SKAA 1213 - Engineering Mechanics

TOPIC 8

KINEMATIC OF PARTICLES

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Outline

- *Introduction*
- *Rectilinear Motion*
- *Curvilinear Motion*
- *Problems*

Introduction

- General Terms & Definition:
- Mechanic Static – equilibrium of a body that is at rest, or the body moves with constant velocity

- Mechanic Dynamics – deals with accelerated motion of a body
  - 1) **Kinematics** – analysis of geometric aspects of a motion
  - 2) Kinetics – analysis of the forces that cause the motion

Introduction

- Dynamic: Kinematic of Particles
- Rectilinear Motion
  - A particle moves in a straight line and does not rotate about its centre of mass.
- Circular Motion (Curvilinear Motion)
  - A particle moves along a path of a perfect circle.
- General Plane Motion (Curvilinear Motion)
  - A particle moves in a plane, which may follow a path that is neither straight nor circular.

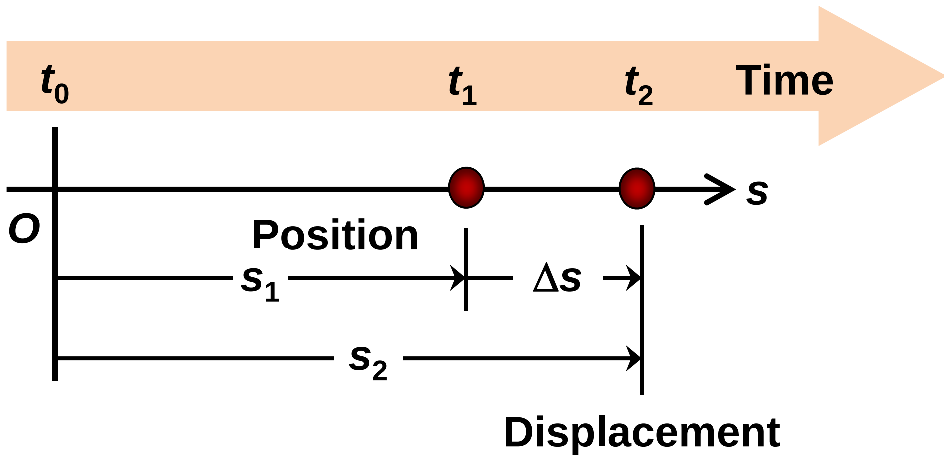
Rectilinear Motion

- Rectilinear Kinematics – specifying the particle's position, velocity, and acceleration at any instant (time factor)

Factor	Symbol	Unit	Remarks
Time	t	seconds (s)	Data may be given in minutes or hours (h)
Position	s	meter (m)	Data may be given in millimeter (mm), kilometer (km)
Velocity	v	m/s	Another common unit is kilometer per hour (km/h)
Acceleration	a	m/s^2	

Rectilinear Motion

- Position:
  - Single coordinate axis, *s*
  - Magnitude of *s* = distance from origin (*O*) to current position (*P*)
  - Direction: +ve = right of origin; -ve = left of origin

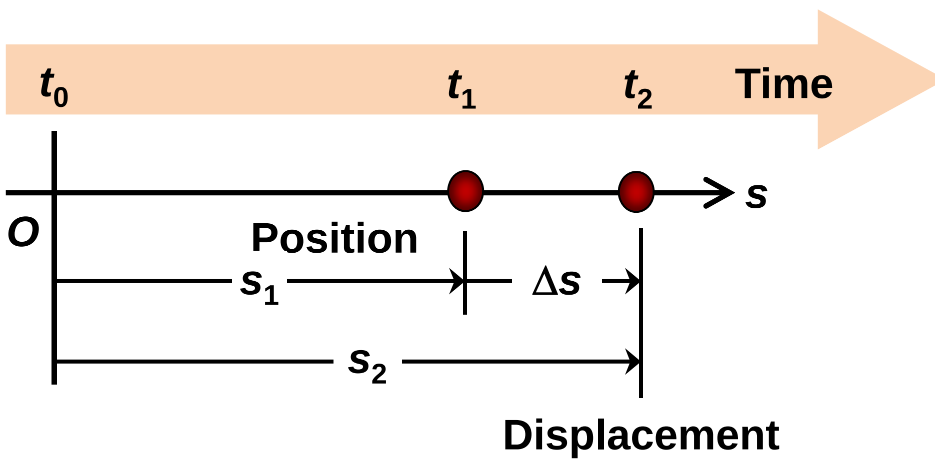


Rectilinear Motion

- Displacement:
  - Change in the particle's position, vector quantity
  - If particle moves from S_1 to S_2:

$$\Delta s = s_2 - s_1$$

  - When Δs is +ve / -ve, particle's position is right / left of its initial position



Rectilinear Motion

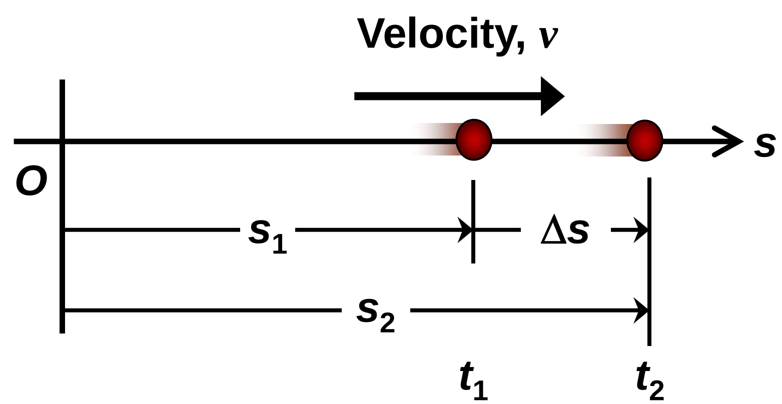
- Velocity:
  - The speed of the changes of positions.
  - Average velocity:

$$\Delta v = \frac{\Delta s}{\Delta t} \qquad \Delta s = s_2 - s_1 \qquad \Delta t = t_2 - t_1$$

  - Instantaneous velocity:

$$v_{ins} = \lim_{\Delta t \to 0} (\Delta s / \Delta t)$$

$$\Rightarrow v = \frac{ds}{dt}$$



Rectilinear Motion

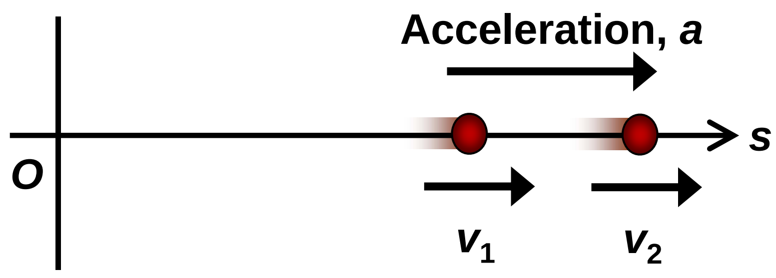
- Acceleration:
  - The speed of the changes of velocities.
  - Average acceleration:

$$\Delta a = \frac{\Delta v}{\Delta t} \qquad \Delta v = v_2 - v_1 \qquad \Delta t = t_2 - t_1$$

  - Instantaneous acceleration:

$$a = \lim_{\Delta t \to 0} (\Delta v / \Delta t)$$

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$



Rectilinear Motion

- Magnitude and directions

Factor	+ve value	-ve value	Zero value
Position, s	Direction to right	Direction to left	-
Velocity, v	Direction to right	Direction to left	Particle stop moving
Acceleration, a	Velocity increased	Velocity decreased	Constant velocity

Rectilinear Motion

- Position, velocity and acceleration as a function of time (*t*):

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$ **Differential**

$$s(t) \Rightarrow v(t) \Rightarrow a(t)$$

$$s(t) \Leftarrow v(t) \Leftarrow a(t)$$

$$s = \int v dt \qquad v = \int a dt$$ **Integration**

Rectilinear Motion

- Function of position, velocity and acceleration without time (t) factor:

$$
\left.
\begin{aligned}
v = \frac{ds}{dt} \;\Rightarrow dt = \frac{ds}{v} \\
a = \frac{dv}{dt} \;\Rightarrow dt = \frac{dv}{a}
\end{aligned}
\right\}
\frac{ds}{v} = \frac{dv}{a} \Rightarrow \; vdv = ads
$$

Rectilinear Motion

- Constant acceleration, a_c:

$$\int_{v_0}^{v} dv = \int_0^t a_c \, dt \Rightarrow v = v_0 + a_c t \quad \left(\overset{+}{\rightarrow}\right)$$

$$\int_{s_0}^{s} ds = \int_0^t v \, dt \Rightarrow s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \quad \left(\overset{+}{\rightarrow}\right)$$

$$\int_{v_0}^{v} v dv = \int_{s_0}^{s} a_c ds \Rightarrow v^2 = v_0^2 + 2a_c (s - s_0) \quad \left(\overset{+}{\rightarrow}\right)$$

Rectilinear Motion

- Summary of Equations:

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad a = \frac{d^2 s}{dt^2} \qquad vdv = ads$$

  - When acceleration is constant:

$$v = v_0 + a_c t$$

$$s = s_o + v_o t + \frac{1}{2} a_c t^2$$

$$v^2 = v_0^2 + 2a_c \left(s - s_0\right)$$

Rectilinear Motion

- A vehicle moves in a straight line such that for a short time its velocity is defined by $v = (0.9t^2 + 0.6t)$ m/s where t is in second.
- When $t = 0$, $s = 0$.
- Determine it position (s) and acceleration (a) when t = 3s.

Rectilinear Motion

- Solution:

Position

When s = 0 when t = 0, we have:

$$\left(\overset{+}{\rightarrow}\right) \quad v = \frac{ds}{dt} = \left(0.9t^2 + 0.6t\right)$$

$$\int_0^s ds = \int_0^t \left(0.9t^2 + 0.6t\right)dt \quad \Rightarrow s\Big|_0^s = \left(0.3t^3 + 0.3t^2\right)\Big|_0^t$$

$$= 0.3t^3 + 0.3t^2$$

When t = 3s:

$$s = 0.3t^3 + 0.3t^2 \qquad = 0.3(3)^3 + 0.3(3)^2$$

$$= 10.8m$$

Rectilinear Motion

- Solution:

Acceleration

Knowing *v* is a function of time *(t),* the acceleration can be determined from *a = dv/dt*

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(0.9t^2 + 0.6t\right)$$

$$= 1.8t + 0.6$$

When *t* = 3s: $a = 1.8t + 0.6 = 1.8(3) + 0.6$

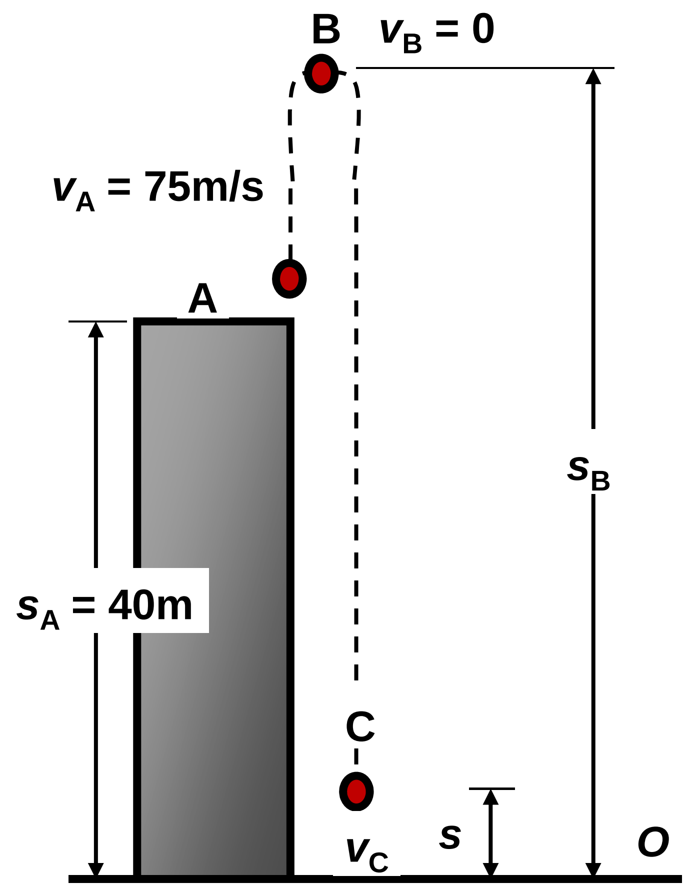
$$= 6 m/s^2$$

Rectilinear Motion

A ball is thrown upward at 75m/s from the top of a 40 m tall building. Determine:

a) Maximum height s_B reached by the ball.

b) The speed of the ball just before it hits the ground.



Rectilinear Motion

- Solution:

Information gathering:

Take origin at "*O*" and upward direction is positive.

Acceleration is constant and due to gravity: $a_C = -9.81 m/s^2$

The ball will reach maximum height at B:

$s = s_B$ ➔ $v_B = 0$ (ball stops moving at maximum height)

From the question we have:

$t = 0$ ➔ $v_A = +75 m/s$, $s_A = +40 m$

Rectilinear Motion

- Solution:

At Point B:

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v_B^2 = v_A^2 + 2a_C(s_B - s_A)$$

$$0^2 = 75^2 + 2(-9.81)(s_B - 40)$$

$$5625 - 19.62\text{s}_\text{B} + 784.8 = 0$$

$$\Rightarrow s_B = \frac{5625 + 784.8}{19.62} = 327 \text{ m}$$

Rectilinear Motion

- Solution:

At Point C:

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v_C^2 = v_B^2 + 2a_C(s_C - s_B)$$

$$v_C^2 = 0 + 2(-9.81)(0 - 327)$$

$$v_c = \sqrt{6415.74}$$

$$\Rightarrow v_c = -80.1\, m/s = 80.1\, m/s\ (\downarrow)$$

Rectilinear Motion

- Erratic Motion:
  - When a particle moves in erratic motion, it can be best described graphically by a series of curves.
  - A graph is used to describe the relationship with any 2 of the factors: *a, v, s, t*
  - Recall kinematic equations:

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad s = \int v\,dt \qquad v = \int a\,dt$$

Rectilinear Motion

- Erratic Motion:
  - The *s-t, v-t* and *a-t* Graphs
  - When given the s-t graph, we can construct the v-t graph and a-t graph, and vice versa:
    - Slope of s-t graph = v;
    - Slope of v-t graph = a;

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

    - Area under a-t graph = v
    - Area under v-t graph = s

$$s = \int v\,dt \qquad v = \int a\,dt$$

Rectilinear Motion

- Erratic Motion:
  - The *s-t*, *v-t* and *a-t* Graphs
  - General behavior of graph:
    - Incline slope → positive
    - Stagnant slope → "0"
    - Decline slope → negative

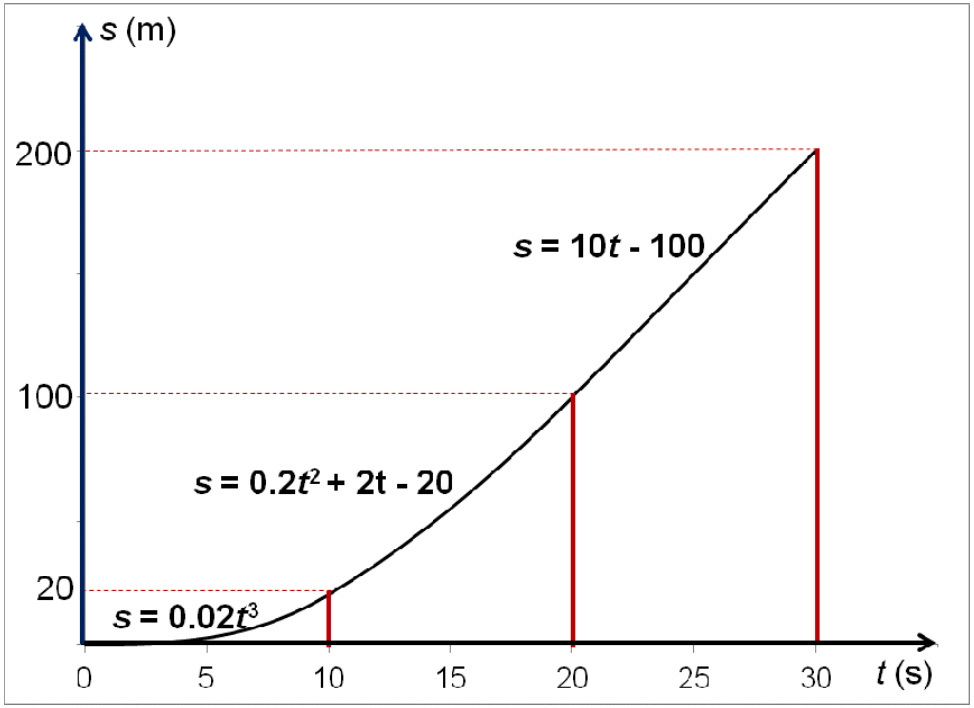
    - Positive area → increase slope
    - Negative area → decrease slope

Rectilinear Motion

- Erratic Motion:
  - Take an example of a bicycle moves along a straight road in the motion with such that it position is described by:
    - $s_1 = 0.02t^3$ from time t = 0s to t = 10s;
    - $s_2 = 0.2t^2 + 2t - 20$ from time t = 10s to t = 20s;
    - $s_3 = 10t - 100$ from time t = 20s to t = 30s;

Rectilinear Motion

- Erratic Motion:
  - With the given information, a *s-t* graph can be constructed:



Rectilinear Motion

- Erratic Motion:
  - By using function v = ds/dt and a = dv/dt:

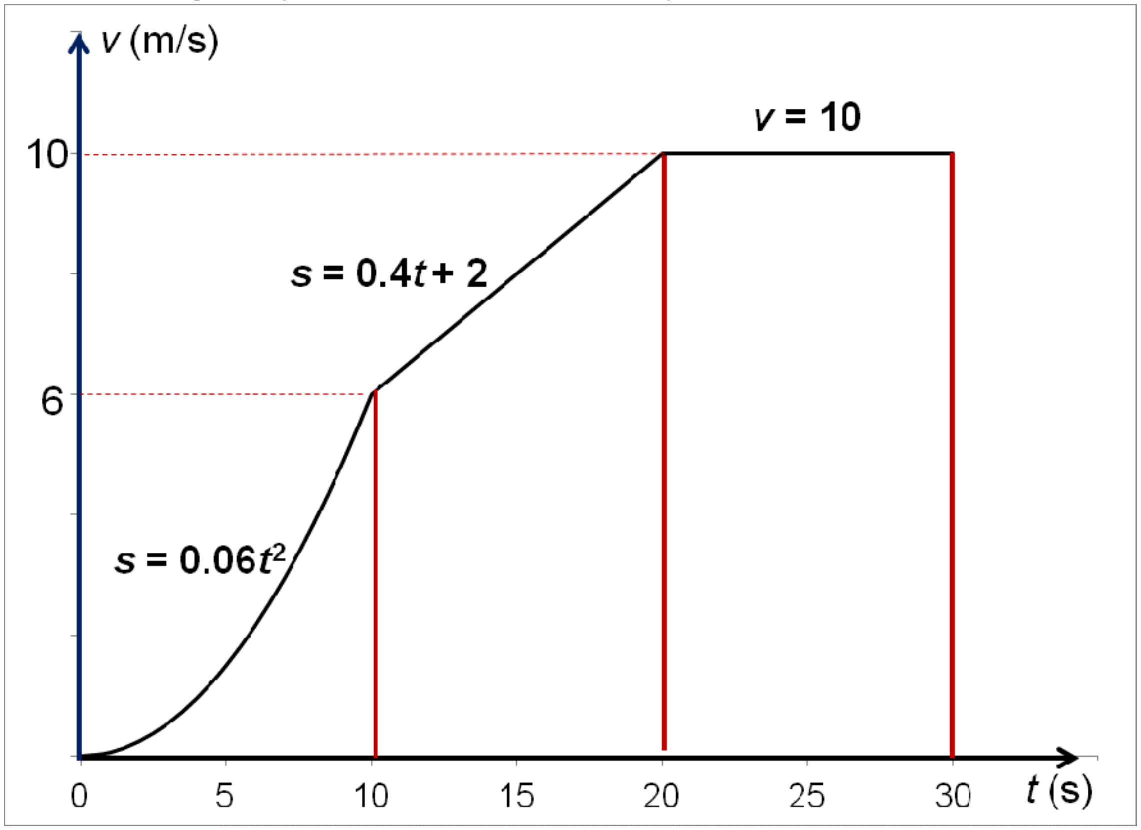
$$0 \le t \le 10; \quad s = 0.02t^3 \quad v = \frac{ds}{dt} = 0.06t^2 \qquad a = 0.12t$$

$$10 \le t \le 20; \quad s = 0.2t^2 + 2t - 20 \quad v = \frac{ds}{dt} = 0.4t + 2 \quad a = 0.4$$

$$20 \le t \le 30; \quad s = 10t - 100 \quad v = \frac{ds}{dt} = 10 \qquad a = 0$$

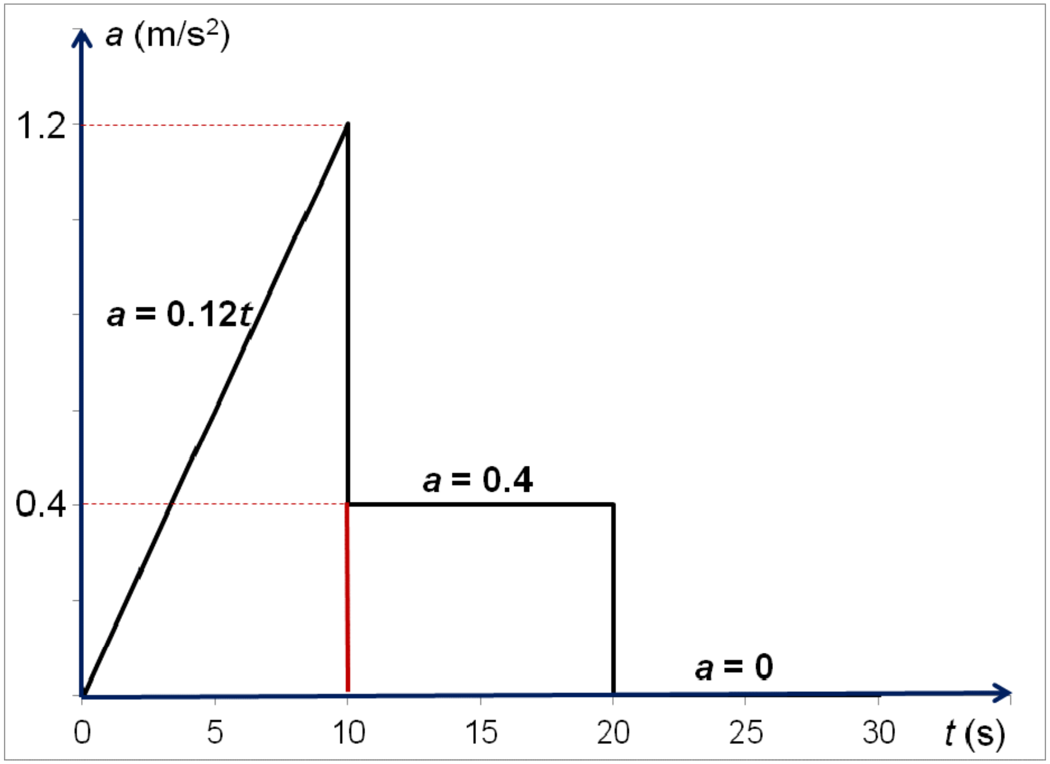
Rectilinear Motion

- Erratic Motion:
  - The *v-t* graph of the bicycle motion:



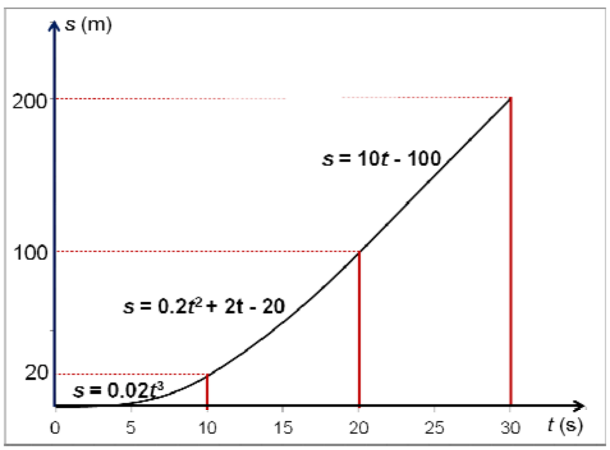
Rectilinear Motion

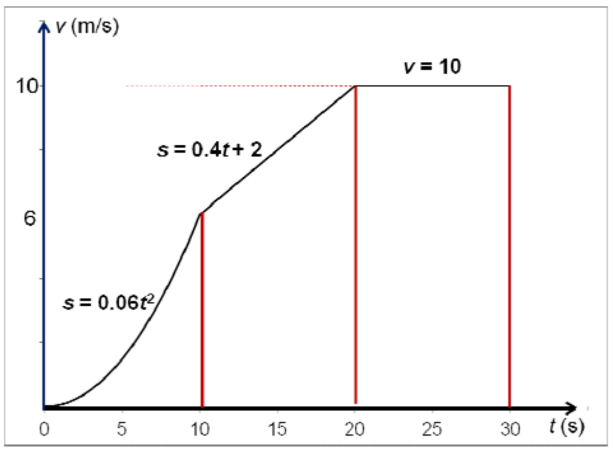
- Erratic Motion:
  - The *a-t* graph of the bicycle motion:

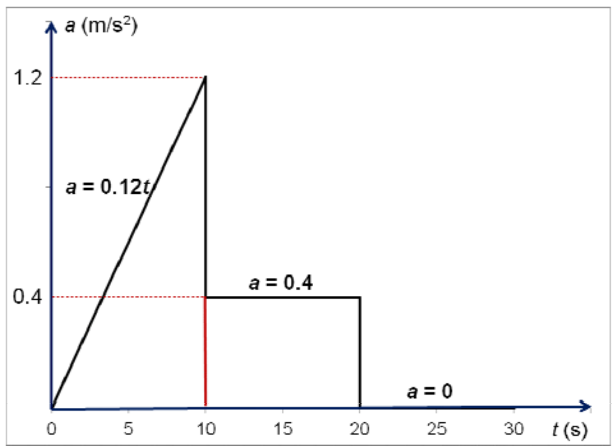


Rectilinear Motion

- Erratic Motion:
  - Comparison between s-t, v-t and a-t graphs:

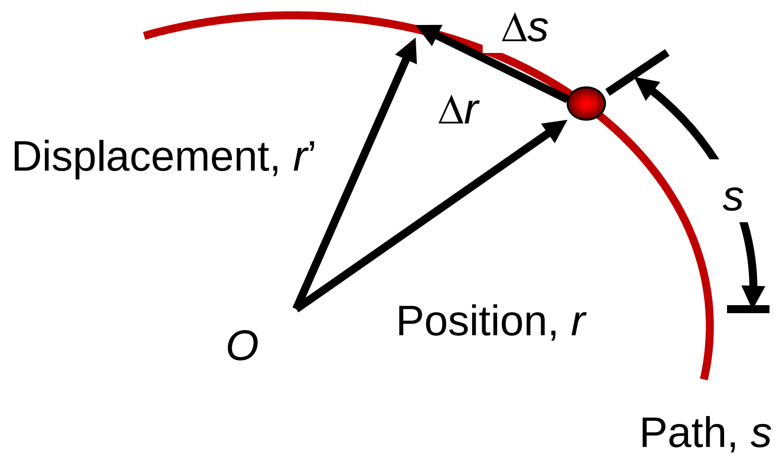






Curvilinear Motion

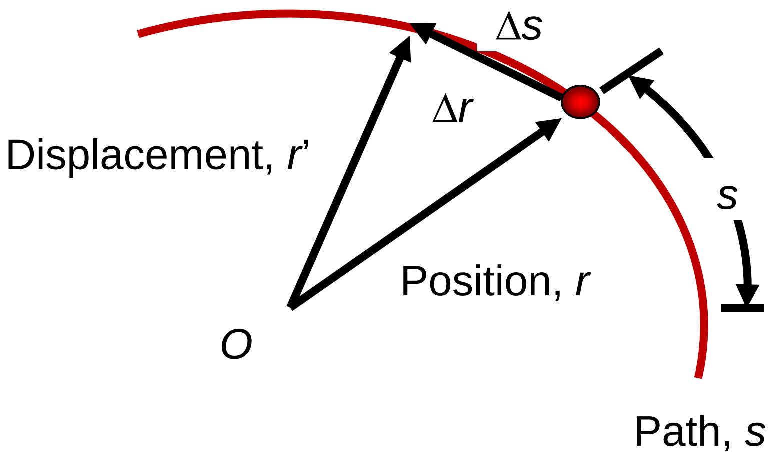
- Introduction:
  - Curvilinear occurs when a particle is moving along a curved path.
  - Position is measured from a fixed point O, by the position vector r = r(t)



Curvilinear Motion

- Introduction:
  - Displacement Δ*r* represents the change in the particle's position.

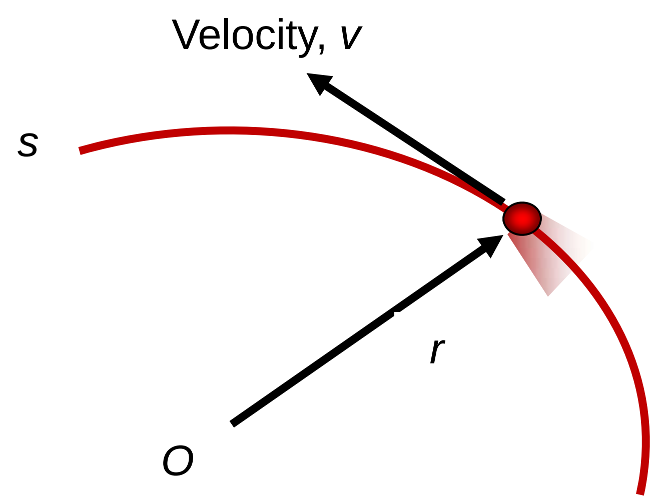
$$r' = r + \Delta r$$



Curvilinear Motion

- Introduction:
  - *Average velocity* is defined as: $v_{avg} = \frac{\Delta r}{\Delta t}$
  - Instantaneous velocity is found when $\Delta t \rightarrow 0$:

$$v = \frac{dr}{dt}$$

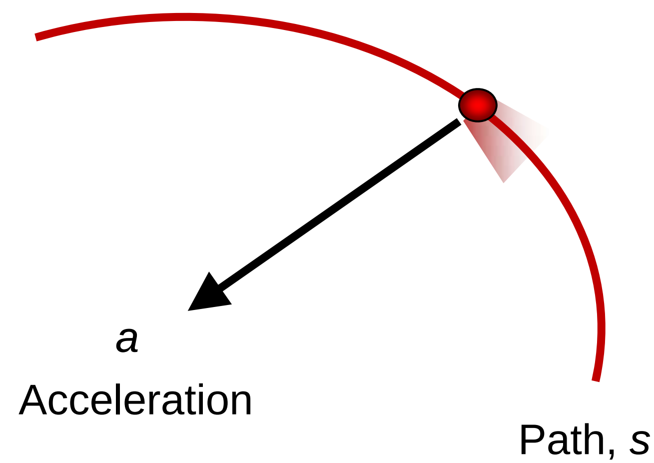


Curvilinear Motion

- Introduction:
  - The average and instantaneous acceleration are:

$$a_{avg} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2 r}{dt^2}$$

  - *a* acts tangent to the hodograph and is not tangent to the path, s



Curvilinear Motion

- Projectile Motion
  - Projectile launched at (x_0, y_0) and path is defined in the x-y plane where y-axis is the vertical axis.
  - Air resistance is neglected
  - The only force exists is the weight downwards
  - Projectile's acceleration always act vertically
    - Constant acceleration: $a_c = g = 9.81\ \text{m/s}^2$
    - $a_x = 0$; $a_y = -g = -9.81\ \text{m/s}^2$

Curvilinear Motion

- Projectile Motion
  - Horizontal motion:
  - Since $a_x = 0$,

$$(\overset{+}{\rightarrow}) \quad v = v_0 + a_c t;$$

$$(\overset{+}{\rightarrow}) \quad x = x_0 + v_0 t + \frac{1}{2} a_c t^2;$$

$$(\overset{+}{\rightarrow}) \quad v^2 = v_0^2 + 2a_c (s - s_0);$$

$$v_x = (v_0)_x$$

$$x = x_0 + (v_0)_x t$$

$$v_x = (v_0)_x$$

Curvilinear Motion

- Projectile Motion
  - Vertical motion:
  - Positive *y* axis is upward, we take $a_y = - g$

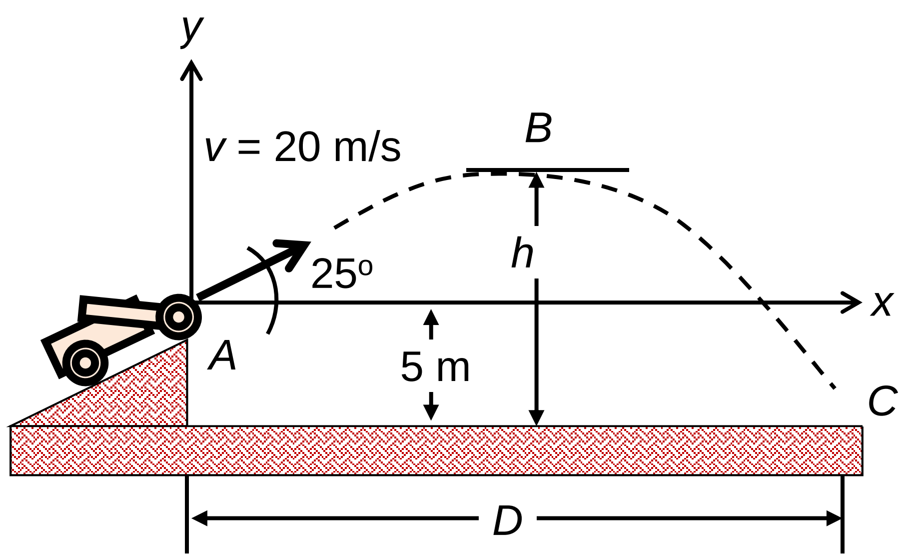
$$(+\uparrow)\quad v = v_0 + a_c t; \qquad v_y = (v_0)_y - gt$$

$$(+\uparrow)\quad y = y_0 + v_0 t + \frac{1}{2} a_c t^2; \qquad y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$$

$$(+\uparrow)\quad v^2 = v_0^2 + 2a_c (y - y_0); \qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

Curvilinear Motion

- Projectile Motion
  - A cyclist jumps off the 25° slope track at 5m height, and with the speed 20 m/s. Calculate the time (*t*) that the cyclist is flying in the air, and the distance (*D*) from point A when he landed on the ground.



Curvilinear Motion

- Projectile Motion
  - Solution:
    - Vertical motion:

$$v_y = v\sin\theta = (20)\sin 25^\circ = 8.45m/s$$

$$y = y_0 + (v_0)_y t - \frac{1}{2}gt^2$$

$$0 = 5 + 8.45t - \frac{1}{2}(9.81)t^2$$

$$4.905t^2 - 8.45t - 5 = 0$$

Curvilinear Motion

- Projectile Motion
  - Solution:
    - Using mathematical solution (take positive answer):

$$4.905t^2 - 8.45t - 5 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-(-8.45) \pm \sqrt{8.45^2 - 4(4.905)(-5)}}{2(4.905)}$$

$$t = 2.19s$$

Curvilinear Motion

- Projectile Motion
  - Solution:
    - Horizontal motion:

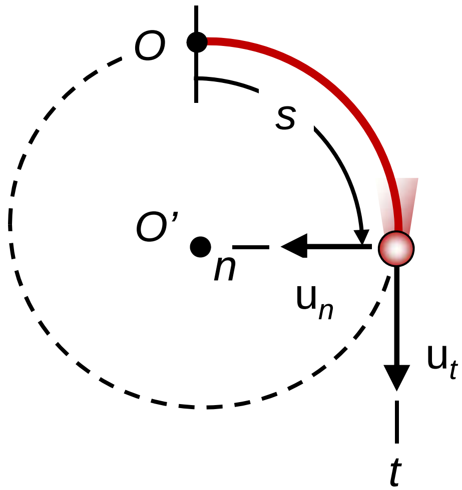
$$v_x = v\cos\theta = (20)\cos 25^\circ = 18.13 m/s$$

$$x_C = x_A + (v_{Ax})t$$

$$D = x_C = 0 + (18.13)(2.19) = 39.7 m$$

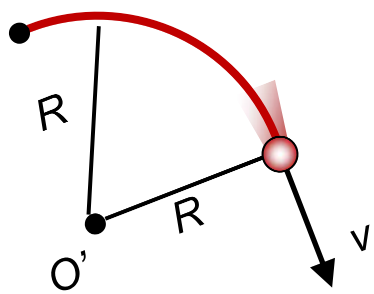
Curvilinear Motion

- Planar Circular Motion:
  - Normal & Tangential Components
    - When a particle moves in planar circular motion, the path of motion can be described using *n* and *t* coordinates, which act normal and tangent to the path.



Curvilinear Motion

- Planar Circular Motion:
  - Normal & Tangential Components
    - Velocity: Particle's velocity *v* has direction that is always tangent to the path



$$\vec{v} = v \cdot \vec{u}_t$$

$$v = \frac{ds}{dt} = \dot{s}$$

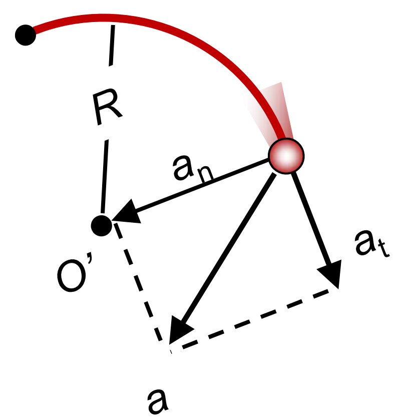
Curvilinear Motion

- Planar Circular Motion:
  - Normal & Tangential Components
    - Acceleration: the time rate of change of velocity

$$\vec{a} = \dot{\vec{v}} = \dot{v}\vec{u}_t + v\dot{\vec{u}}_t$$

$$\vec{a} = a_t\vec{u}_t + a_n\vec{u}_n$$

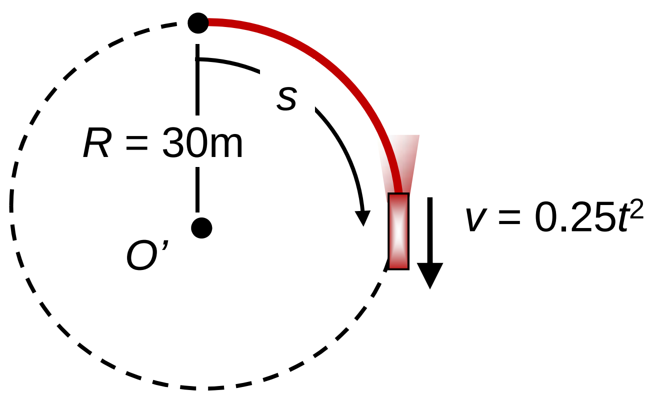
where $a_t = \dot{v}$ or $a_t ds = v dv$ and $a_n = \frac{v^2}{R}$



- Magnitude: $a = \sqrt{a_t^2 + a_n^2}$

Curvilinear Motion

- Planar Circular Motion:
  - Starting from rest, a motor travels around a circular path of R = 30 m at a speed that increases with time: $v = 0.25t^2$ m/s. Find the magnitudes of the boat's velocity and acceleration at the time t = 3 s.



Curvilinear Motion

- Planar Circular Motion:
  - Solution:
    - 1) Calculate the velocity at t = 3s
    - The magnitude is given by: $v = 0.25t^2$ m/s.
    - At t = 3s: $v = 0.25t^2 = 0.25(3)^2 =$ 2.25 m/s

Curvilinear Motion

- Planar Circular Motion:
  - Solution:
    - 2) Calculate the tangential and normal components of acceleration and then the magnitude of the acceleration vector.
    - Tangential Component:

$$a_t = \dot{v} = \frac{d}{dt}\left(0.25t^2\right) = 0.5t$$

    - At t = 3s: a_t = 0.5(3) = 1.5 ms^{-2}

Curvilinear Motion

- Planar Circular Motion:
  - Solution:
    - 2) Normal Component:

$$a_n = \frac{v^2}{\rho} = \frac{\left(0.25t^2\right)^2}{30}$$

- At t = 3s: $a_n = \frac{v^2}{\rho} = \frac{\left(0.25(3)^2\right)^2}{30} = 0.169 m/s^2$

Curvilinear Motion

- Planar Circular Motion:
  - Solution:
    - acceleration vector is:

$$a = a_t u_t + a_n u_n = \dot{v} \cdot u_t + \frac{v^2}{\rho} \cdot u_n$$

    - Magnitude of acceleration:

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + 0.169^2} = 1.509 m/s^2$$

Curvilinear Motion

- Angular motion
  - Angular motion equation:

$$v = v_0 + a_c t; \qquad \omega = \omega_0 + \alpha t;$$

$$y = y_0 + v_0 t + \frac{1}{2} a_c t^2; \qquad \theta = \omega_0 t + \frac{1}{2} \alpha t^2;$$

$$v^2 = v_0^2 + 2a_c (y - y_0); \qquad \omega^2 = \omega_0^2 + 2\alpha\theta;$$

Problem P1

- A position of a particle is given by $s = 12 + 6t^2 - t^3$, where s is distance in meter and t is time in second. Determine:
  a) velocity of the particle when the particle is at 17 m.
  b) maximum velocity of the particle.
  c) acceleration of the particle when it stops momentary.
  d) position of the particle when the velocity is maximum, and
  e) travelling distance by the particle in 3 seconds.

Problem P2

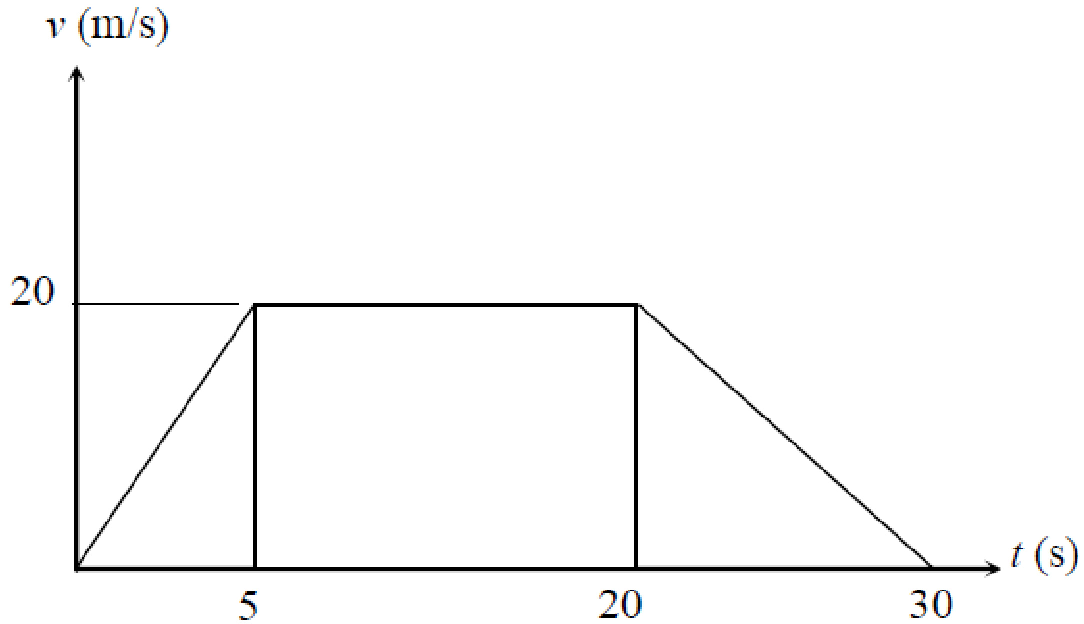
- A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$s, the particle is located 10 m to the left of the origin.
  a) Determine the acceleration when $t = 4$s.
  b) Calculate the displacement from $t = 0$ s to $t = 10$ s.
  c) Analyze and the distance the particle travels during this time period.
  d) Sketch the movement of the particle.

Problem P3

- A ball is thrown upward from the ground. The initial velocity is 15 m/s. Calculate:
  - The height and its velocity after 2 seconds.
  - The maximum height that the ball reached.

Problem P4

- The *v*-*t* graph of a car while traveling along a road is shown in following Figure. Draw the *s*-*t* and *a*-*t* graphs for the motion.



Problem P5

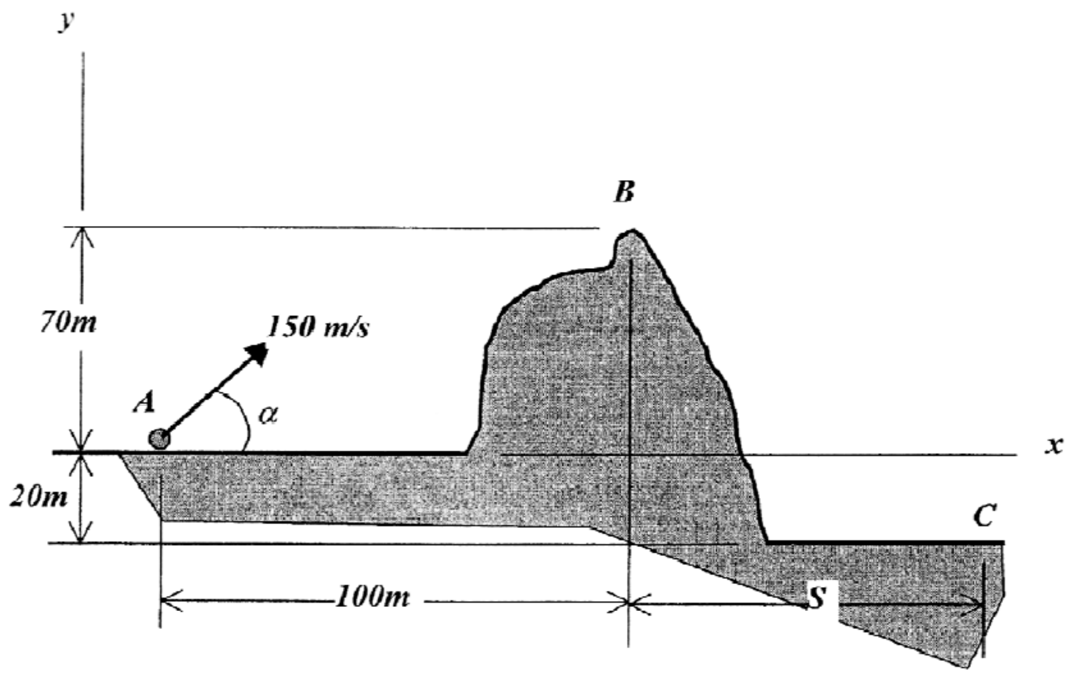
- A projectile launched from point A with an initial velocity of 150 m/s at an angle of α to the horizontal axis. The projectile passes over the peak of a hill (point B) at a vertical height of 70 m above point A, and fall to the ground at point C which is 20 m vertically below point A. When passing point B the vertical component of the projectiles' velocity is upwards.

Problem P5 (cont.)

a) Determine the angle, α.

b) The velocity of the projectile when passing point B.

c) The horizontal distance, *s*.



Problem 6

- A car races around a horizontal circular track with a radius of 80 m. Starting from rest, the car increases its speed at a constant rate of 2 m/s^2. Find the time (t) needed for it to reach an acceleration of 3 m/s^2. What is its speed at this instant?

The End