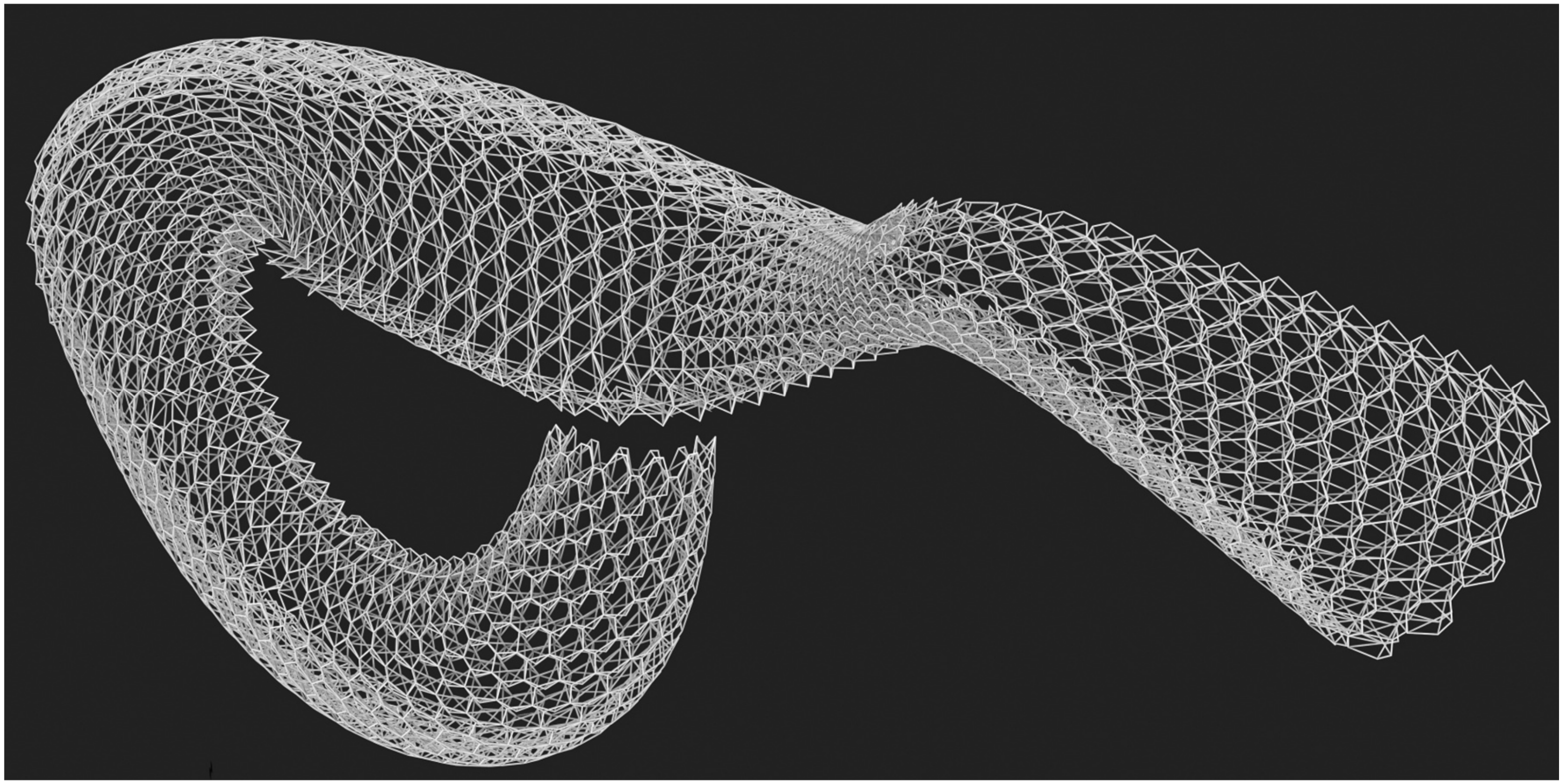
STATICALLY DETERMINATE SPACE TRUSSES

Introduction

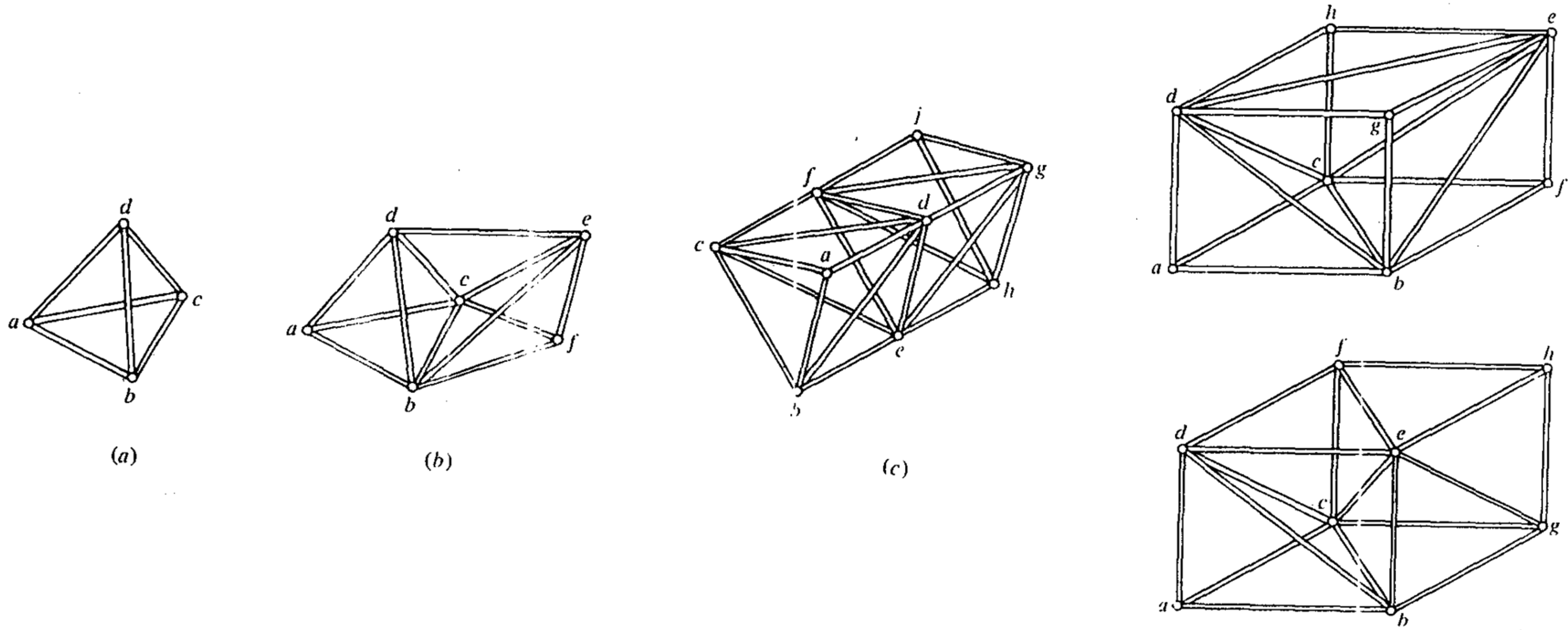
A space truss is a truss that cannot be represented as a planar truss.



Types of Space Truss

1. Simple Space Truss

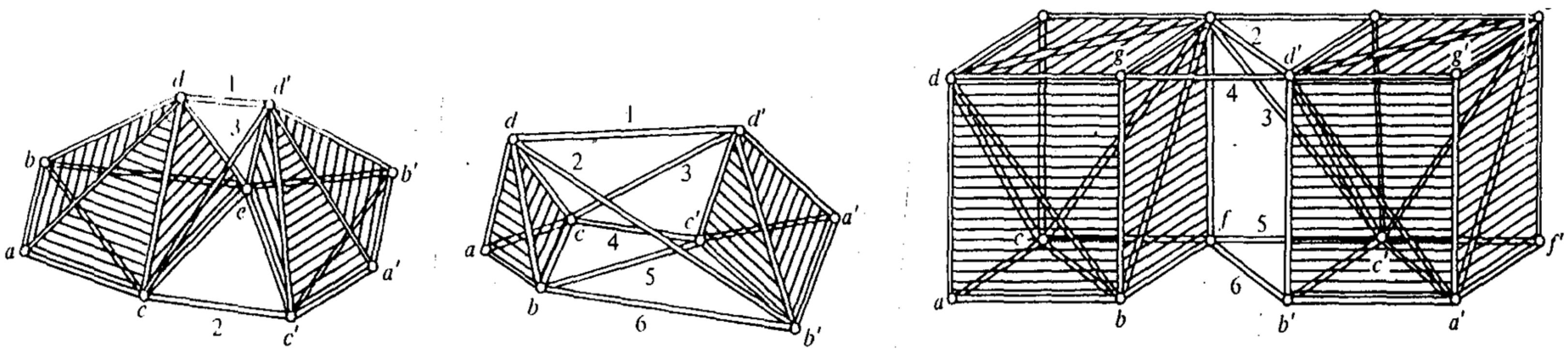
This truss is constructed from a tetrahedron. The truss can be enlarged by adding three members.



Types of Space Truss

2. Compound Space Truss

This truss is constructed by combining two or more simple truss.

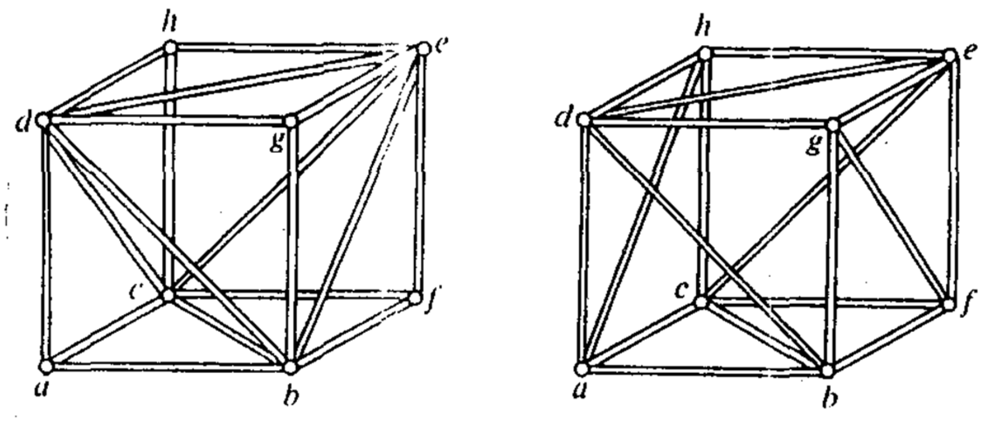


(a) (b) (c)

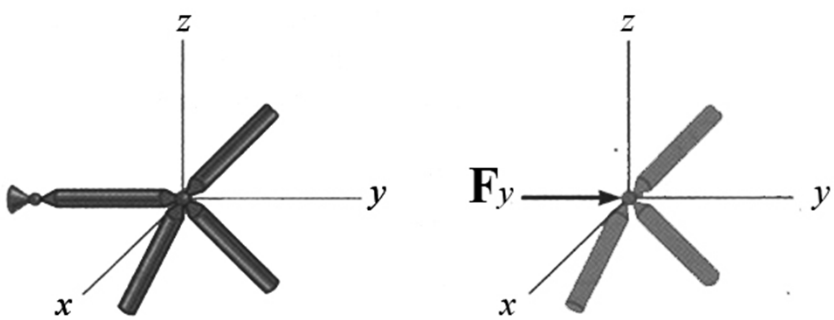
Types of Space Truss

3. Complex Space Truss

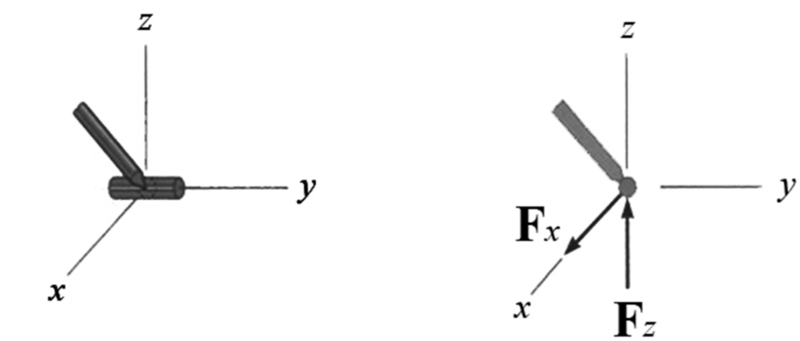
Complex truss is a truss that cannot be classified as simple truss or compound truss.



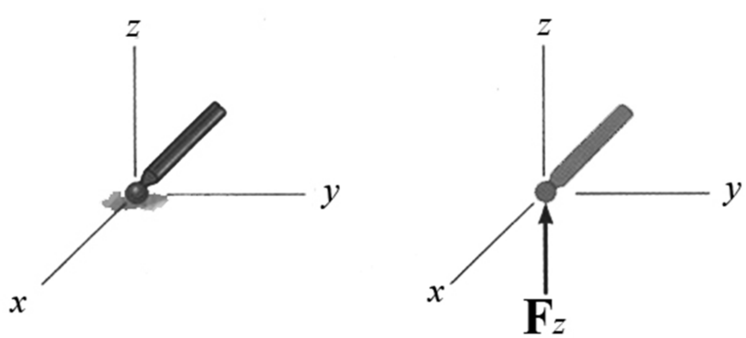
Types of Support



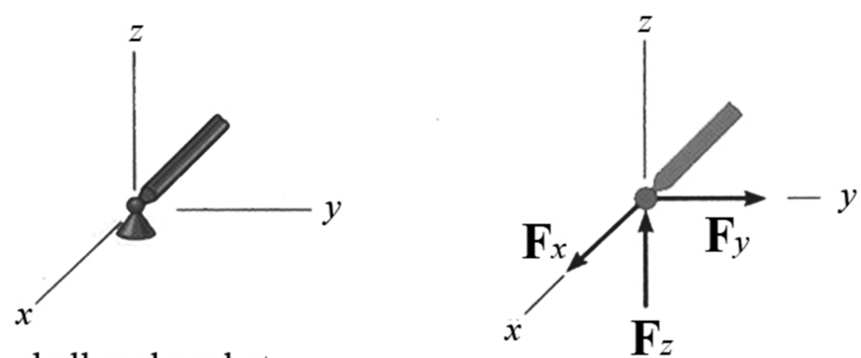
short link



slotted roller constrained in a cylinder



roller



ball-and-socket

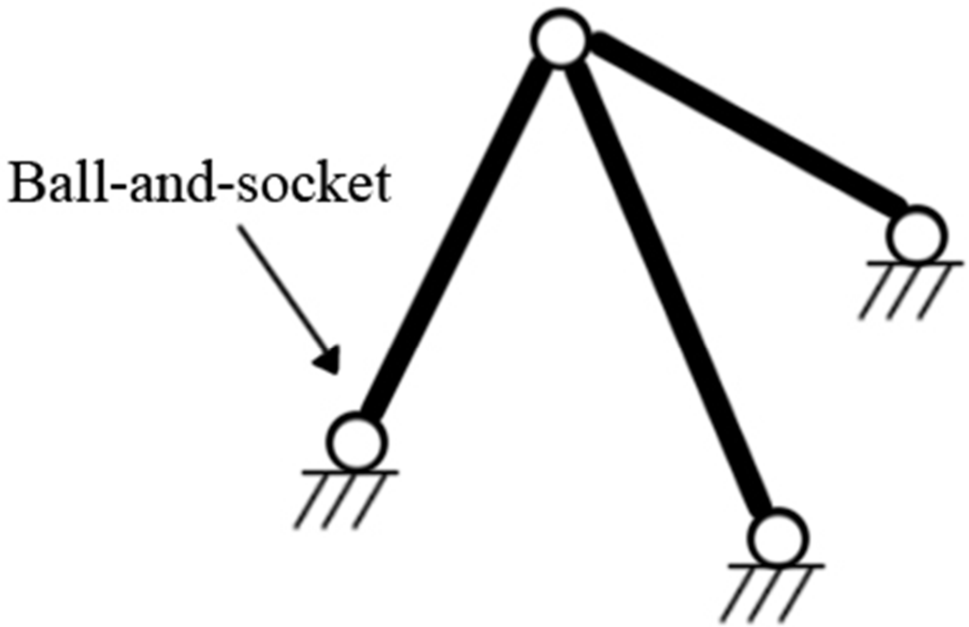
Determinancy and Stability

$b + r = 3j$	determinate truss
$b + r > 3j$	indeterminate truss
$b + r < 3j$	unstable truss

Externally, if

$r < 6$	unstable truss
$r = 6$	determinate if truss is stable
$r > 6$	indeterminate truss

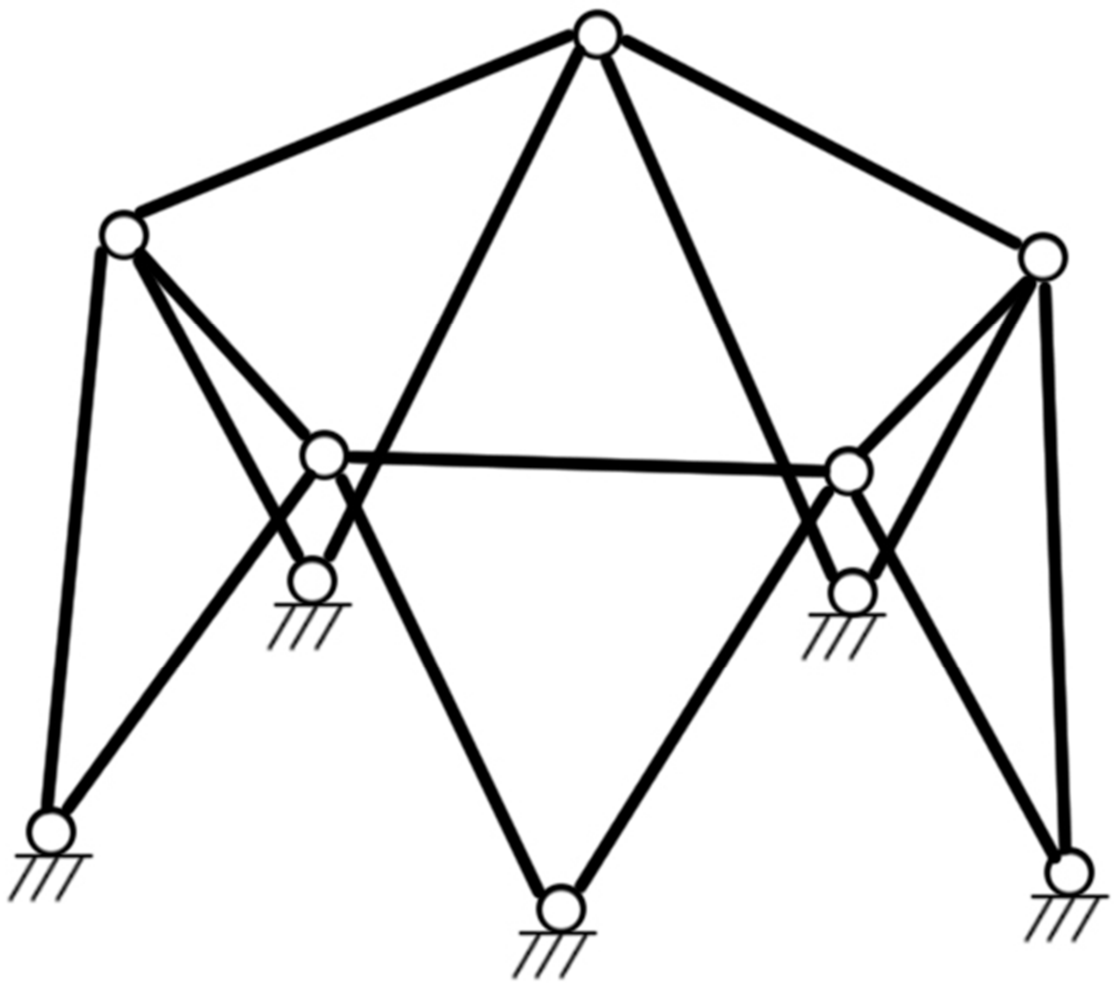
Example 1



$$b = 3$$
$$j = 4$$
$$r = 9$$
$$b + r = 12\ ,\ 3j = 12$$
$$\therefore b + r = 3j$$

Determinate Truss

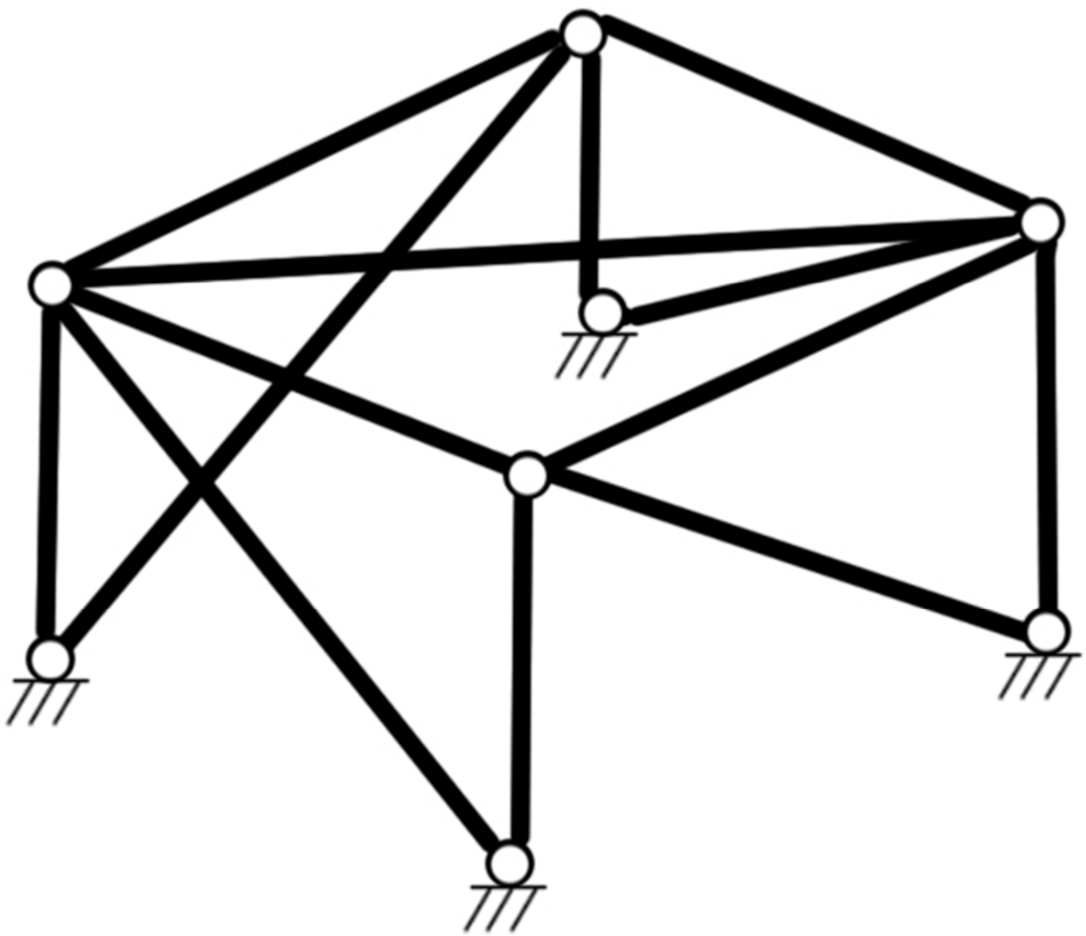
Example 2



$b = 15$
$j = 10$
$r = 15$

$b + r = 30$
$3j = 30$

Example 3



$b = 13$
$j = 8$
$r = 12$

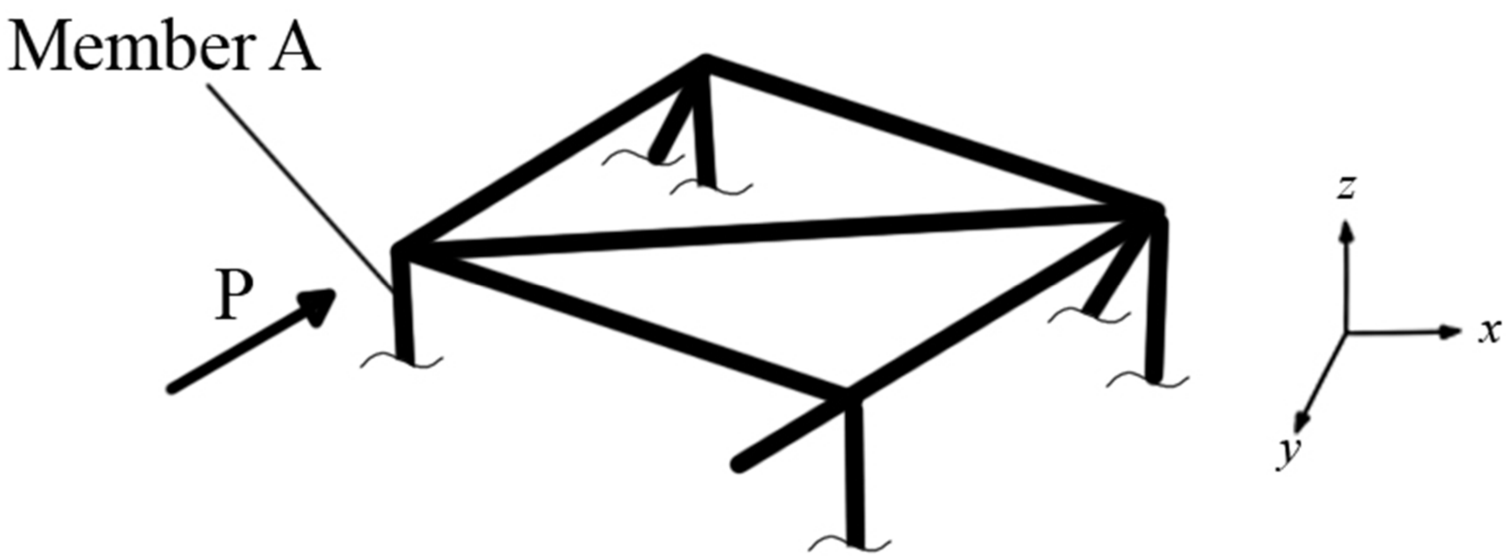
$b + r = 25,\ 3j = 24$
$\therefore\ b + r > 3j$

Indeterminate Truss in the first degree

Zero Force Members

THEOREM 1:

If all members and external force except one member at a joint, (say, member *A*) lie in the same plane, then, the force in member *A* is zero.

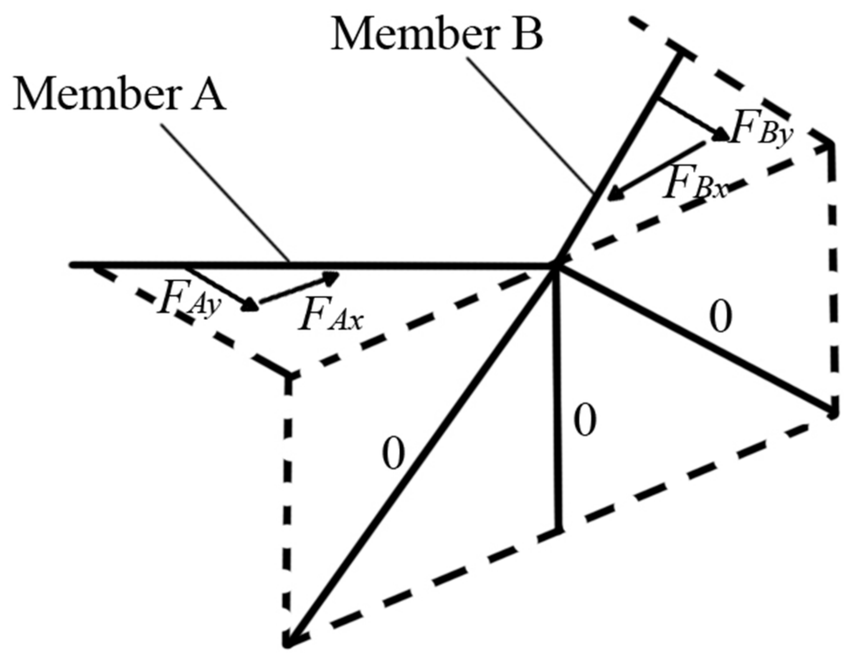


The force in member *A* is zero.

Zero Force Members

THEOREM 2:

If all members at a joint has zero force except for two members, (say member *A* and *B*), and both members (*A* and *B*) do not lie in a straight line, then the force in member *A* and *B* are zero.



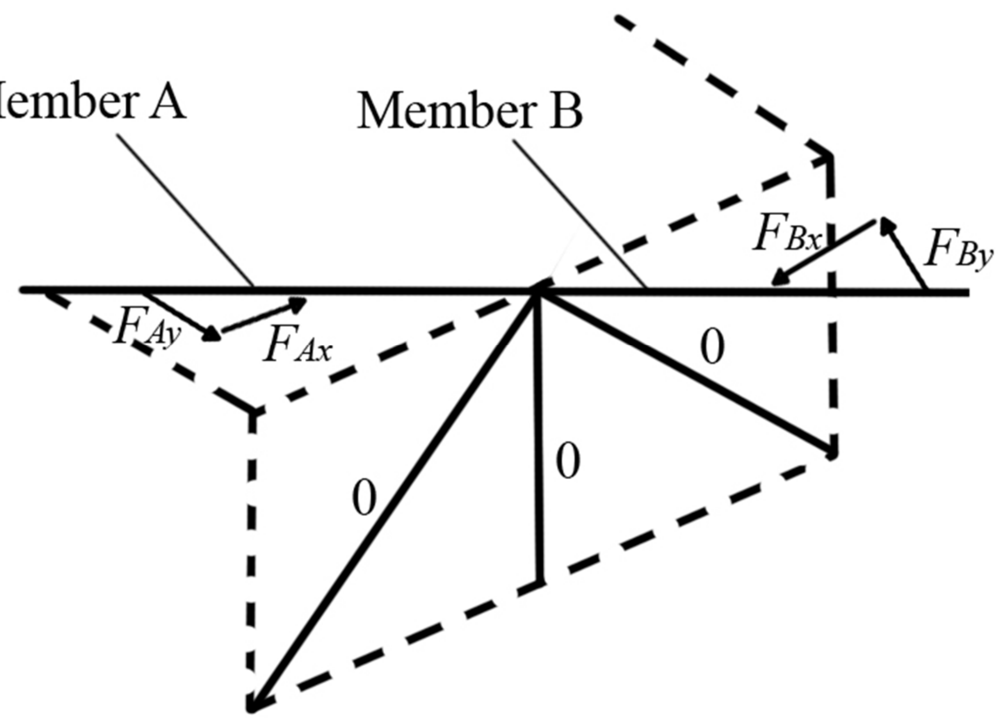
Both members *A* and *B* has zero force because both members do not lie in a straight line.

Zero Force Members

If members *A* and *B* lie in a straight line, then, the forces in these members might not be zero. In fact, referring to the example below,

$F_{Ax} = -F_{Bx}$

$F_{Ay} = -F_{By}$

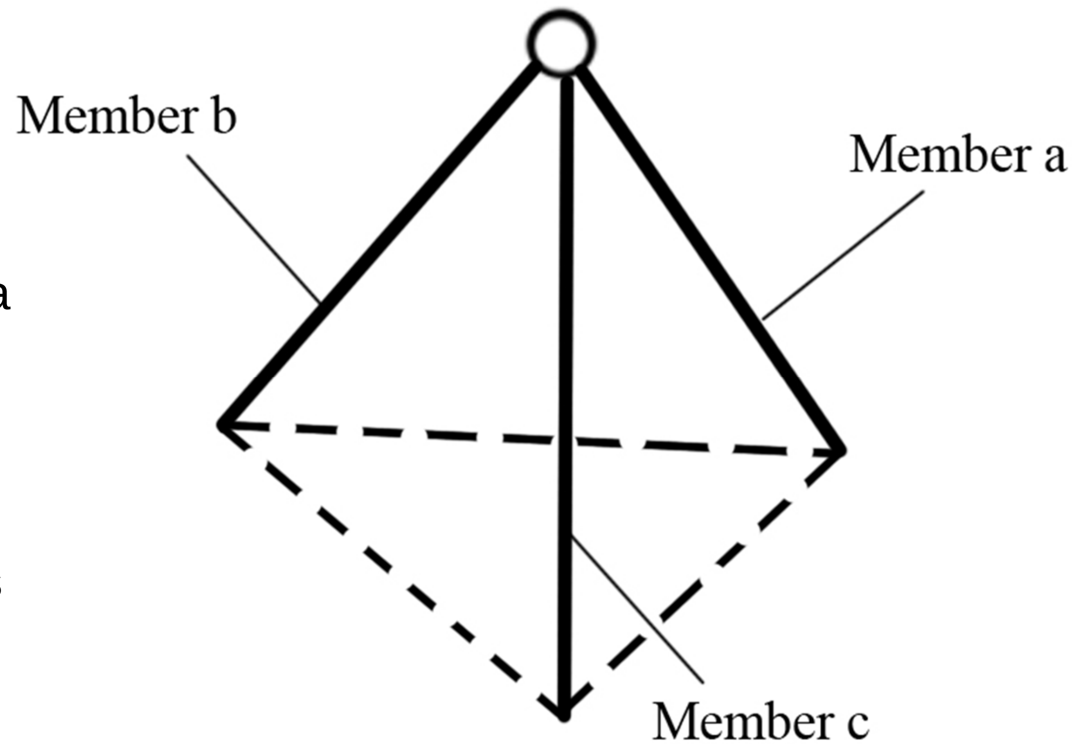


Zero Force Members

THEOREM 3:

If three members at a joint do not lie in the same plane and there is no external force at that joint, then the force in the three members is zero.

Three members connected at a joint has zero force. A plane can consists of two members, say member *a* and *b.* Thus, no force can balance the component of member *c* that is normal to the plane.



Example 4

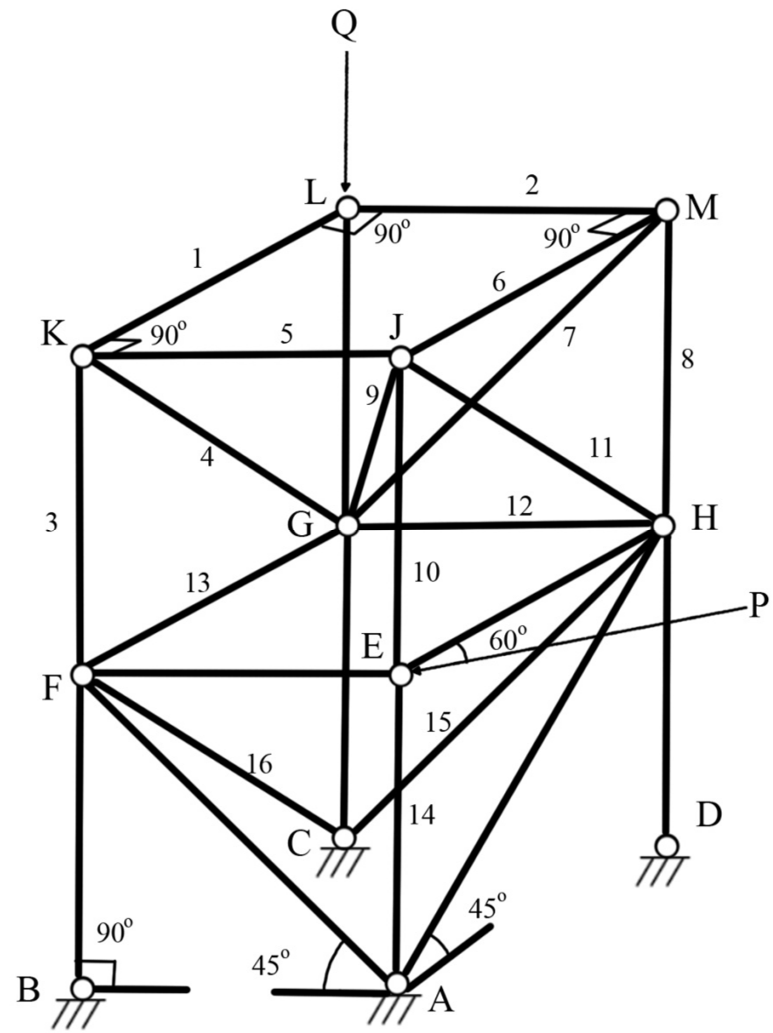
Identify the members of the space truss that has zero force.

Theorem 1: Joint *L*,
F_1=0 $F_2 = 0$

Theorem 3: Joint *K*,
F_3=F_4=F_5=0, and F_1=0

Theorem 3: Joint *M*,
F_6=F_7=F_8=0, and F_2=0

Theorem 3: Joint *J*,
F_9=F_{10}=F_{11}=0, and F_5= F_6=0

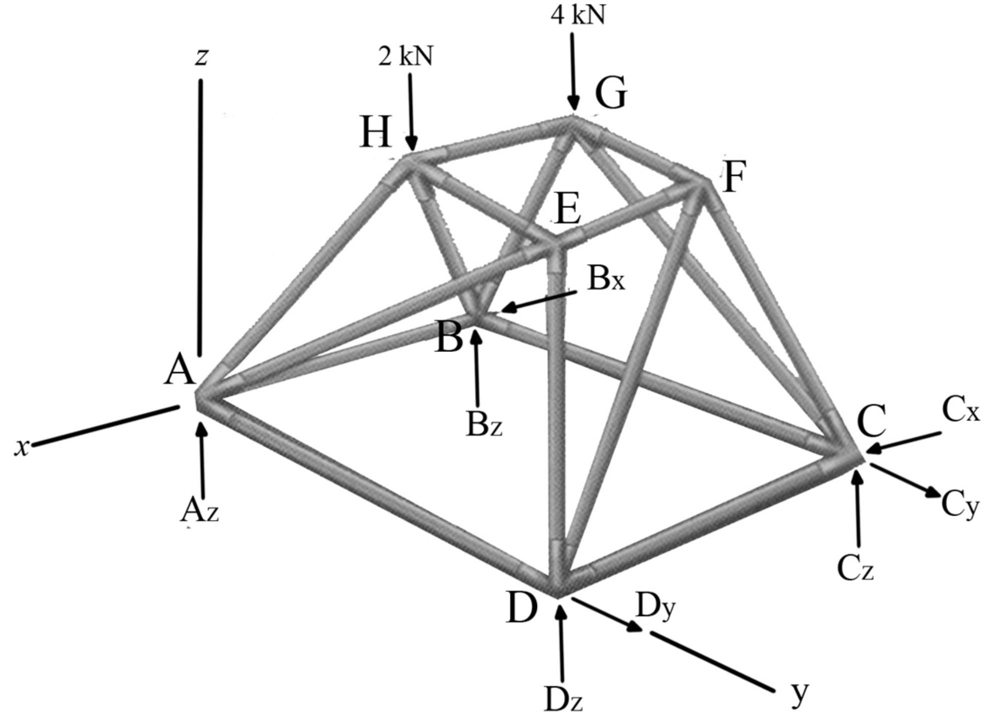


Example 5

Joint *F*:

Members *FE, FC, FD* lie in a plane, except member *FG.* Thus, member *FG* has zero force. (Theorem 1)

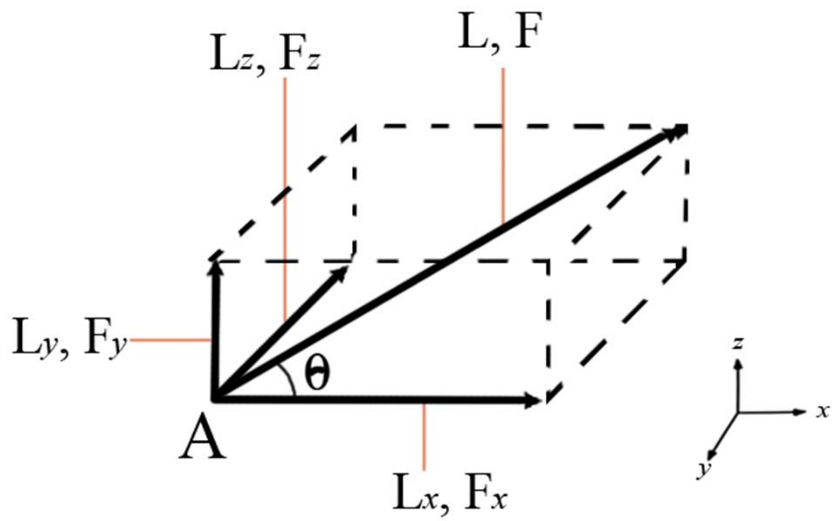
Members *FE, FC, FD* have zero force. (Theorem 3)



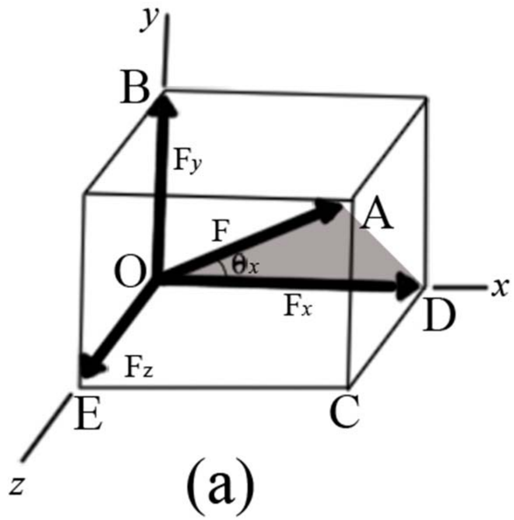
Joint *E*:

Members *ED, EA, EH* have zero force. (Theorem 3)

Tension Coefficient Method



The component of length, *L,* and force, *F* in the *x, y, z* direction.



$$F_x = F\text{kos}\,\theta_x$$

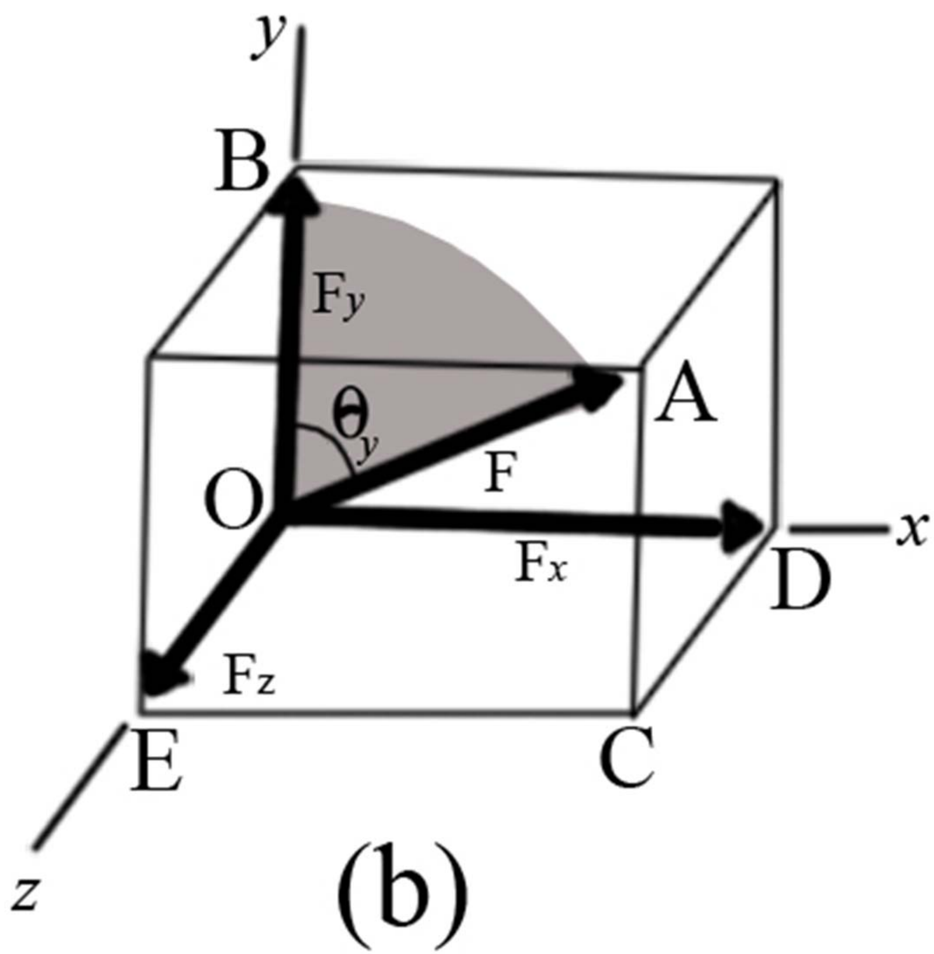
$$= F\,(L_x/L) = (F/L)L_x$$

$$F_x = tL_x$$

t = F/L

t = tension coefficient

Tension Coefficient Method

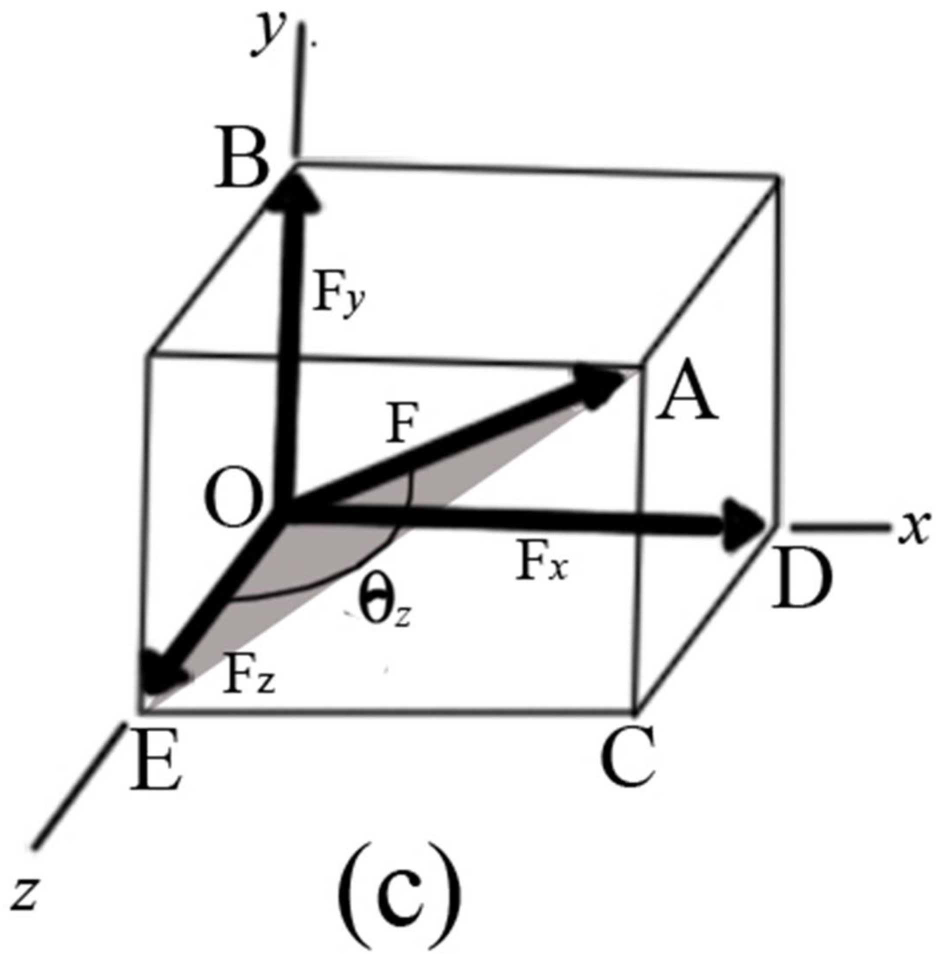


$$F_y = F\text{kos}\,\theta_y$$

$$= F\,(L_y/L) = (F/L)L_y$$

$$F_y = tL_y$$

Tension Coefficient Method



$$F_z = F\cos\theta_z$$

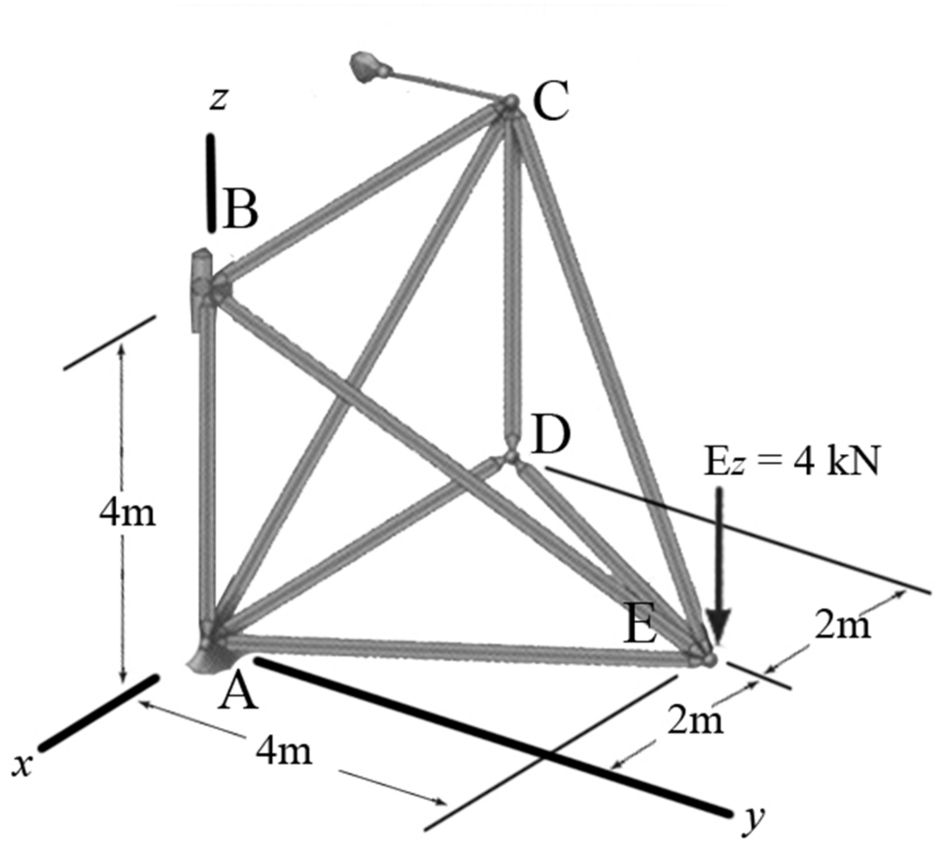
$$= F(L_z/L) = (F/L)L_z$$

$$F_z = tL_z$$

Example 6

Determine the force in each member of the space truss shown.

1. Start with joints where there are only 3 members.



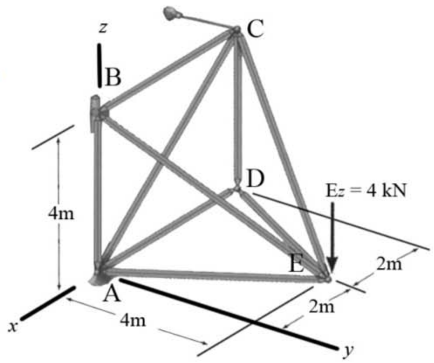
Joint *D*

Theorem 3: Three members at a joint and no external force. Thus , all members have zero forces. $t_{DC} = t_{DA} = t_{DE} = 0$

Example 6 (cont.)

Substitute $t_{DE} = 0$

$t_{DE} = 0$



Joint E

Members	L_x (m)	L_y (m)	L_z (m)	L (m)	t kN/m	F kN
EC	-2	-4	4	4.47		
EB	2	-4	4	6		
ED	-2	-4	0	4.47	0	
EA	2	-4	0	4.47		
Force (kN)	0	0	-4			

$\Sigma F_z = 0 \Rightarrow 4t_{EB} + 4t_{EC} - 4 = 0$ Eq. (1)

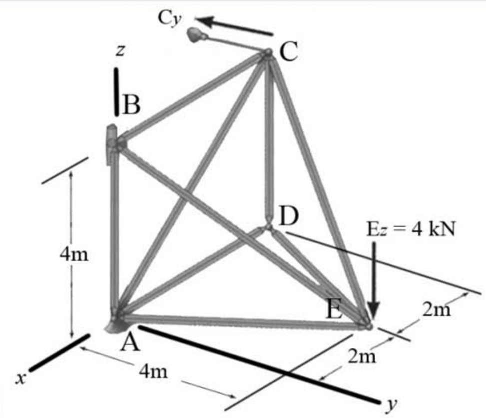
$\Sigma F_x = 0 \Rightarrow -2t_{EC} + 2t_{EB} - 2t_{ED} + 2t_{EA} = 0$ Eq. (2)

$\Sigma F_y = 0 \Rightarrow -4t_{EC} - 4t_{EB} - 4t_{ED} - 4t_{EA} = 0$ Eq. (3)

Solve eq.(1), (2), (3): $t_{EC} = 0$; $t_{EA} = -1$ kN/m, $t_{EB} = 1$kN/m

$F = t \times L$; $F_{EC} = 0\ kN$; $F_{EA} = -4.47\ kN$

Joint C has 3 unknowns, as $t_{CE}, = t_{CD} = 0$ Thus, by Theorem 3, t_{CB}, t_{CA} and C_y will be zero.



Joint C

Members	L_x (m)	L_y (m)	L_z (m)	L (m)	t kN/m	F kN
CE	2	4	-4	6	0	
CA	4	0	-4	5.66		
CD	0	0	-4	4	0	
CB	4	0	0	4		
Force (kN)	0	$-C_y$	0			

Can be omitted as explained above

$\Sigma F_z = 0 \Rightarrow 4t_{CA} = 0$; $t_{CA} = 0$

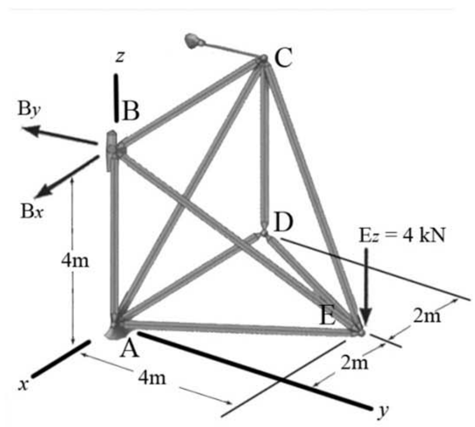
$\Sigma F_x = 0 \Rightarrow 4t_{CA} + 4t_{CB} = 0$; $t_{CB} = 0$

$\Sigma F_y = 0 \Rightarrow -C_y = 0$; $C_y = 0$

$F = t \times L$; $F_{CA} = 0\ kN$; $F_{CB} = 0\ kN$

Example 6 (cont.)

Joint B has 3 unknowns



Joint B

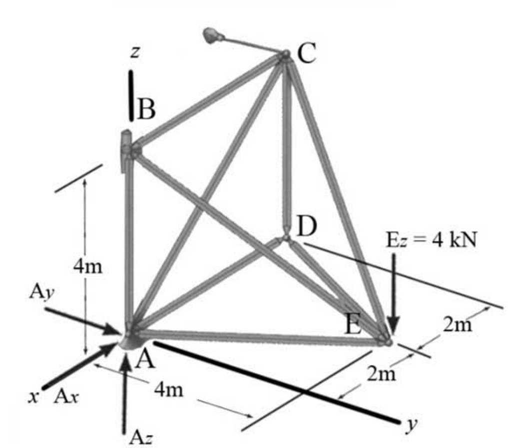
Members	L_x (m)	L_y (m)	L_z (m)	L (m)	t kN/m	F kN
BC	-4	0	0	4	0	
BA	0	0	-4	4		
BE	-2	-4	-4	6	1	
Force (kN)	B_x	$-B_y$	0			

$\Sigma F_x = 0 \Rightarrow -4t_{BC} - 2t_{BE} + B_x = 0\ ;\ B_x = 2\text{kN}$

$\Sigma F_y = 0 \Rightarrow -4t_{BE} - B_y = 0\ ;\ B_y = -4\text{kN}$

$\Sigma F_z = 0 \Rightarrow -4t_{BA} - 4t_{BE} = 0\ ;\ t_{BA} = -1\ \text{kN/m}$

$F = t \times L\ ;\ F_{BA} = -4\ kN$



Joint A

Members	L_x (m)	L_y (m)	L_z (m)	L (m)	t kN/m	F kN
AB	0	0	4	4	-1	
AC	4	0	4	5.66	0	
AD	-4	0	0	4	0	
AE	-2	4	0	4.47	-1	
Force (kN)	$-A_x$	A_y	A_z			

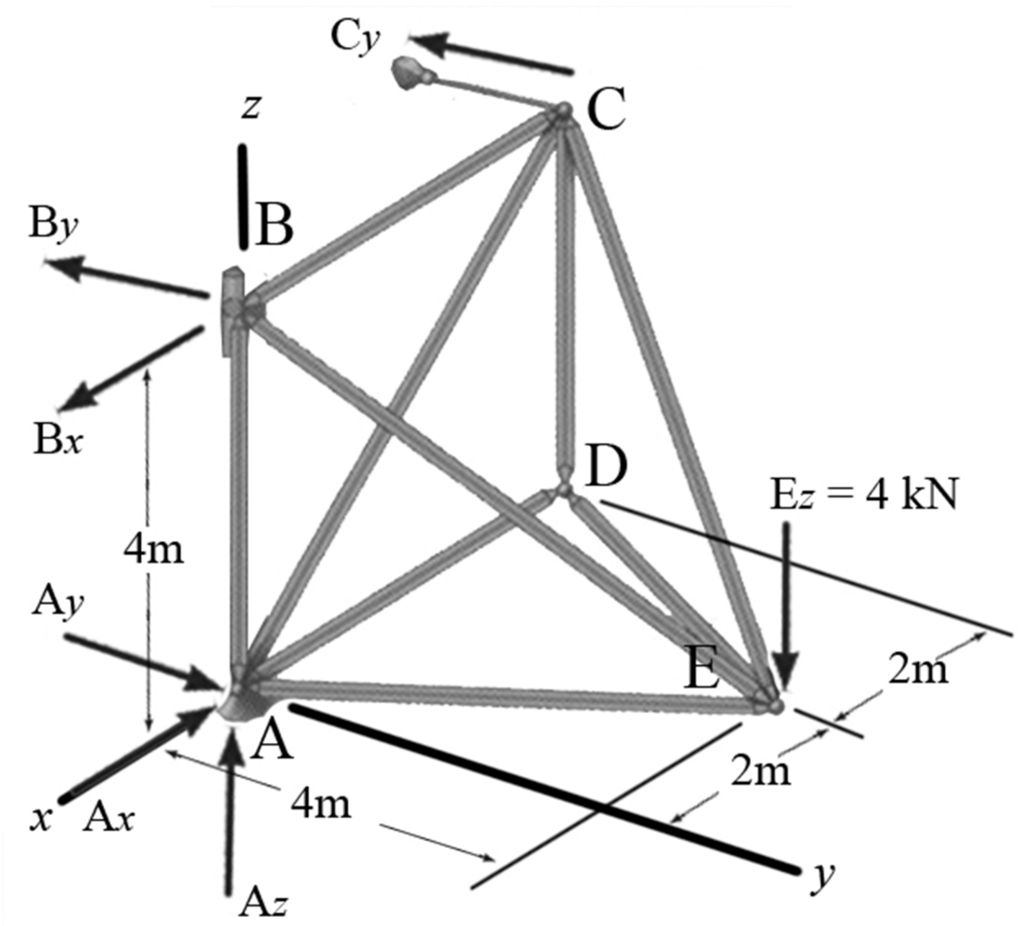
$\Sigma F_x = 0 \Rightarrow -4t_{AC} - 4t_{AD} - 2t_{AE} - A_x = 0\ ;\ A_x = -2\text{kN}$

$\Sigma F_y = 0 \Rightarrow 4t_{AE} + A_y = 0\ ;\ A_y = -4\text{kN}$

$\Sigma F_z = 0 \Rightarrow 4t_{AB} + 4t_{AC} + A_z = 0\ ;\ A_z = 4\ \text{kN}$

Example 6 (cont.)

To obtain the reaction:



$\sum M_y = 0;$
$-4(2) + B_x(4) = 0$
$B_x = 2$ kN

$\sum M_z = 0;$
$C_y = 0$

$\sum F_y = 0;$
$A_y - 4 = 0$
$A_y = 4$ kN

$\sum F_z = 0;$
$A_z - 4 = 0$
$A_z = 4$ kN

$\sum M_x = 0;$
$B_y(4) - 4(4) = 0$
$B_y = 4$ kN

$\sum F_x = 0;$
$2 - A_x = 0$
$A_x = 2$ kN

References

1. Hibbeler, R.C., Mechanics Of Materials, 8th Edition in SI units, Prentice Hall, 2011.
2. Gere dan Timoshenko, Mechanics of Materials, 3rd Edition, Chapman & Hall.
3. Yusof Ahmad, ‘Mekanik Bahan dan Struktur’ Penerbit UTM 2001