

SKAA 1213 - Engineering Mechanics

TOPIC 11

IMPULSE AND MOMENTUM

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Outline

- *Introduction*
 - *Principles of Impulse and Momentum*
 - *Impacts*
 - *Problems*
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Introduction

- Impulse:
 - Definition: measurement of the effect of a force during the time the force acts.
 - Linear impulse: $I = \int F dt$
 - The impulse acts in the same direction as the force,
 - Magnitude has unit of force-time, N.s

Introduction

- Momentum:
 - Linear momentum: $L = mv$
 - Magnitude mv has unit of mass-velocity, **kg.m/s**

Introduction

- Consider the Second Law of Newton:

$$\sum \mathbf{F} = m\mathbf{a}; \quad \mathbf{a} = \frac{d\mathbf{v}}{dt}$$

$$\Rightarrow \sum \mathbf{F} dt = m d\mathbf{v}$$

- Rearranging the terms and integrating between the limits $\mathbf{v} = \mathbf{v}_1$ at $t = t_1$ and $\mathbf{v} = \mathbf{v}_2$ at $t = t_2$

$$\sum \int_{t_1}^{t_2} \mathbf{F} dt = m \int_{\mathbf{v}_1}^{\mathbf{v}_2} d\mathbf{v}$$

$$\Rightarrow \sum \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

Introduction

- The equation is called *principle of linear impulse and momentum*
- The equation is rewritten in the form:

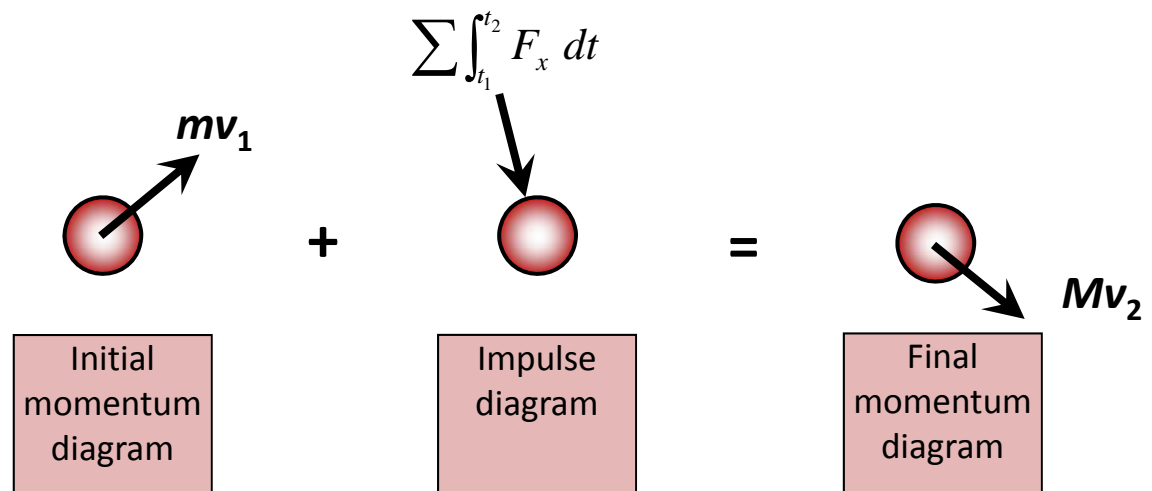
$$m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2$$

Principle of Impulse and Momentum

- *Principle of linear impulse and momentum in its x, y, z components:*

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$



Principle of Impulse and Momentum

- Conservation of Momentum:

$$\sum m_i (\mathbf{v}_i)_1 = \sum m_i (\mathbf{v}_i)_2$$

$$m_A (\mathbf{v}_A)_1 + m_B (\mathbf{v}_B)_1 = m_A (\mathbf{v}_A)_2 + m_B (\mathbf{v}_B)_2$$

$$m_A (\mathbf{v}_A)_1 + m_B (\mathbf{v}_B)_1 = (m_A + m_B) \mathbf{v}_2$$

Principle of Impulse and Momentum

- **Recall “Work and Energy”...**

- Principle of Work and Energy: $T_1 + \sum U_{1-2} = T_2$

- Conservation of Energy: $T_1 + V_1 = T_2 + V_2$

- Kinetic energy: $T = \frac{1}{2}mv^2$

- Potential energy: $V_g = W\Delta y$

Principle of Impulse and Momentum

- **Summary:**

- Principle of Impulse and Momentum

$$m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2$$

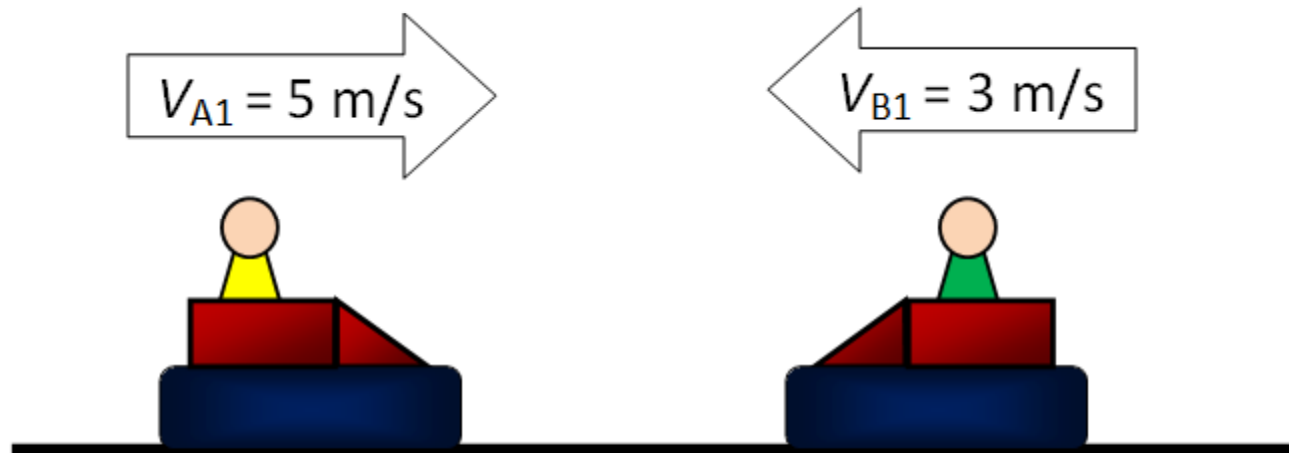
- Conservation of Momentum:

$$m_A(\mathbf{v}_A)_1 + m_B(\mathbf{v}_B)_1 = m_A(\mathbf{v}_A)_2 + m_B(\mathbf{v}_B)_2$$

$$m_A(\mathbf{v}_A)_1 + m_B(\mathbf{v}_B)_1 = (m_A + m_B)\mathbf{v}_2$$

Principle of Impulse and Momentum

- Two bumper cars *A* and *B* each have a mass of **200 kg** and velocities as shown before they freely collide head on. If no energy is lost during the collision, determine their velocities after collision.



Principle of Impulse and Momentum

- Solution:
 - Conservation of momentum:

$$(+ \rightarrow) \quad m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$(200)(5) + (200)(-3) = (200)(v_A)_2 + (200)(v_B)_2$$

$$\Rightarrow (v_A)_2 = 2 - (v_B)_2 \quad (1)$$

Principle of Impulse and Momentum

- Solution:
 - Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m_A (v_A)_1^2 + \frac{1}{2} m_B (v_B)_1^2 + 0 = \frac{1}{2} m_A (v_A)_2^2 + \frac{1}{2} m_B (v_B)_2^2 + 0$$

$$\Rightarrow (v_A)_2^2 + (v_B)_2^2 = 34 \quad (2)$$

Principle of Impulse and Momentum

- Solution:

- Sub Eq. (1) into (2):

$$(v_A)_2^2 + (v_B)_2^2 = (2 - v_B)_2^2 + (v_B)_2^2 = 34$$

$$\Rightarrow 2(v_B)_2^2 - 4(v_B)_2 - 30 = 0$$

- Solve using equation: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$(v_B)_2 = 5 \text{ m/s} \rightarrow$$

$$(v_A)_2 = -3 \text{ m/s or } 3 \text{ m/s} \leftarrow$$

Impact

- *Impact* occurs when two bodies collide and cause impulsive forces to be exerted between them.
 - *Central impact* occurs when the direction of motion of the mass centers of the two colliding particles is along the *line of impact*
 - *Oblique impact* occurs when one or both of the particles is at an angle with the *line of impact*

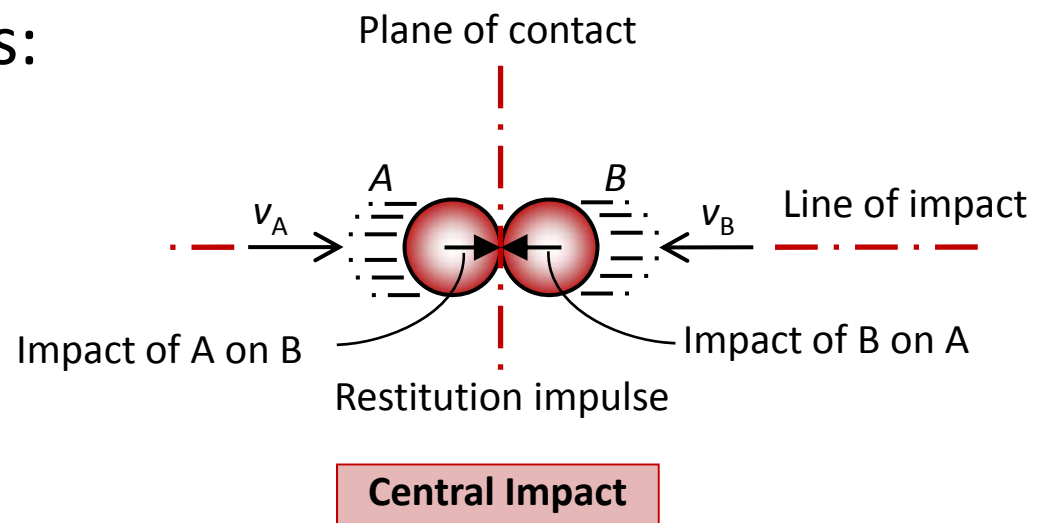
Impact

- **Central Impact:**

- Ratio of the restitution impulse to the deformation impulse is called the *coefficient of restitution*

- Can be expressed in terms of the particles' initial and final velocities:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$



Impact

- **Central Impact:**

- When $(v_A)_1 > (v_B)_1$, collision will occur
- During collision, particles undergo a period of deformable
- Only at maximum deformation will both particles have common velocity as their relative motion is zero

Impact

- **Central Impact:**

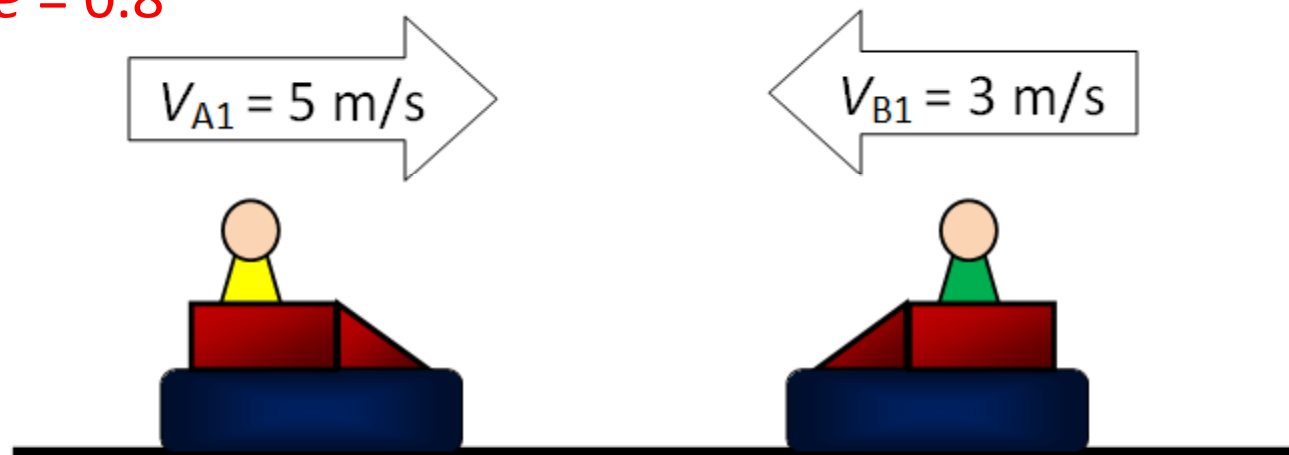
- Afterward a *period of restitution* occurs, particles will return to their original shape or remain permanently deformed
- Equal but opposite *restitution impulse* $\int \mathbf{R} dt$ pushes the particle apart from one another
- After separation the particles will have the final momentum, where $(v_B)_2 > (v_A)_2$

Impact

- Consider the same example, The two 200-kg bumper cars A and B collide head on. Determine their velocities after collision if the **coefficient of restitution** for the disks is:

a) $e = 1.00$

b) $e = 0.8$



Impact

- Solution (a)
 - Conservation of momentum:

$$(+ \rightarrow) \quad m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$(200)(5) + (200)(-3) = (200)(v_A)_2 + (200)(v_B)_2$$

$$\Rightarrow (v_A)_2 = 2 - (v_B)_2 \quad (1)$$

Impact

- Solution (a)
 - Coefficient of Restitution:

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = 1.0$$

$$\frac{(v_B)_2 - (v_A)_2}{5 - (-3)} = 1.0$$

$$\Rightarrow (v_B)_2 - (v_A)_2 = 8 \quad (2)$$

Impact

- Solution (a)

– **Put (1) into (2):**

$$(v_A)_2 = 2 - (v_B)_2 \quad (1)$$

$$(v_B)_2 - (v_A)_2 = 8 \quad (2)$$

$$\Rightarrow (v_B)_2 - [2 - (v_B)_2] = 8$$

$$(v_B)_2 = \frac{8+2}{2} = 5m/s \quad (\rightarrow)$$

Impact

- Solution (a)
 - From (1):

$$\begin{aligned}(v_A)_2 &= 2 - (v_B)_2 \\ &= 2 - 5 \\ &= -3m/s \quad (\leftarrow)\end{aligned}$$

Impact

- Solution (b)
 - Coefficient of Restitution:

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = 0.8$$

$$\frac{(v_B)_2 - (v_A)_2}{5 - (-3)} = 0.8$$

$$\Rightarrow (v_B)_2 - (v_A)_2 = 6.4 \quad (2)$$

Impact

- Solution (b)

– **Put (1) into (2):**

$$(v_A)_2 = 2 - (v_B)_2 \quad (1)$$

$$(v_B)_2 - (v_A)_2 = 6.4 \quad (2)$$

$$\Rightarrow (v_B)_2 - [2 - (v_B)_2] = 6.4$$

$$(v_B)_2 = \frac{6.4 + 2}{2} = 4.2 \text{ m/s} \quad (\rightarrow)$$

Impact

- Solution (b)
 - From (1):

$$\begin{aligned}(v_A)_2 &= 2 - (v_B)_2 \\ &= 2 - 4.2 \\ &= -2.2 \text{ m/s} \quad (\leftarrow)\end{aligned}$$

Impact

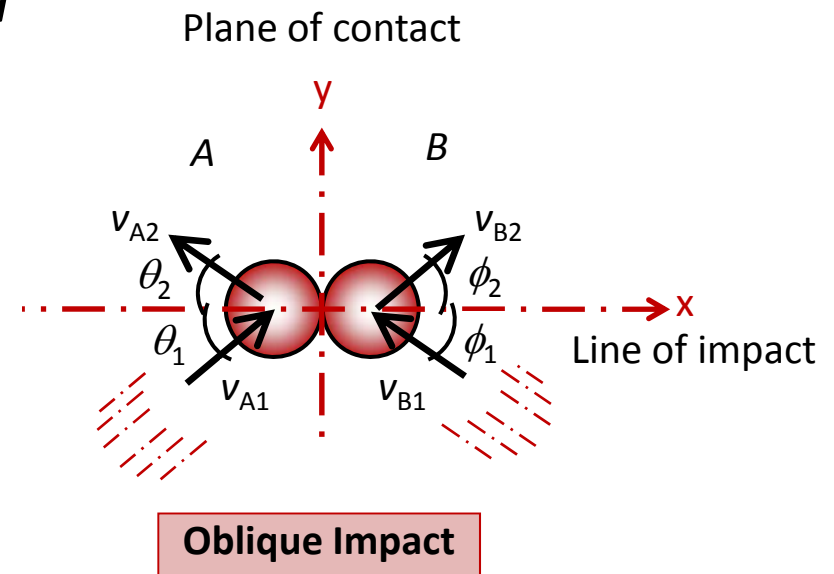
- **Oblique Impact**

- When oblique impact occurs, particles move away from each other with velocities having unknown directions and unknown magnitudes
 - Provided the initial velocities are known, four unknown are present in the problem
 - Unknown are $(v_A)_2$, $(v_B)_2$, ϑ_2 and Φ_2 , or x and y components of the final velocities
-

Impact

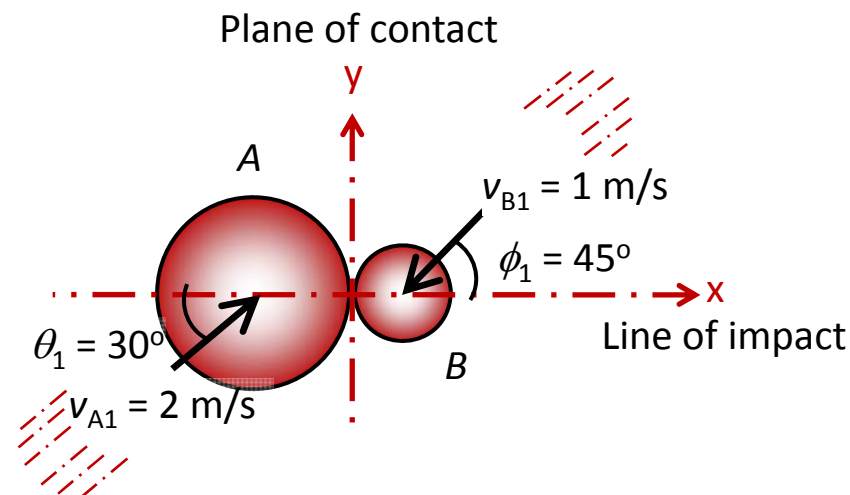
- **Oblique Impact**

- When y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act *only in the x direction*



Impact

- Two balls A and B , having mass of 2 kg and 1 kg respectively, collide with the velocities shown. If the coefficient of restitution for the disks is $e = 0.75$, determine the x and y components of the final velocity of each disk just after collision.



Impact

- Solution

$$(v_{Ax})_1 = 2 \cos 30^\circ = 1.73m / s$$

$$(v_{Ay})_1 = 2 \sin 30^\circ = 1.00m / s$$

$$(v_{Bx})_1 = -1 \cos 45^\circ = -0.707m / s$$

$$(v_{By})_1 = -1 \sin 45^\circ = -0.707m / s$$

Impact

- Solution

– Conservation of 'x' momentum:

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) m_A (v_{Ax})_1 + m_B (v_{Bx})_1 = m_A (v_{Ax})_2 + m_B (v_{Bx})_2$$

$$2(1.73) + 1(-0.71) = 2(v_{Ax})_2 + 1(v_{Bx})_2$$

$$\Rightarrow 2(v_{Ax})_2 + (v_{Bx})_2 = 2.75 \quad (1)$$

Impact

- Solution
 - Coefficient of restitution 'x':

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

$$0.75 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{1.73 - (-0.71)}$$

$$\Rightarrow (v_{Bx})_2 - (v_{Ax})_2 = 1.83 \quad (2)$$

Impact

- Solution

$$2(v_{Ax})_2 + (v_{Bx})_2 = 2.75 \quad (1)$$

$$(v_{Bx})_2 - (v_{Ax})_2 = 1.83 \quad (2)$$

– Solve two equation simultaneously:

$$(v_{Ax})_2 = 0.31m/s \rightarrow$$

$$(v_{Bx})_2 = 2.14m/s \rightarrow$$

Impact

- Solution

– Conservation of 'y' momentum:

$$\left(+ \uparrow\right) \quad m_A (v_{Ay})_1 = m_A (v_{Ay})_2$$

$$\Rightarrow \quad (v_{Ay})_2 = 1.00m/s \uparrow$$

$$\left(+ \uparrow\right) \quad m_B (v_{By})_1 = m_B (v_{By})_2$$

$$\Rightarrow \quad (v_{By})_2 = -0.707m/s = 0.71m/s \downarrow$$

Problem P1

- A 2 kg ball *A* is traveling horizontally to the right at 25 m/s and strikes a 8 kg ball *B* that is at rest. If the coefficient of restitution between ball *A* and *B* is $e = 0.8$, determine the velocity and direction of both the ball *A* and ball *B* after the strike.

Problem P2

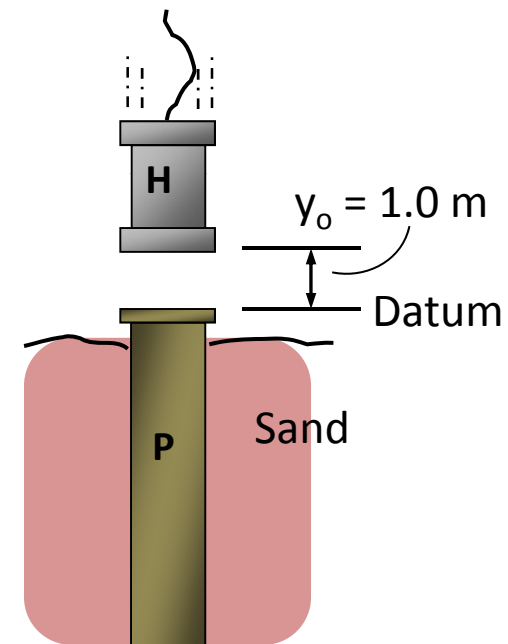
- The three balls shown have a mass $m = 200$ N each. If A has a speed, $v_A = 30$ m/s just before a direct collision with B, determine the speed of ball C after collision. The coefficient of restitution between each ball is $e = 0.9$. Neglect the size of each ball.



Problem P3

An 500-kg rigid pile P is driven into the ground using a 150-kg hammer H . The hammer falls from rest at a height $y_0 = 1.0$ m and strikes the top of the pile.

Determine the impulse which the hammer imparts on the pile if the pile is surrounded entirely by loose sand so that after striking, the hammer does not rebound off the pile.

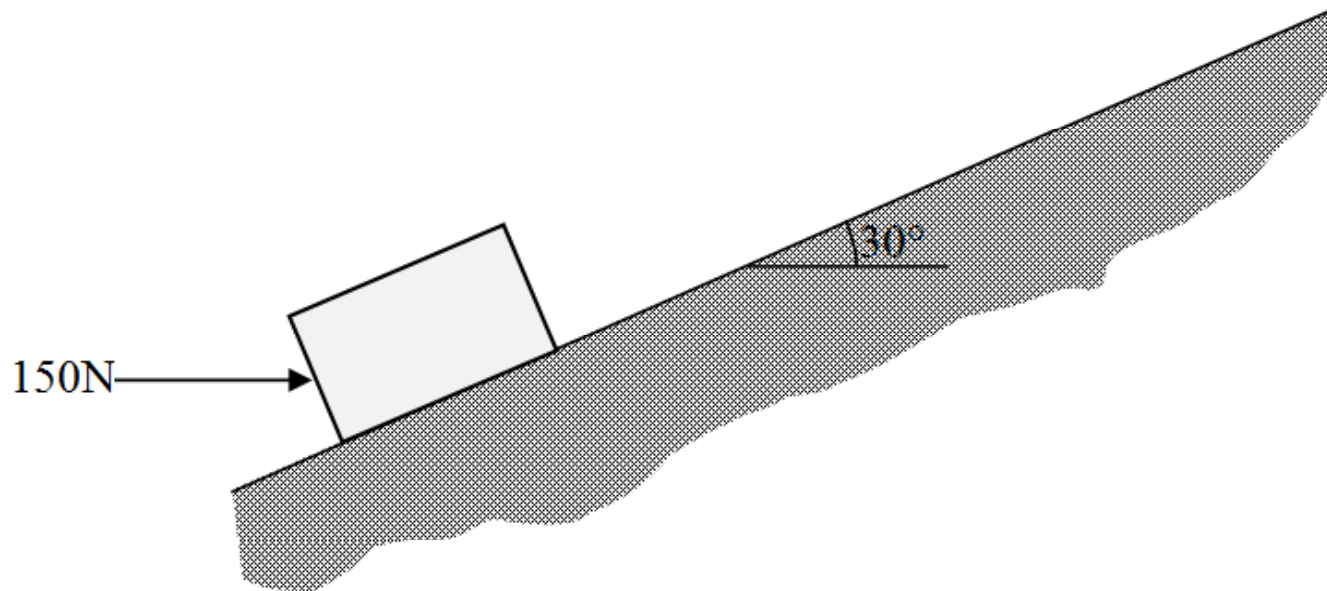


Problem P4

- a 10 kg block being pushed up an inclined plane by a horizontal 150 N force. Friction force between the surface of the inclined plane and the block is determined to be 20 N. After a period of t seconds, the block moved up the inclined plane a distance of s meters and the velocity of the block changed from a state of rest to 2.5 m/s.

Problem P4 (cont.)

- Determine the distance, s using the principle of work and energy.
- Determine the time, t using the principle of impulse and momentum





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