

SEL4223 Digital Signal Processing

FIR Filter Design

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Introduction

- The design will be based on zeros instead of poles where all poles will only exist at $z = 0$
- Thus, the difference equation of an FIR filter does not consists of past or future output sequence where an FIR filter of length M with input $x[n]$ and output $y[n]$ can be represented as follow

$$\begin{aligned}y[n] &= b_0x[n] + b_1x[n - 1] + \cdots + b_{M-1}x[n - M + 1] \\ &= \sum_{k=0}^{M-1} b_kx[n - k]\end{aligned}$$

- Designing FIR filter is actually a process of finding the filter coefficients b_k .

Stability and causality

- Not an issue for the IIR filter design since the analog filter was already design to be both stable and causal and the Bilinear transformation technique preserved the causality and stability of the analog filter when converted to the digital filter.
- For FIR filter, the filter is **always stable** since ROC of FIR system always include unit circle in the z-plane.
- However, causality is the issue need to be taken care of in designing the FIR filter.

FIR filter design technique

- Below are the three common methods:
 - Windowing technique
 - Frequency Sampling technique
 - Optimum technique
- In this module, only windowing technique will be discussed. Thus the following pages are discussing the windowing technique.

Windowing Technique

- For IIR filter design based on bilinear transformation, the filter is design based on established analog filter such as Butterworth and Chebyshev filter.
- In contrast, designing FIR filter based on windowing technique is basically done by **truncating ideal filter (the desired filter) which is an IIR type (filter length = ∞) into FIR type.**
- The truncation process is done at time-domain.
- DTFT is then use to analyze the frequency response characteristic of the filter such as ripple and transformation bandwidth.

Filter Design Procedure (Step 1)

Step 1

- Determine filter characteristic (ω_p , ω_s , ω_c , N , window function)

In windowing technique, δ_p , δ_s are determined by the window function where $\delta_p = \delta_s$. Thus the notation is simplified with δ .

Similar to IIR filter, transition bandwidth of FIR filter depends on the value of N . Thus, if N is specified, no need to specify ω_p and ω_s and vice versa.

$\omega_c = \frac{\omega_s + \omega_p}{2}$. This is where $|H(\omega)| = 0.5$ or at -6.0206 dB

Filter Design Procedure (Step 2)

Step 2

- Obtain impulse response of the desired filter (ideal filter)

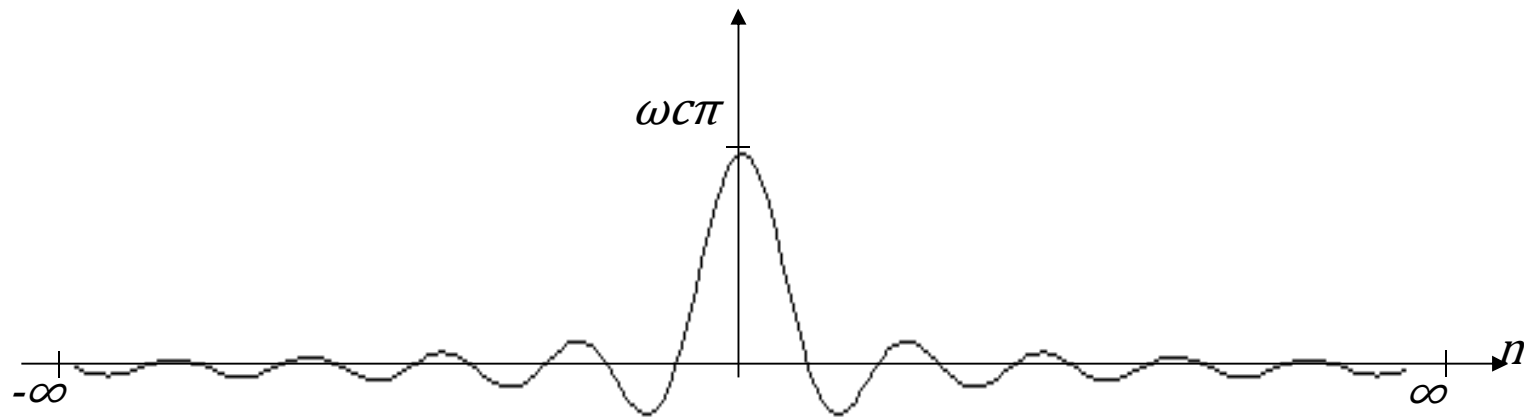
$$H_d(\omega) = \begin{cases} 1 & \text{for } |\omega| \leq \omega_c \\ 0 & \text{for } \omega_c < |\omega| \leq \pi \end{cases}$$

$$h_d[n] = \frac{1}{2} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega$$

$$= \begin{cases} \frac{\sin(\omega_c n)}{\pi n} & \text{for } n \neq 0 \\ \frac{\omega_c}{\pi} & \text{for } n = 0 \end{cases}$$

Filter Design Procedure (Step 2)

$$h_d[n] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n)$$



Filter Design Procedure (Step 3)

Step 3

- Multiply $h_d[n]$ with the chosen window function $w[n]$.

This is to reduce the infinite length of signal $h_d[n]$ into finite as this is an FIR filter. Hence, the length of the FIR filter will follow the length of the window function.

Thus the filter length, M is equals to $N + 1$ where N is the filter order.

Filter Design Procedure (Step 3)

Below are few of the window functions with length of M

Rectangular: $w[n] = 1,$ $-\frac{M-1}{2} \leq n \leq \frac{M-1}{2}$

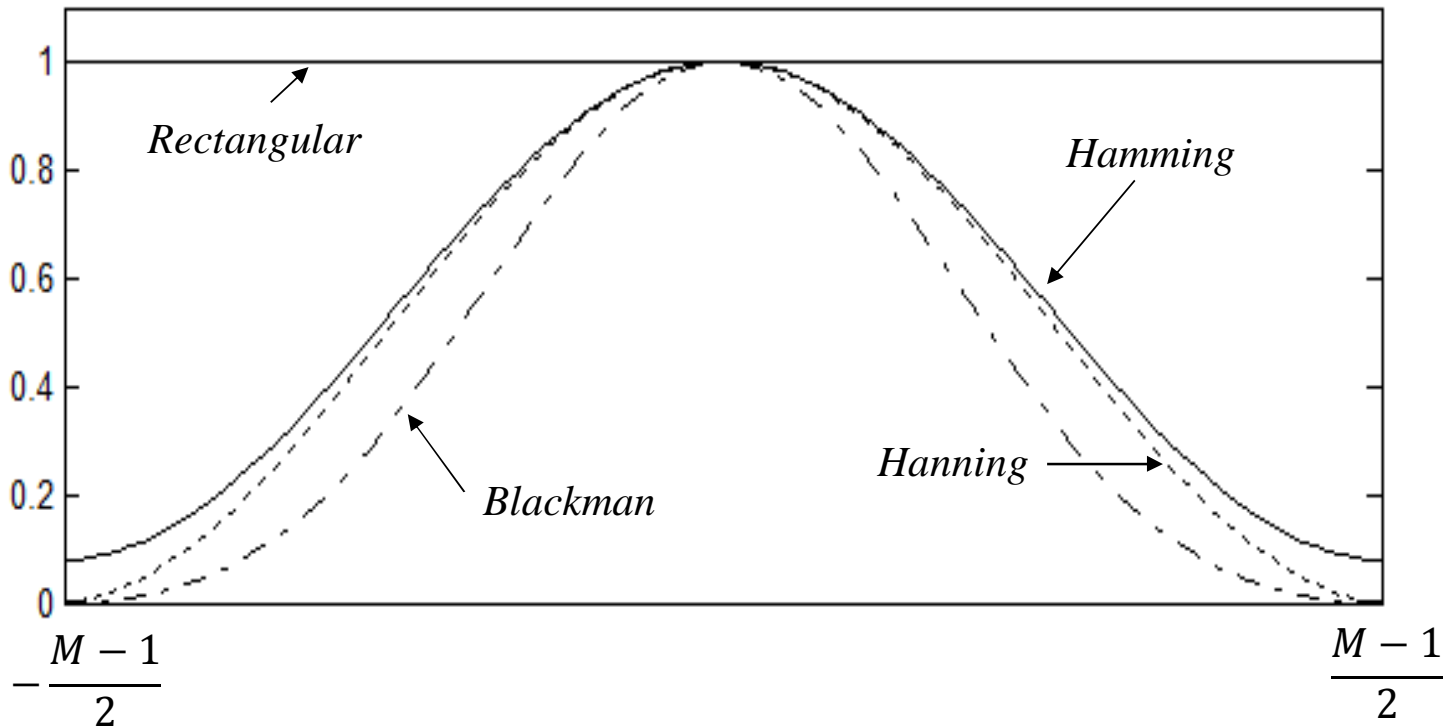
Hamming: $w[n] = 0.54 + 0.46\cos\left(\frac{2\pi n}{M-1}\right),$ $-\frac{M-1}{2} \leq n \leq \frac{M-1}{2}$

Hanning: $w[n] = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi n}{M-1}\right)\right],$ $-\frac{M-1}{2} \leq n \leq \frac{M-1}{2}$

Blackman: $w[n] = 0.42 + 0.5\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos\left(\frac{4\pi n}{M-1}\right),$ $-\frac{M-1}{2} \leq n \leq \frac{M-1}{2}$

Filter Design Procedure (Step 3)

- Below is the plot of the window function



Filter Design Procedure (Step 3)

- The multiplication process is called truncation process where

$$h[n] = h_d[n] \cdot w[n]$$

- Ripple of the FIR filter depend on the window function. Thus, the ripple is constant through all filter order. The ripple δ and transition bandwidth $\Delta\omega$ for several window functions are listed in table below

Type of window	δ (dB)	$\Delta\omega$
Rectangular	-20.9 dB	$1.84\pi/(M - 1)$
Hanning	-43.9 dB	$6.22\pi/(M - 1)$
Hamming	-54.5 dB	$6.64\pi/(M - 1)$
Blackman	-75.3 dB	$11.12\pi/(M - 1)$

Filter Design Procedure (Step 4)

Step 4

- Delay $h[n]$ with n_d to obtain a causal impulse response ($h[n] = 0$ for $n < 0$)

n_d can be computed as follow

$$n_d = \frac{M-1}{2}$$

From there, the impulse response of a causal FIR filter can be represented as

$$h[n] = h_d \left[n - \frac{M-1}{2} \right] \cdot w \left[n - \frac{M-1}{2} \right]$$

Example 1

Design a 20th order FIR lowpass filter based on windowing technique where cutoff frequency of the filter is $\omega_c = 0.5\pi \text{ rad}$. Design the filter based on rectangular, Hamming, Hanning and Blackman window.

Solution

- Step 1:** Determine filter characteristic. From the question, $\omega_c = 0.5\pi \text{ rad}$, $N = 20$ and four type of window for four different window must be used.
- Step 2:** Obtain impulse response of the desired filter (the ideal filter version)

$$h_d[n] = 0.5 \text{sinc}(0.5\pi n)$$

Example 1 (cont.)

Step 3&4: Multiply $h_d[n]$ with the window function $w[n]$ and delay with n_d

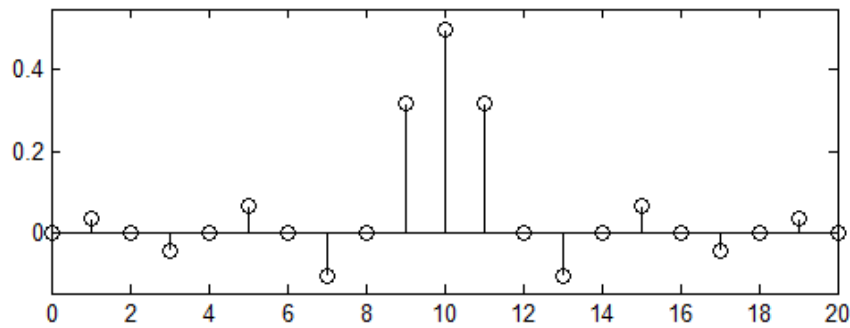
$$M = N + 1 = 21$$

$$n_d = \frac{M-1}{2} = \frac{N}{2} = 10$$

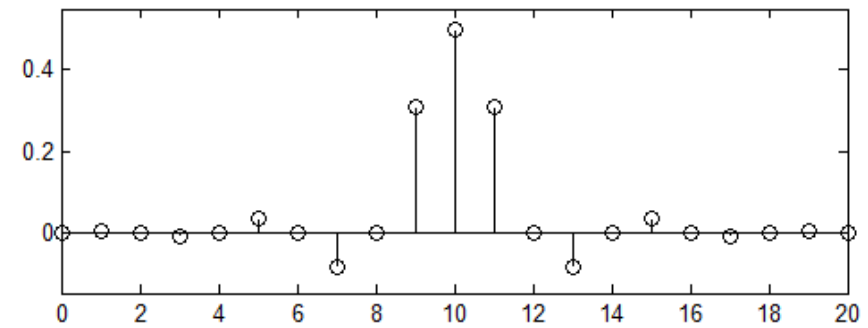
$$h[n] = 0.5 \text{sinc}(0.5\pi(n - 10)) \cdot w[n - 10]$$

Example 1 (cont.)

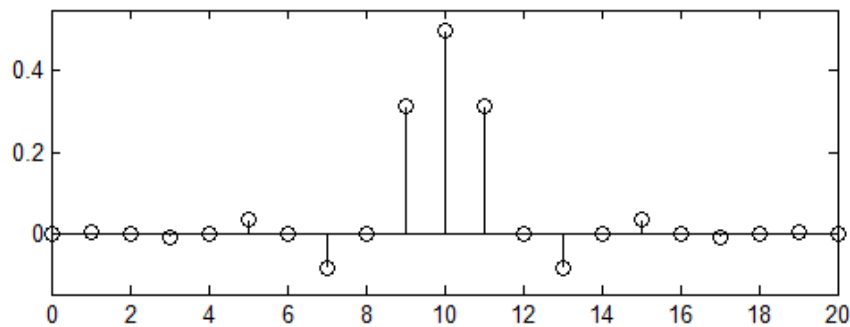
Rectangular



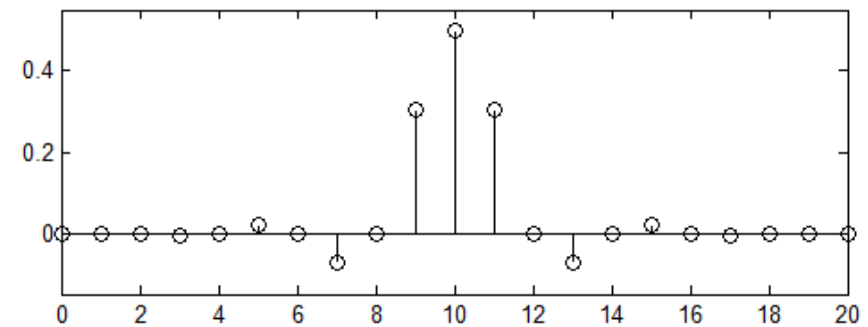
Hanning



Hamming

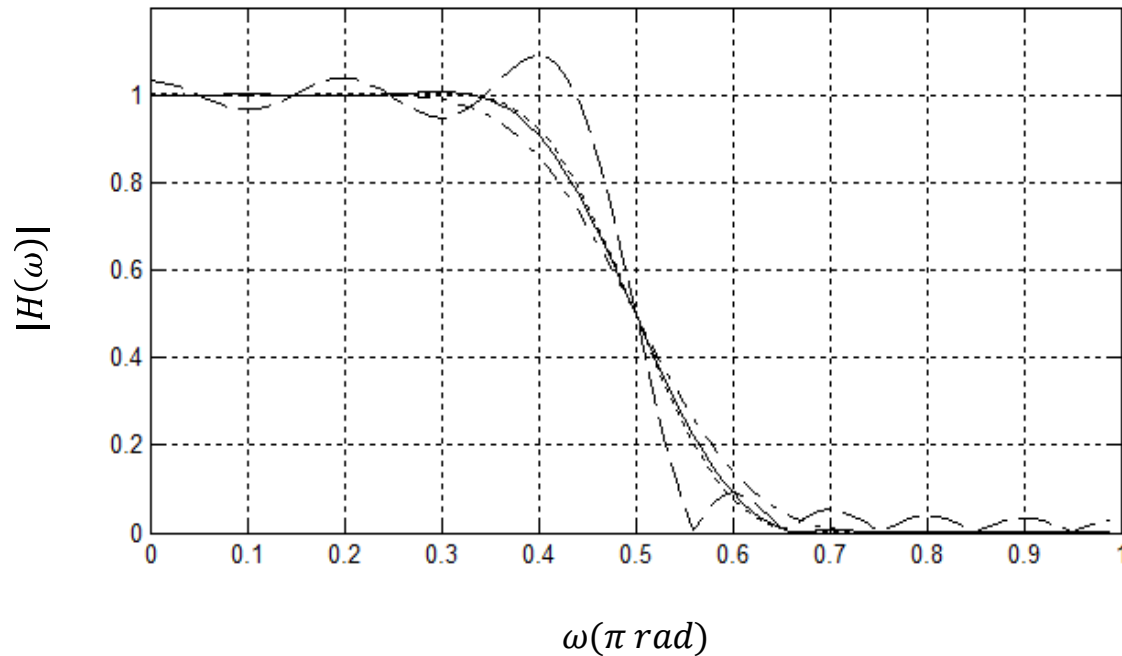


Blackman

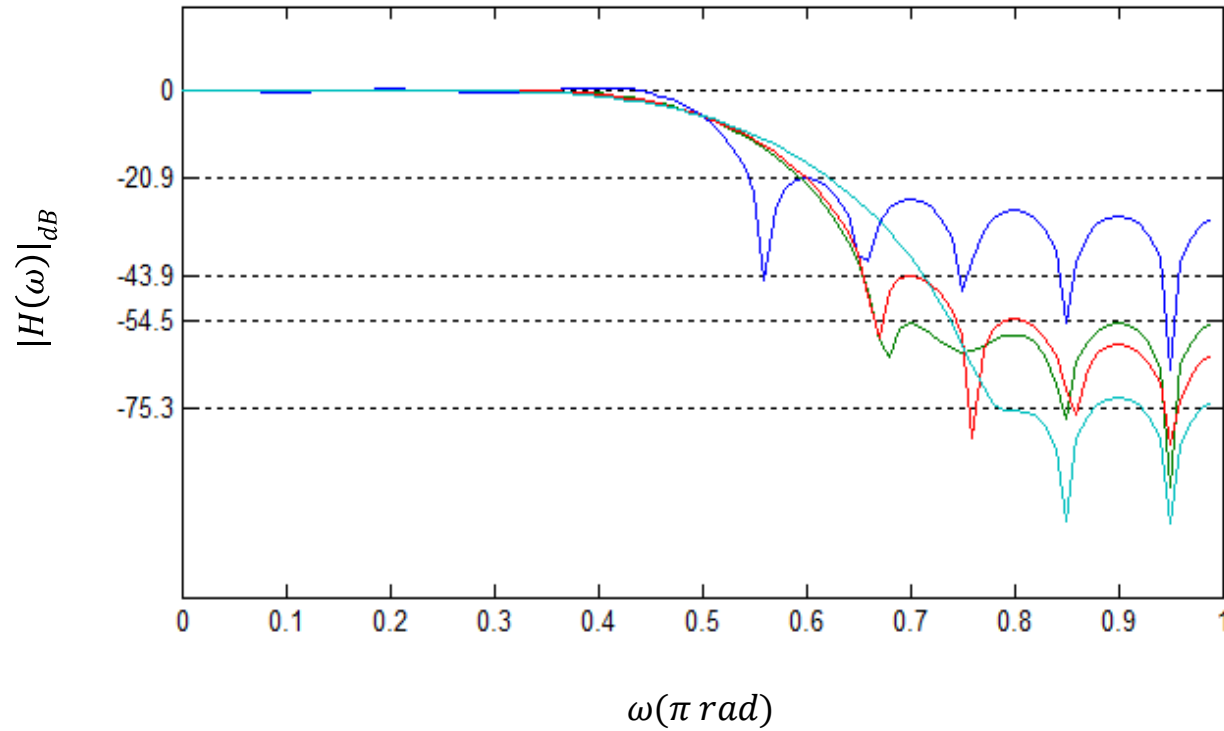


Example 1 (cont.)

The magnitude spectrum and magnitude dB of these filters are shown in the following figures.



Example 1 (cont.)



Example 2

Design an FIR lowpass filter based on rectangular window with filter characteristic below

$$\omega_p = 0.2\pi, \quad \omega_s = 0.4\pi$$

Solution

Step 1: Determine filter characteristic.

$$\omega_c = \frac{0.2\pi + 0.4\pi}{2} = 0.3\pi$$

Step 2: Obtain impulse response of the desired filter (the ideal filter version)

$$h_d[n] = 0.3 \text{sinc}(0.3\pi n)$$

Example 2 (cont.)

Step 3&4: Multiply $h_d[n]$ with the window function $w[n]$ and delay with n_d

$$\Delta\omega = \frac{1.84\pi}{M-1} = 0.4\pi - 0.2\pi = 0.2\pi$$

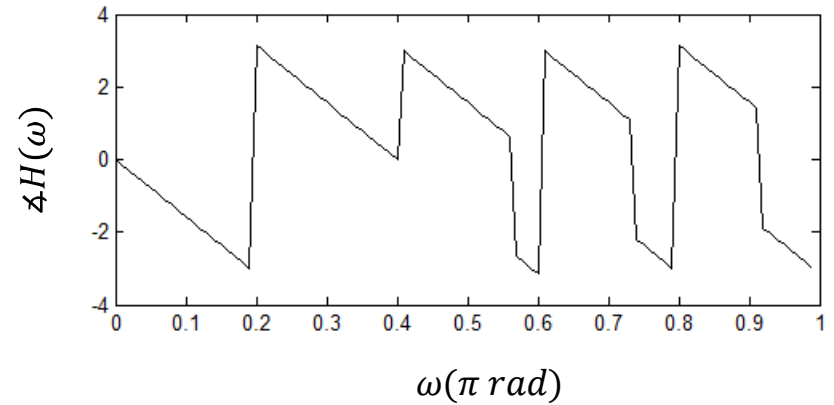
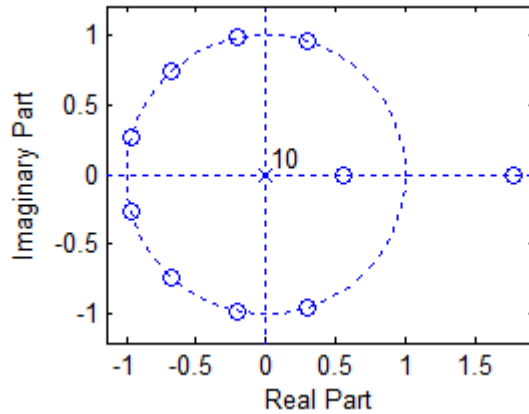
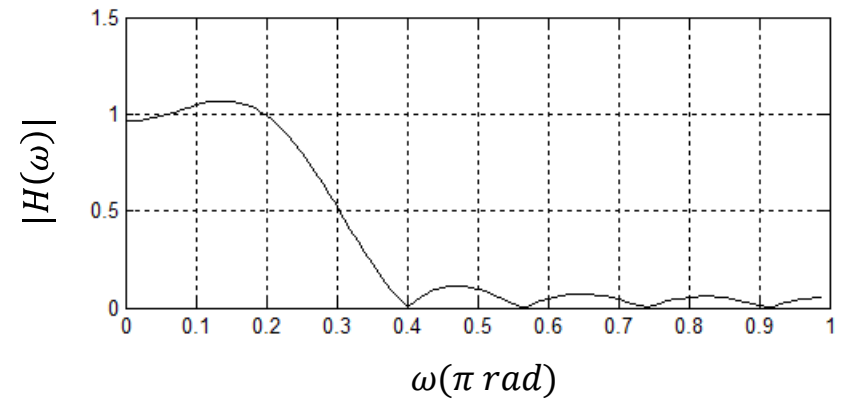
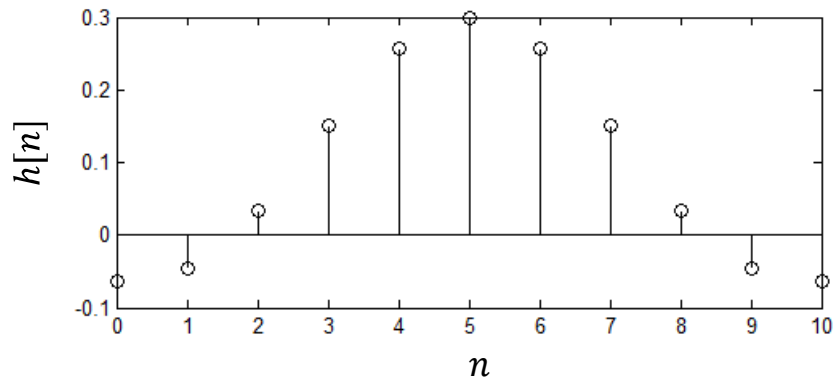
$$M = \frac{1.84\pi}{0.2\pi} + 1 = 10.2 \approx 11$$

$$n_d = \frac{M-1}{2} = 5$$

$$h[n] = 0.3 \text{sinc}(0.3\pi(n - 5)) \cdot w[n - 5]$$

$$h[n] = \begin{cases} 0.3 \text{sinc}(0.3\pi(n - 5)) & \text{for } 0 \leq n \leq 10 \\ 0 & \text{elsewhere} \end{cases}$$

Example 2 (cont.)



Quiz 1

- Design an FIR lowpass filter with below specifications based on windowing technique:

$$\delta = 0.007$$

$$\Delta\omega = 0.6\pi$$

$$M \leq 12$$

$$\omega_c = 1.5708$$

References

- 1) John G. Proakis, Dimitris K Manolakis, “Digital Signal Processing: Principle, Algorithm and Applications”, Prentice-Hall, 4th edition (2006).
- 2) Sanjit K. Mitra, “Digital Signal Processing-A Computer Based Approach”, McGraw-Hill Companies, 3rd edition (2005).
- 3) Alan V. Oppenheim, Ronald W. Schaffer, “Discrete-Time Signal Processing”, Prentice-Hall, 3rd edition (2009).