SKAA 1213 - Engineering Mechanics

TOPIC 10

WORK AND ENERGY

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Outline

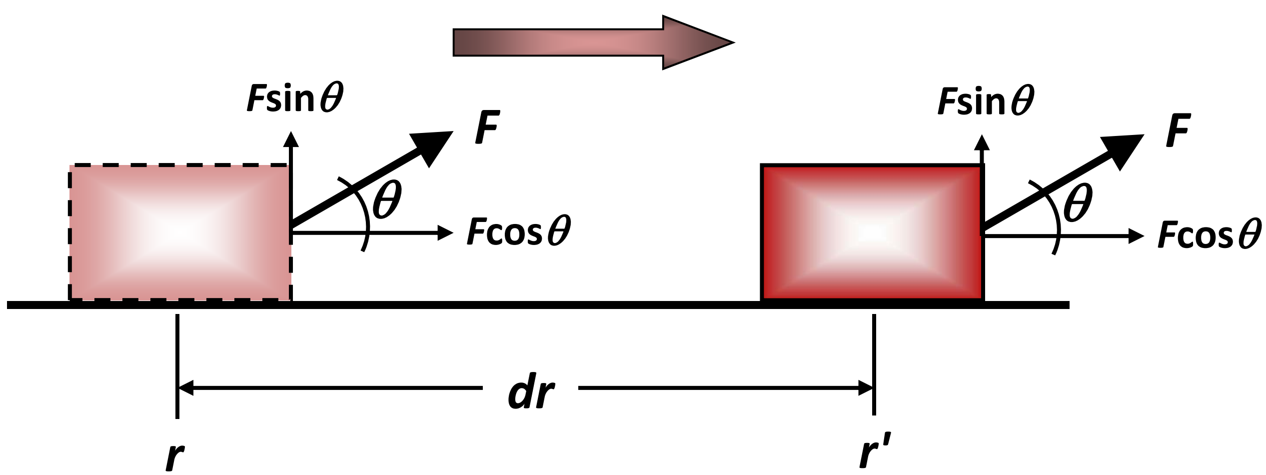
- *Introduction*
- *The Work of a Force*
- *Principle of Work and Energy*
- *Conservative Forces and Potential Energy*
- *Conservation of Energy*
- *Problems*

Introduction

- A force (F) does work (U) on a particle when it undergoes a displacement (dr) in the direction of the force.

$$U = F \cdot dr$$

$$U = F\cos\theta \cdot dr = Fdr\cos\theta$$



The Work of a Force

- When particle undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ (or s_1 to s_2), the work can be determined by integration.
- If **F** is expressed as a function of position, $F = F(s)$,

$$U_{1-2} = \int_{r_1}^{r_2} F.dr = \int_{s_1}^{s_2} F\cos\theta\, ds$$

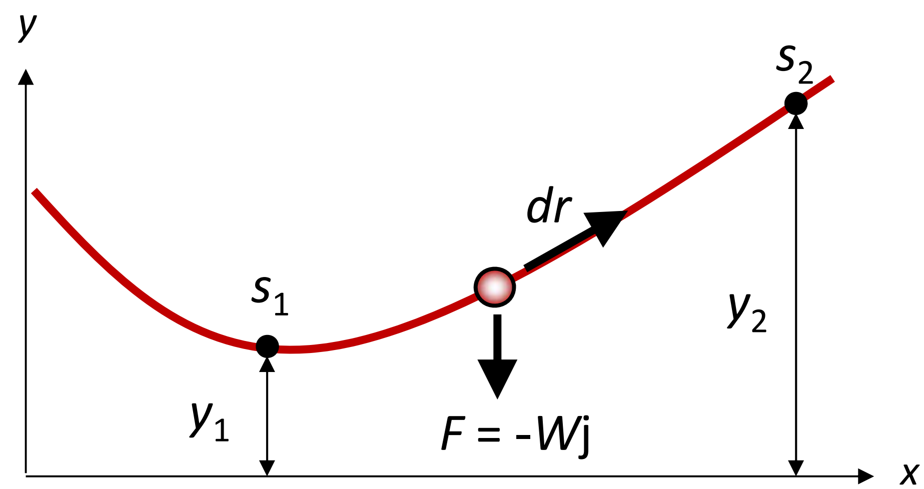
The Work of a Force

- ***Work of a Weight***
  - Consider a particle which moves up along the path s from s_1 to position s_2

$$U_{1-2} = \int F.dr = \int_{r_1}^{r_2} (-W\tilde{j}).(dx\tilde{i} + dy\tilde{j} + dz\tilde{k})$$

$$= \int_{y_1}^{y_2} -W dy = -W(y_2 - y_1)$$

$$U_{1-2} = -W\Delta y$$

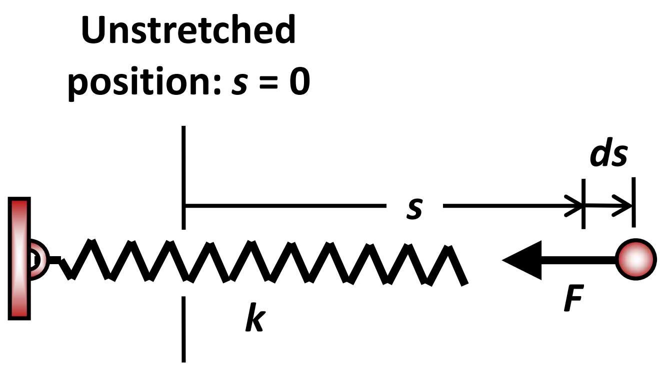


The Work of a Force

- ***Work of a Spring Force***
  - Magnitude of force in a linear elastic spring when displaced a distance *s* from unstretched position is $F_s = k_s$
  - The work is ***negative*** since F_s acts in the opposite sense to *ds*.

$$U_{1-2} = \int_{s_1}^{s_2} F_s\, ds = \int_{s_1}^{s_2} -ks\, ds$$

$$U_{1-2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

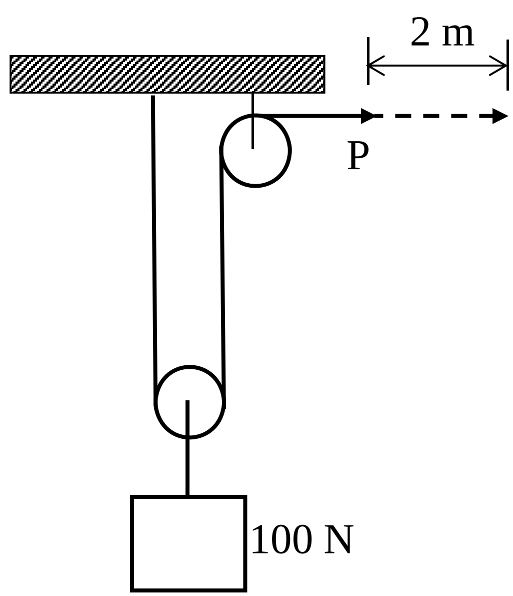


The Work of a Force

- Summary:
- Work: $$U_{1-2} = F\cos\theta(s_2 - s_1)$$
- Work of a weight: $$U_{1-2} = -W\Delta y$$
- Work of a spring: $$U_{1-2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

The Work of a Force

- Force P slowly lifts the 100 N weight as it moves 2 m to the right. Determine the work done.

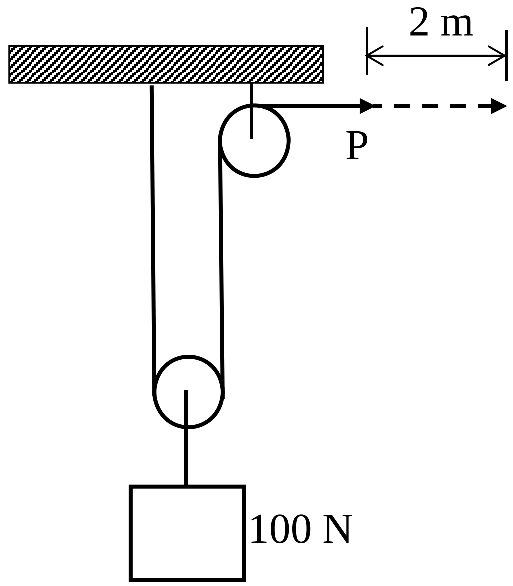


The Work of a Force

- Method 1:
  – Looking at the right pulley:

$$P = \frac{100}{2} = 50\,N$$

$$\therefore U = Fs = (50)(2) = 100\ J$$

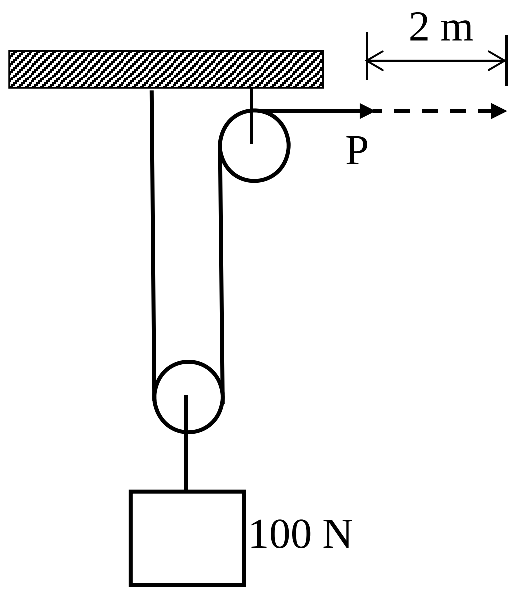


The Work of a Force

- Method 2:
  - Looking at the bottom pulley, because there are 2 cables, the distance moved by the 100 N is:

$$\frac{2\ m}{2} = 1\ m$$

$$\therefore U = Fs = (100)(1) = 100\ J$$



Principle of Work and Energy

- Consider a particle *P* path measured with an inertial coordinate system
- For the particle in the tangential direction,

$$\sum F_t = ma_t \qquad \sum \int_{s_1}^{s_2} F_t \, ds = \int_{v_1}^{v_2} mv \, dv$$

$$\sum \int_{s_1}^{s_2} F_t \, ds = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- For *principle of work and energy* for the particle:

$$\sum U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Principle of Work and Energy

- The principle is often also expressed as:

$$T_1 + \sum U_{1-2} = T_2$$

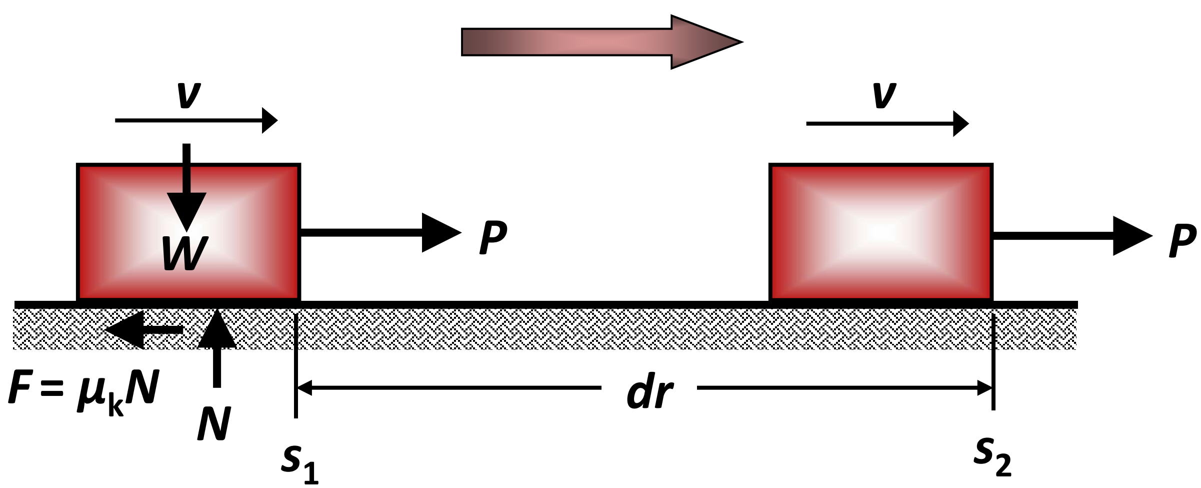
$$T_1 = \frac{1}{2} m v_1^2 \qquad\qquad T_2 = \frac{1}{2} m v_2^2$$

- Definition:
  - The particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from initial to its final position is equal to the particle's final kinetic energy

Principle of Work and Energy

- ***Work of Friction Caused by Sliding.***
  - When an applied force **P** just balances the resultant frictional force ($\mu_k N$), due to equilibrium a constant velocity **v** is maintained:

$$\frac{1}{2}mv^2 + Ps - (\mu_k N)s = \frac{1}{2}mv^2$$



Principle of Work and Energy

- Summary:
- Principle of Work and Energy: $T_1 + \sum U_{1-2} = T_2$
- Kinetic energy: $T = \frac{1}{2} m v^2$
- Work of friction: $\frac{1}{2} m v^2 + Ps - (\mu_k N)s = \frac{1}{2} m v^2$
- Work of spring: $U_{1-2} = -\left(\frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2 \right)$

Principle of Work and Energy

- A 1500 kg automobile is traveling down the 15° inclined road at a speed of 6 m/s. if the driver jams on the brakes, causing his wheels to lock, determine how far *s* his tires skid on the road. The coefficient of the kinetic friction between the wheels and the road is $\mu_k = 0.3$

Principle of Work and Energy

- Solution:
  - *Work (Free-Body Diagram)*

    N$_A$ does no work as and the weight 1500 kg, is displaced *s* sin 15°. Applying equation of equilibrium normal to the road,

$$\sum F_n = 0; \quad ; \quad N_A - 1500(9.81)\cos 15^\circ = 0$$

$$\Rightarrow N_A = 14213.6N$$

$$F_k = \mu_k N_A = 0.3(14213.6) = 4264.1N$$

Principle of Work and Energy

- Solution:
  - *Principle of Work and Energy*

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}mv_1^2 + \left[U_W + U_k\right] = \frac{1}{2}mv_2^2$$

$$\frac{1}{2}(1500)(6)^2 + \left\{14715(s\sin 15^\circ) - (4264.1)s\right\} = 0$$

$$\Rightarrow s = 59.27\text{m}$$

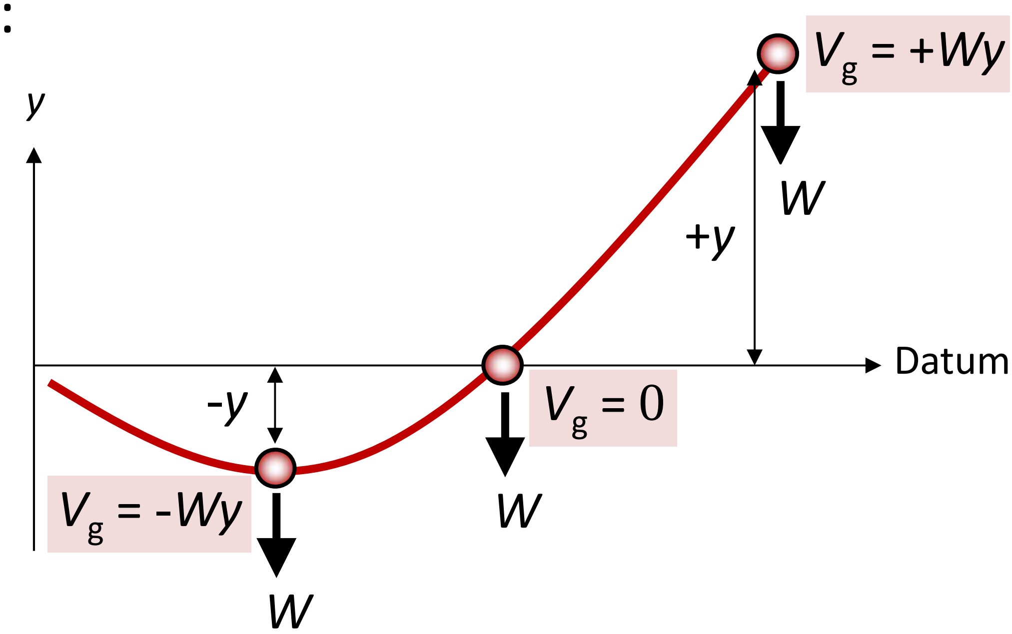
Conservative Forces and Potential Energy

- ***Conservative Force***
  - Defined by the work done in moving a particle from one point to another that is *independent of the path* followed by the particle
- ***Energy***
  - Energy is defined as the capacity for doing work
  - *Kinetic energy* is associated with the *motion* of the particle

Conservative Forces and Potential Energy

- ***Gravitational Potential Energy***
  - When a particle is located a distance *y above* a datum, the weight **W** has positive *gravitational potential energy* V_g
  - If y is positive upward, gravitational potential energy of the particle of weight W is:

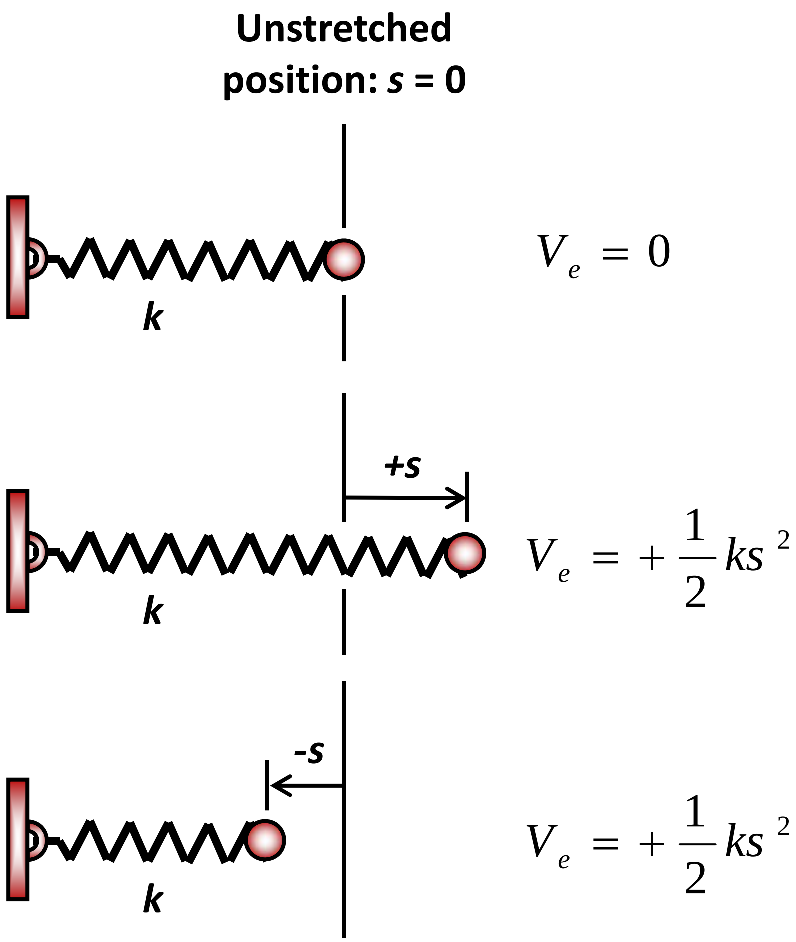
$$V_g = Wy$$



Conservative Forces and Potential Energy

- ***Elastic Potential Energy***
  - When an elastic spring is elongated or compressed from un-stretched position, the elastic potential energy *is*

$$V_e = +\frac{1}{2}ks^2$$



Conservation of Energy

- Work done by the *conservative forces* can be written in terms of the difference in their potential energies
- If *only conservative forces* are applied to the body when moving from state to state, we have *conservation of energy* (or *conservation of mechanical energy)*

$$T_1 + V_1 = T_2 + V_2$$

Conservation of Energy

- Summary:
- Gravitational potential energy: $V_g = Wy$
- Elastic potential energy : $V_e = +\frac{1}{2}ks^2$
- Conservation of energy: $T_1 + V_1 = T_2 + V_2$

Conservation of Energy

- A 1500 N lift moves upward from the eight floor to the twelve floor, with the height for each floor is 3 m. What is the increase in the potential energy of the lift?

Conservation of Energy

- Solution:
  - Use the eight floor as datum:
  - Total distance travelled = (12-8) x 3 = 12m

$$V_g = Wh$$

$$= (1500\,N)(12\,m)$$

$$\underline{V_g = 18\,000\,J}$$

Problem P1

- A lift carrying 8 boxes at 500 N each. As it goes up, a box is left at each floor or the first box at floor 2 and the last at floor nine. The floor height for each storey is 3.5 m and the lift weight is 5 kN. Determine the work done in lifting the lift and the boxes.

Problem P2

- A car, 1500 kg, is moving downhill resulting in the elevation drop of 400 m. Determine its decrease in potential energy?

Problem P3

- Johnson is 60 kg and he bungee jumps off the platform at *A*. The initial downward speed is 2 m/s. The stiffness of the elastic cord is k = 5 kN/m. The length of the cord is l_o. By taking *B* as the datum and neglecting the size of Johnson,

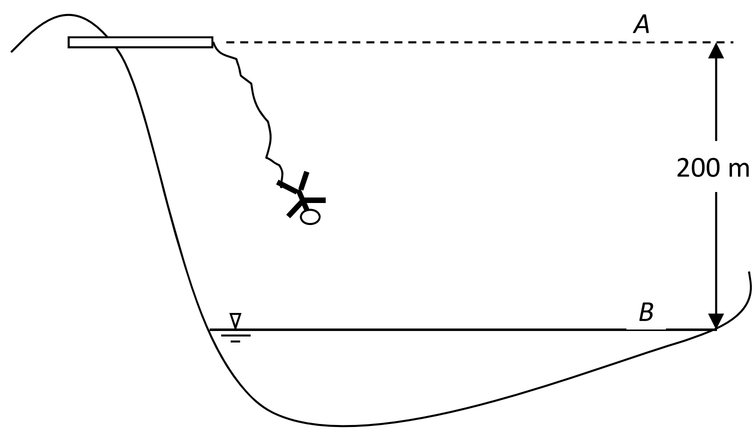


FIGURE P3

Problem P3

a) What is the gravitational potential energy, V_g of Johnson at position *A* and *B*?

b) Calculate the elastic potential energy, V_e of the elastic cord when Johnson is at the position *B*. Give the answer in term of l_o.

c) Determine the required un-stretched length of the cord, so that Johnson stops momentarily just above the surface of the water.

Problem P4

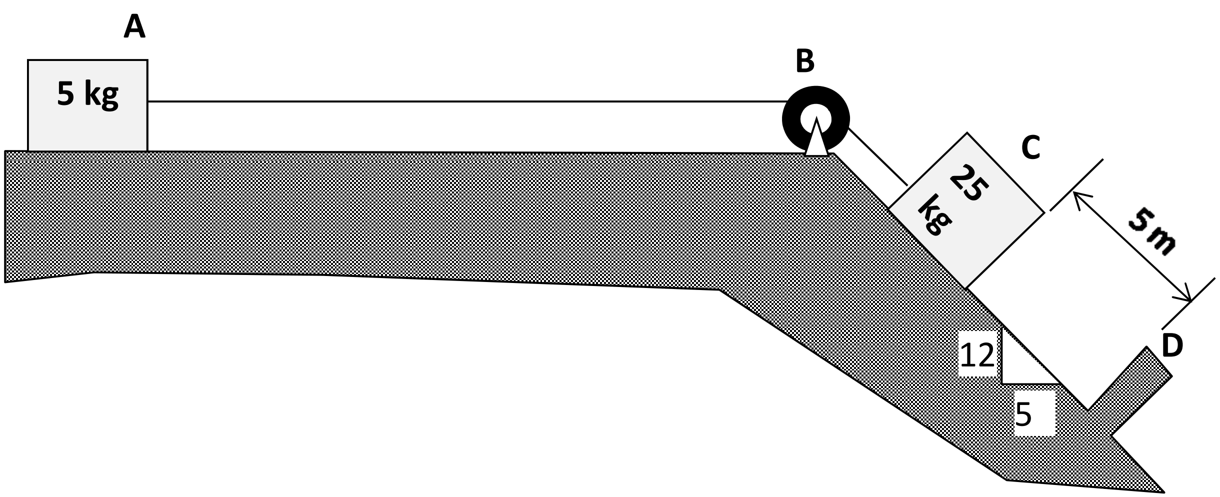
- A 5 kg mass placed on a rough surface at position A and is connected to a 25 kg mass located on an inclined rough surface at C. Both masses are connected by an inelastic weightless rope passing through a frictionless pulley at B. The 25 kg mass is initially held in position at C and is then released to slide down the inclined surface. The coefficient of static, μ_s and kinetic friction, μ_k between both the masses and the rough surface is 0.25 and 0.20 respectively.

Problem P4

a) Show that both masses will move as soon as the 25 kg mass is released.

b) Determine the acceleration of both masses.

c) Using principle of work and energy , determine the distance travelled by the 5 kg mass before coming to a stop after the 25 kg mass comes to a stop at D.



Problem P5

- A horizontal force *F* is pushing a 10 kg block; firstly, along a rough horizontal surface AB, and then up an inclined rough surface until the block stops at C as shown in Figure P5(a). The force *F* remains horizontal as it pushes the block up the inclined plane. The value of the coefficient of kinetic friction (μ_k) between the block and both the horizontal and inclined surface is 0.25. Figure P5(b) and Figure P5(c) shows the *a*-*t* and *v*-*t* graphs of the block as it is in motion from A to C.

Problem P5

FIGURE P5(a)

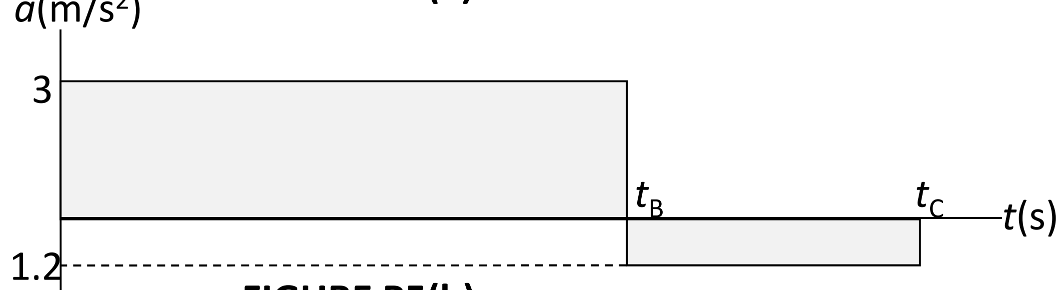


FIGURE P5(b)

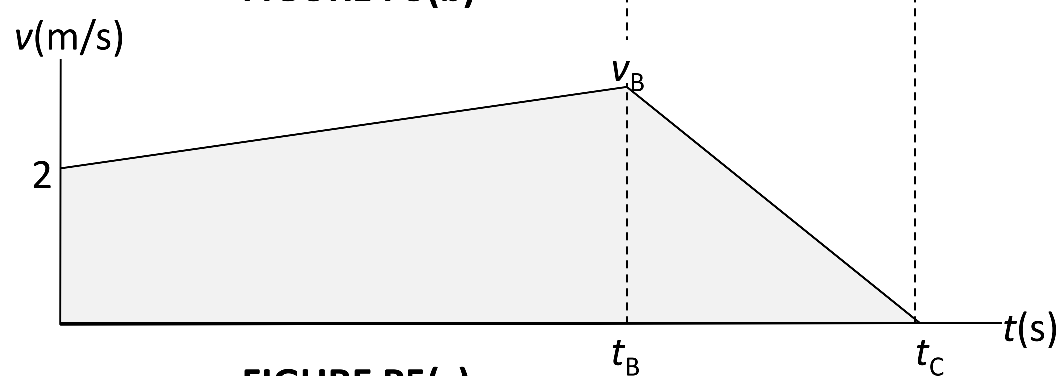


FIGURE P5(c)

Problem P5

a) Determine the angle θ of the inclined plane.

b) Using the principle of work and energy, determine:

   i. the velocity of the block as it passes by point B (v_B) given that the distance from A to B is 30 m,

   ii. the distance from B to C.

c) Validate your answers in part (b) by using rectilinear kinematics.

The End