SKAA 1213 - Engineering Mechanics

TOPIC 2

RESULTANT AND RESOLUTION OF FORCES

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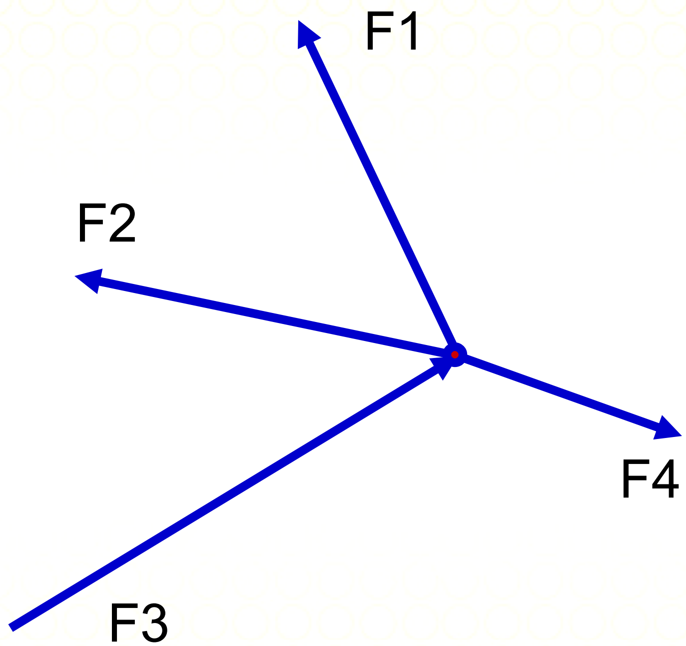
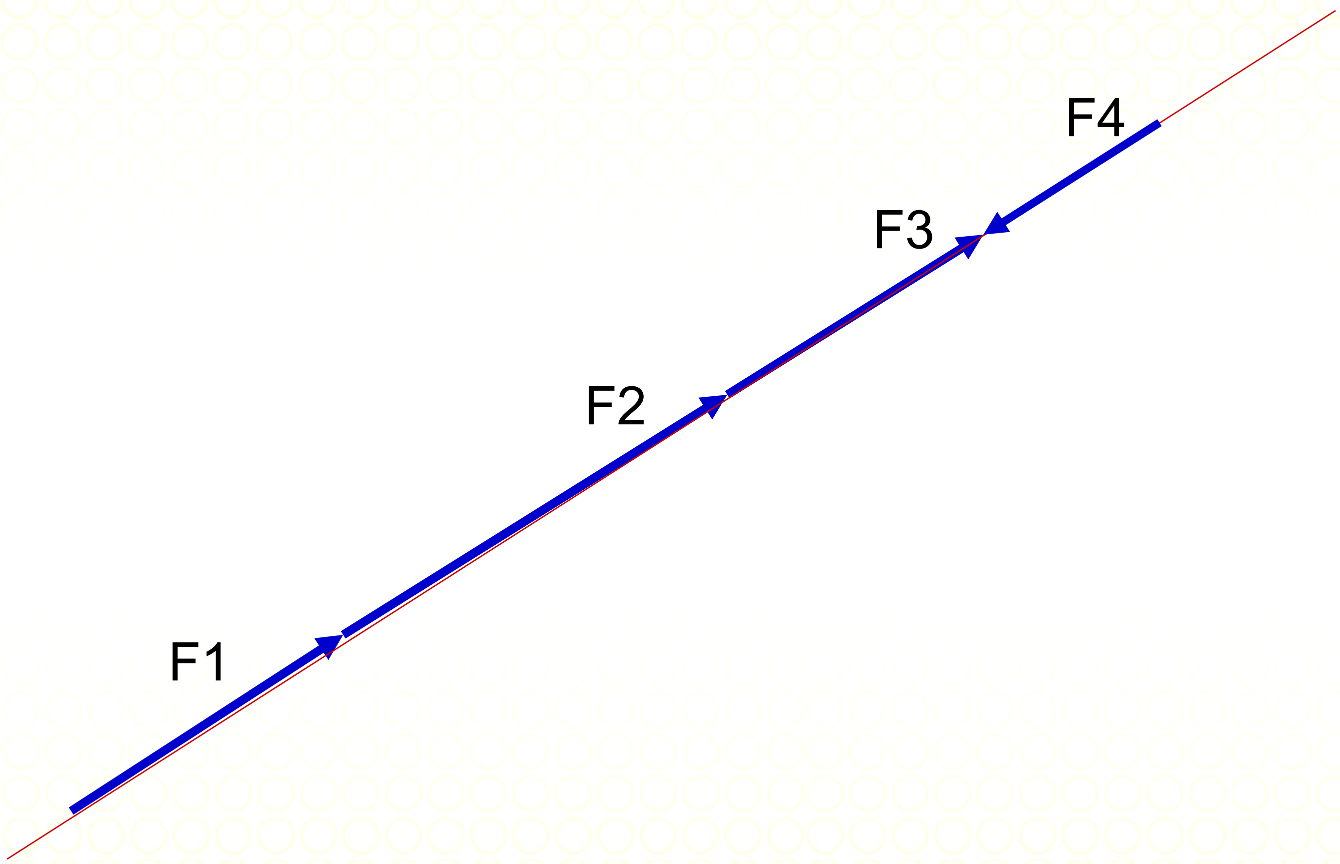
Concurrent Forces -

forces acting at a point



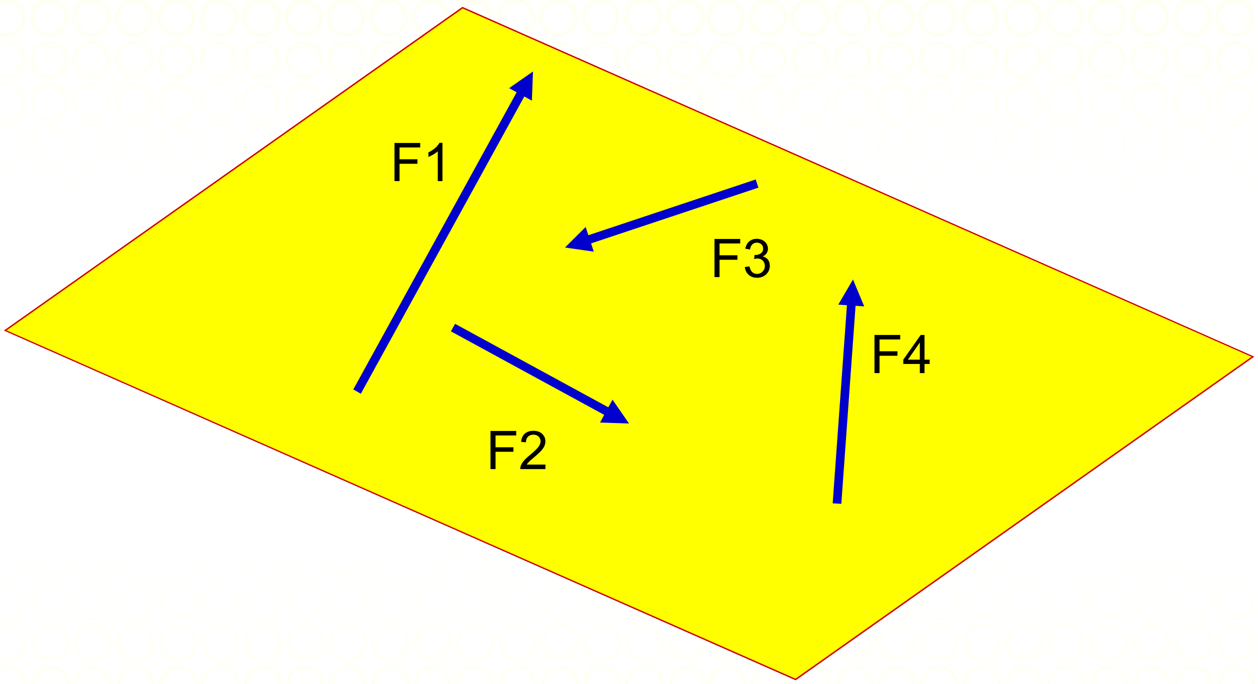
Collinear Forces –

forces acting in the same line



Coplanar Forces –

forces acting in a same plane



Force is a vector, therefore parallelogram law is applicable.

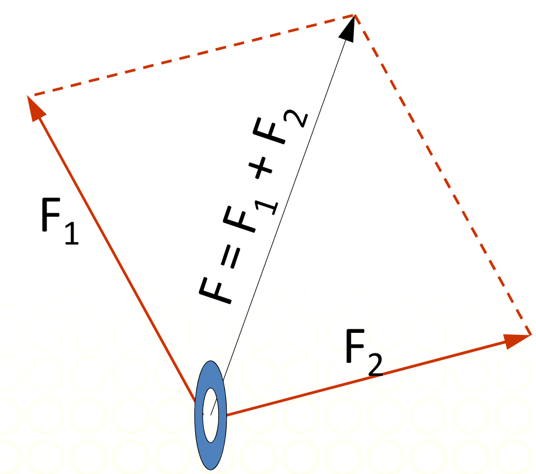
Used to;

1) To find **resultant** force.
2) Resolving a known force into two **components**.

Vector Addition of Forces

If only **two** forces are added, the resultant the **forces** acting at a point can be determined by;

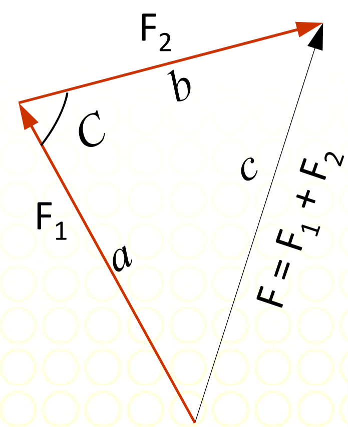
Parallelogram law



Apply the sine and cosine laws.

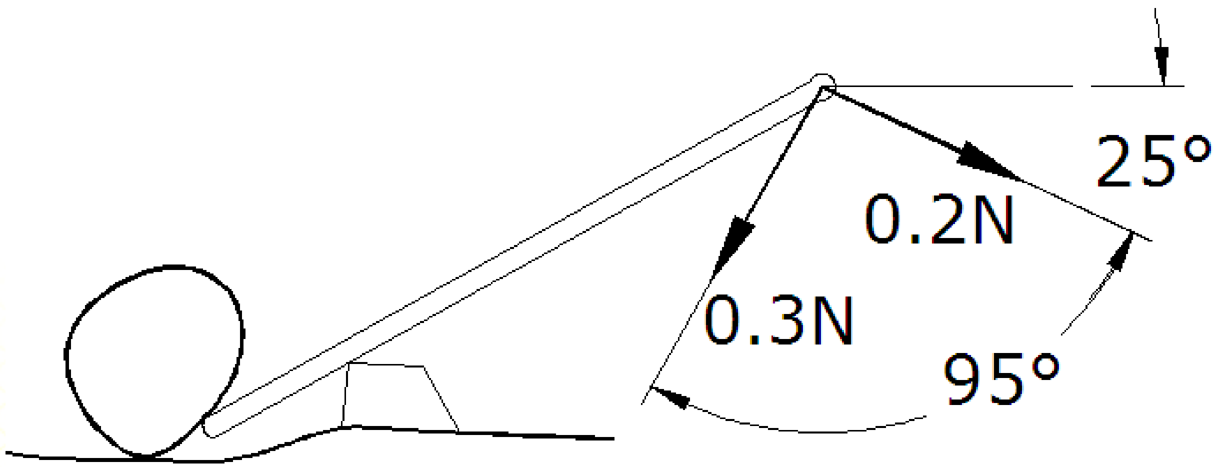
Sine Law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Law: $c^2 = a^2 + b^2 - 2ab \cos C$



Example 1

Determine the magnitude of the resultant force on the lever shown and its direction measured counterclockwise from the positive x axis. *[Answer: R = 0.346 N , β = 275.3°]*



Example 2

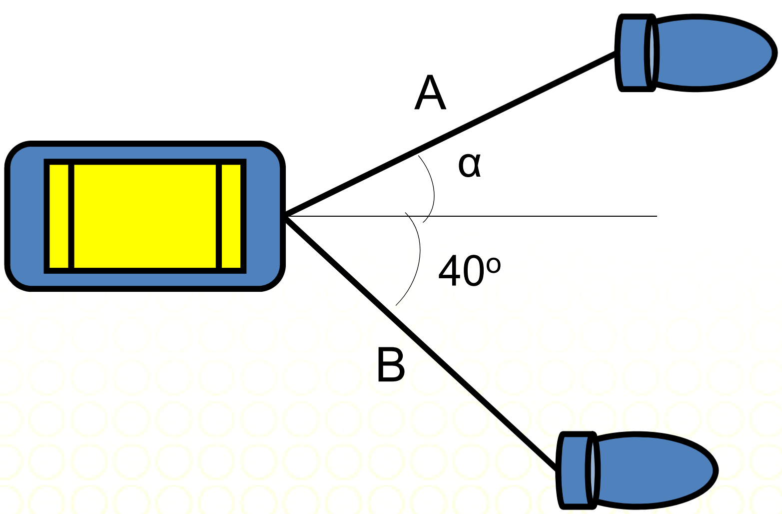
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 30 kN directed along the axis of the barge,;

(a) determine the tension in each of the ropes when α = 35°,

(b) determine α for which the tension in rope A is minimum.

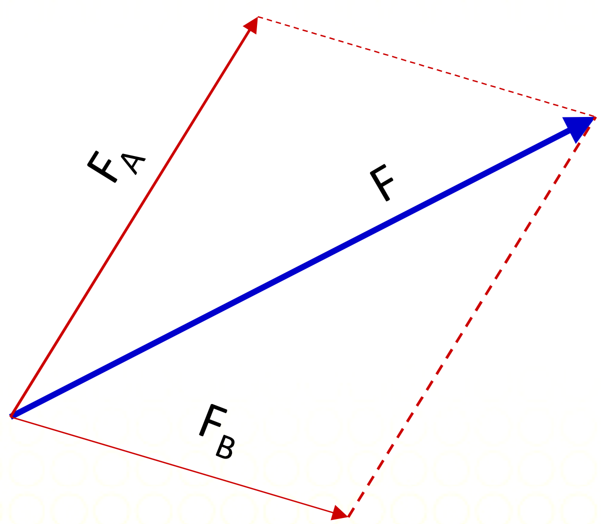
(a) $T_A = \mathbf{20}\ kN$, $T_B = \mathbf{17.8}\ kN$

(b) $T_A = \mathbf{19.3}\ kN$, $T_B = \mathbf{23.0}\ kN$

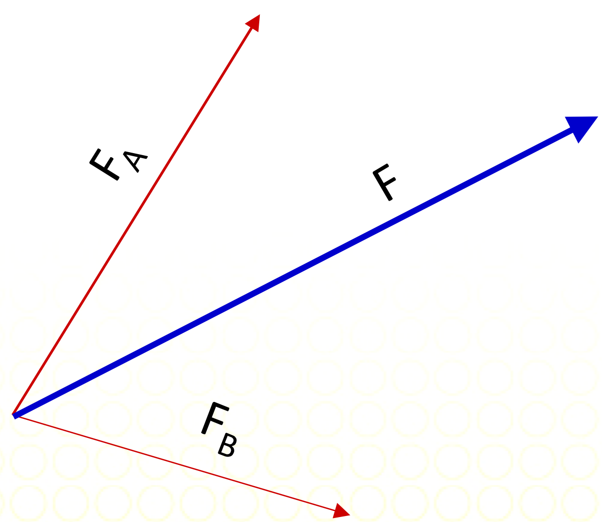


Resolution of Forces

- A single force can be broken into two separate components.
- The two components can be determined by using the parallelogram law.
- It is the reverse of resultant.



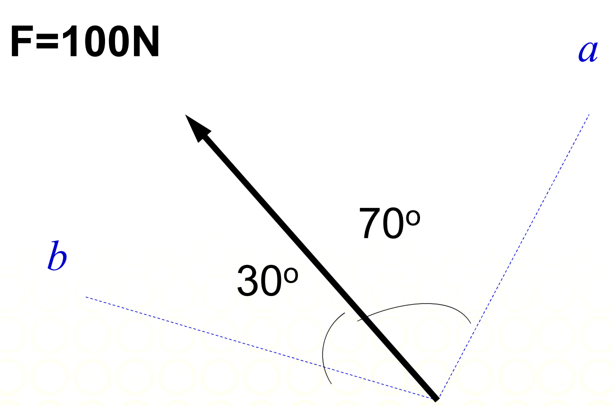
Force F can be replaced by F_A and F_B that produce the same effect



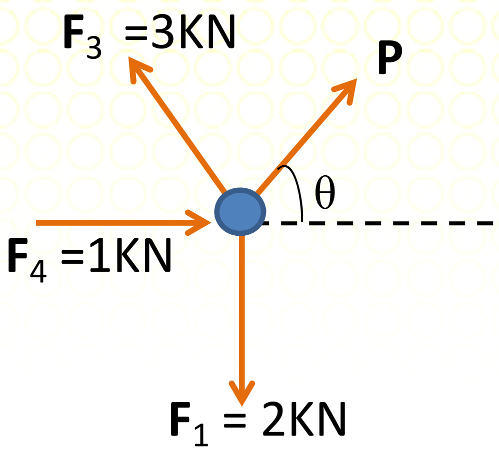
Example 3

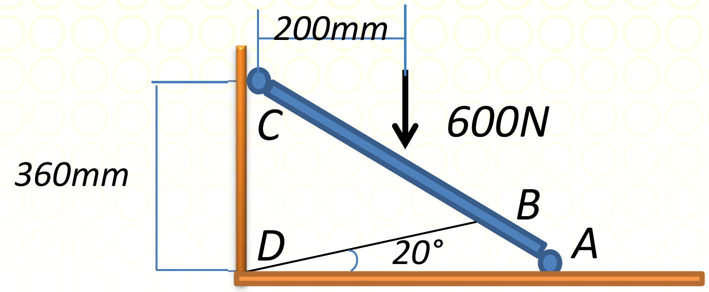
Determine the components of force F=100N for the axis system of *a* and *b* as shown.

*[Answer : F_A = **50.8** N , F_B = **95.4** N]*

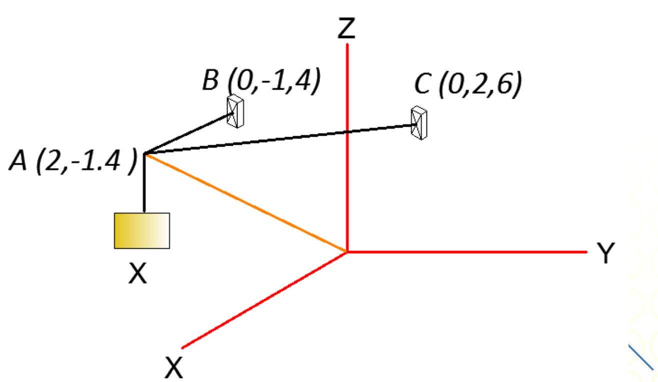


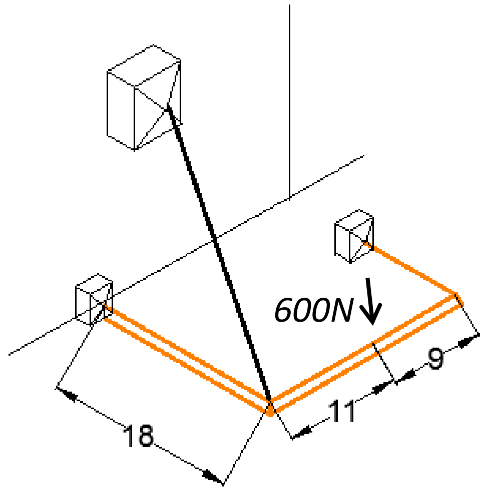
Scalar notation is used to solve problems for coplanar forces – 2D





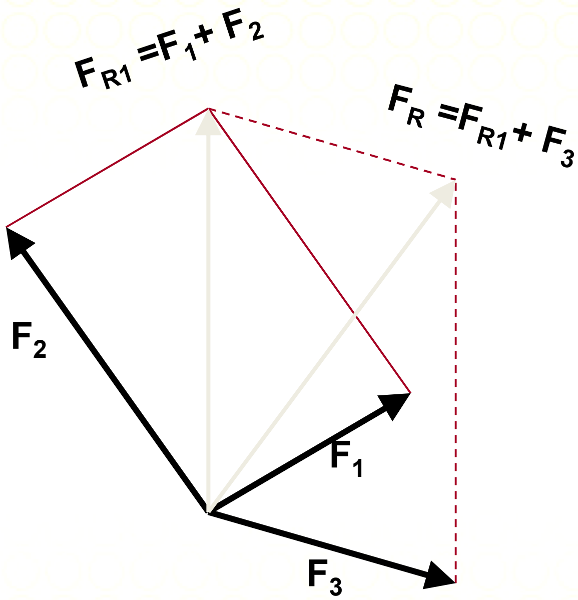
Cartesian vector notation is used in three dimensional problems.





Resultant of more than 2 Forces

If **more than two** forces are added;



It requires extensive geometric and trigonometric calculation to determine the magnitude and direction of the resultant.

An easier way is to use **Rectangular-component method.**

The sense of **direction** is represented graphically by the **arrow head** .

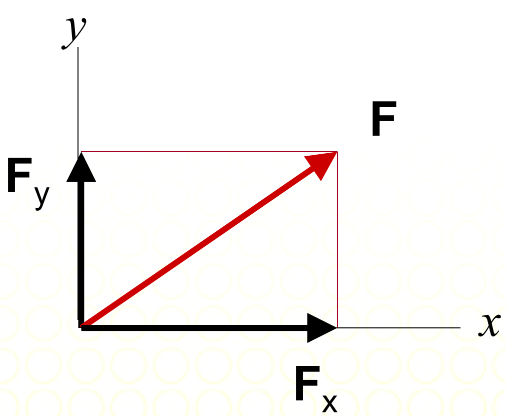
For analytical work, establish a notation for representing the sense of direction of the rectangular components.

This can be done by either **Scalar Notation** or **Cartesian Vector Notation**.

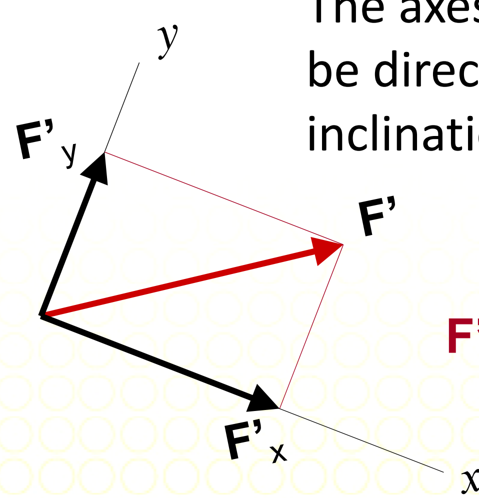
Rectangular Components of Coplanar Force

Method

- Sum up the **components** of each force along specified axes algebraically, and then form a resultant.
- Resolved each force into its **rectangular components** $\mathbf{F}_x$ and $\mathbf{F}_y$ which lie along the *x* and *y* axes.



$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$



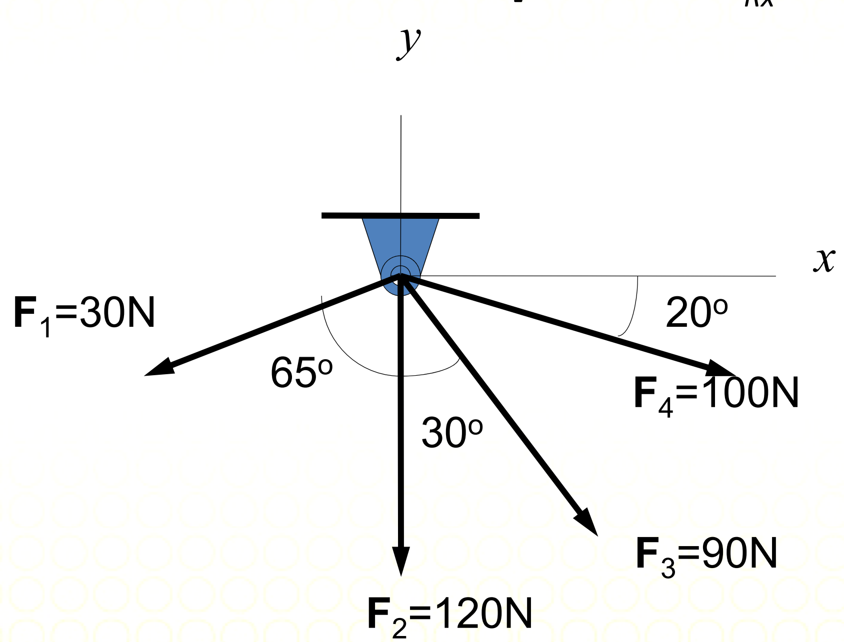
The axes may in general be directed in any inclination.

$\mathbf{F}' = \mathbf{F}'_x + \mathbf{F}'_y$

Example 4

Four forces act on an eye bolt. Determine the resultant of the forces on the bolt.

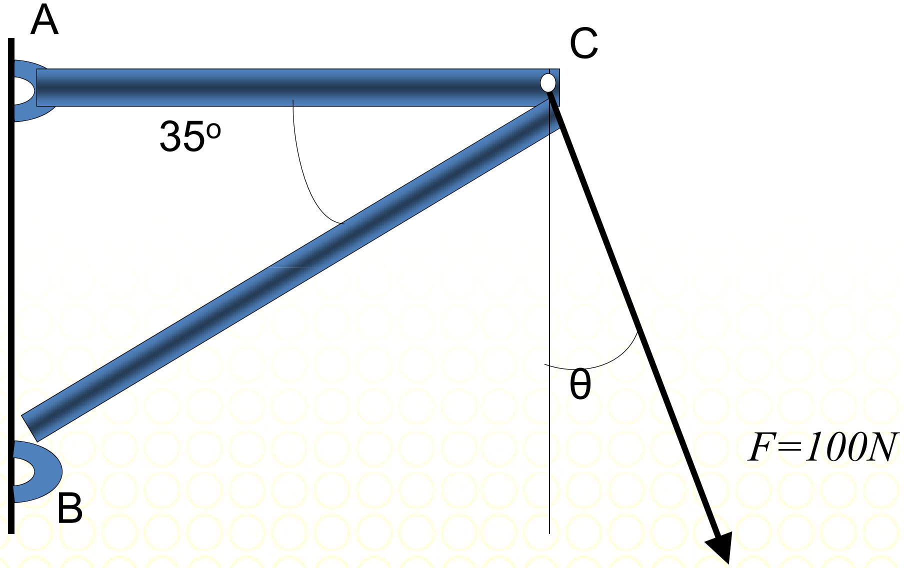
[ANSWER F_{Rx}= 111.8N , F_{Ry}= -244.8N , F_R = 269 N , Θ = - 65.5^o]



Example 5

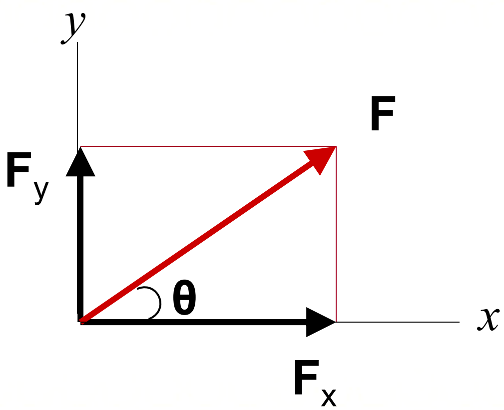
Resolve the force F acting at C into two components acting along members AC and BC, if θ = 20°. Determine θ, so that the component F_{AC} is directed toward C and has a magnitude of 150N.

[Answer : F_{CB} = 163.8 N , F_{AC} = 168.4 N , θ = 4.3°]



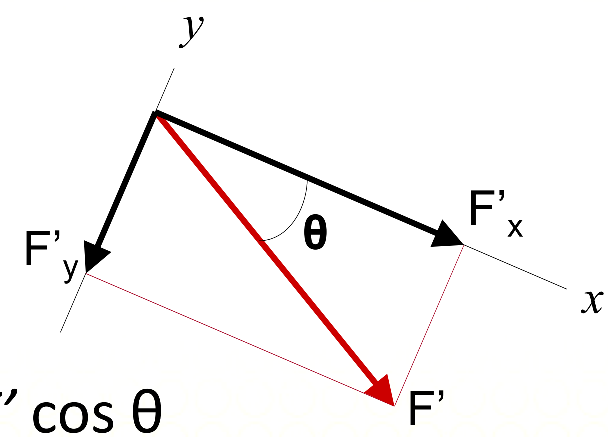
1.Scalar Notation

- Since the *x* and *y* axes have designated +ve and –ve directions, the magnitude and directional sense of the components of a force can be expressed in terms of **algebraic scalars.**
- the component is represented by +ve scalar ***F*** if the sense of direction is along the +ve axis and vice versa.



$F_x = F\cos\theta$

$F_y = F\sin\theta$



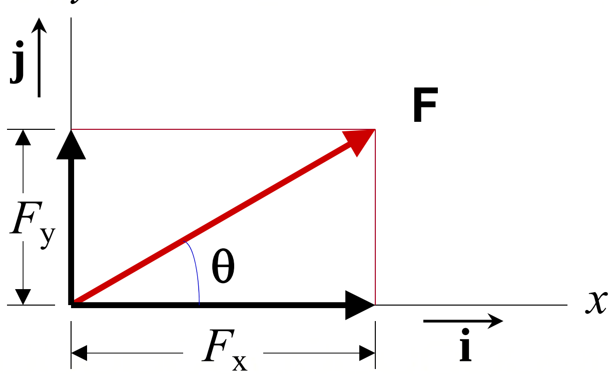
$F'_x = F'\cos\theta$

$F'_y = -F'\sin\theta$ (– ve opposite to the direction of +ve *y*-axis)

2. Cartesian Vector Notation

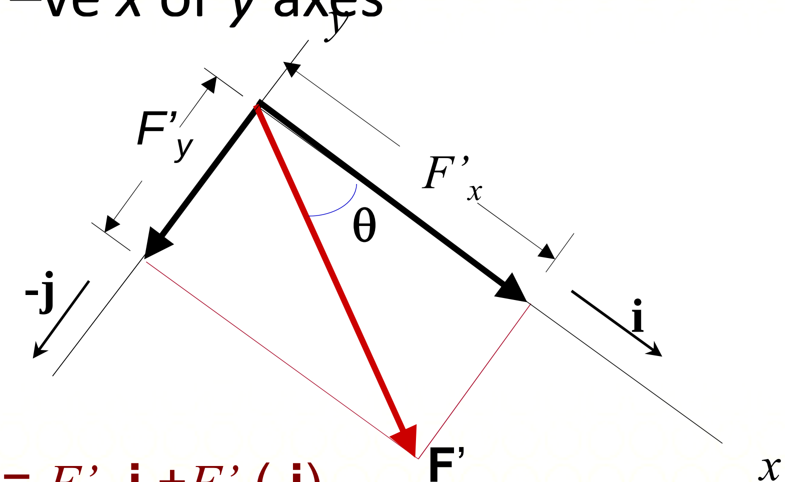
- In 2D, the Cartesian **unit vectors** **i** and **j** are used to show the direction of the *x* and *y* axes

- The unit vectors have a dimensionless magnitude, and it's described analytically by + and – signs, depending whether they are pointing along the +ve or –ve *x* or *y* axes



$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

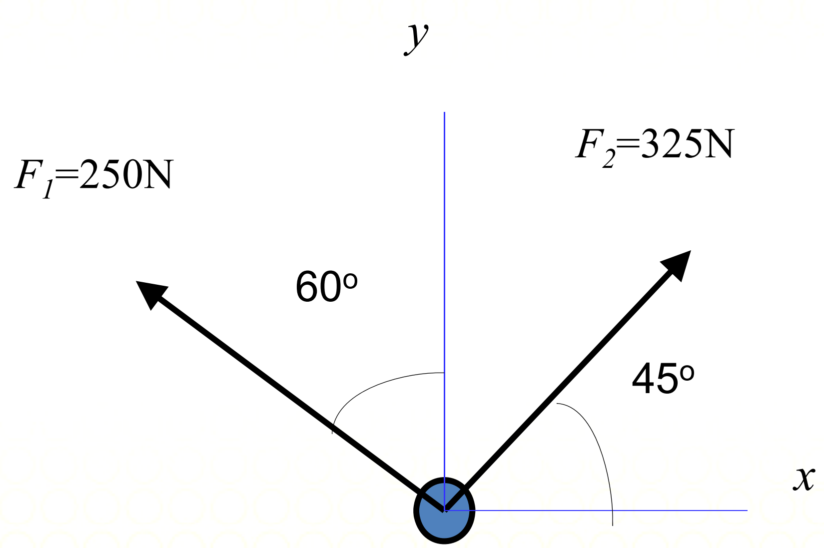


$$\mathbf{F}' = F'_x\,\mathbf{i} + F'_y(-\mathbf{j})$$
$$\mathbf{F}' = F'_x\,\mathbf{i} - F'_y\,\mathbf{j}$$

$$F'_x = F'\cos\theta$$
$$F'_y = F'\sin\theta$$

Example 6

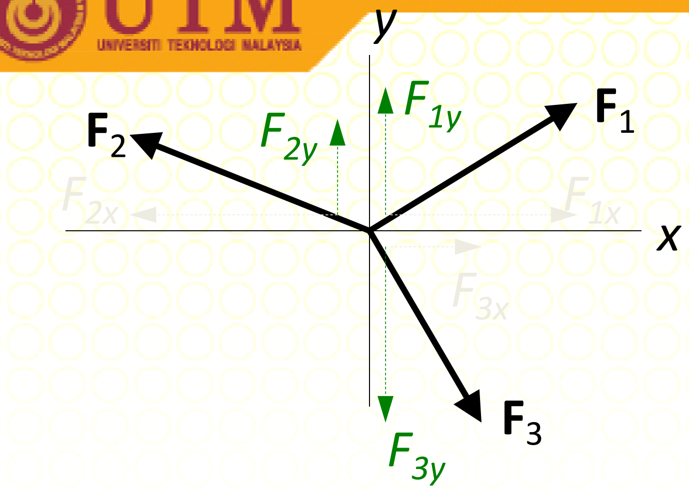
Determine the magnitude and orientation of the resultant force using Cartesian vector method. *[Answer : $F_R = 13.3\, i + 354.8\, j$]*



Coplanar Resultant Force [2D]

(Rectangular-Component Method)

- Either of the two methods (***Scalar*** or ***Cartesian vector***) can be used to determined the resultants.
- Each force is resolved into x and y components and total up all the components using scalar algebra.
- Apply parallelogram law to obtain resultant force by adding the resultant of the x and y components.



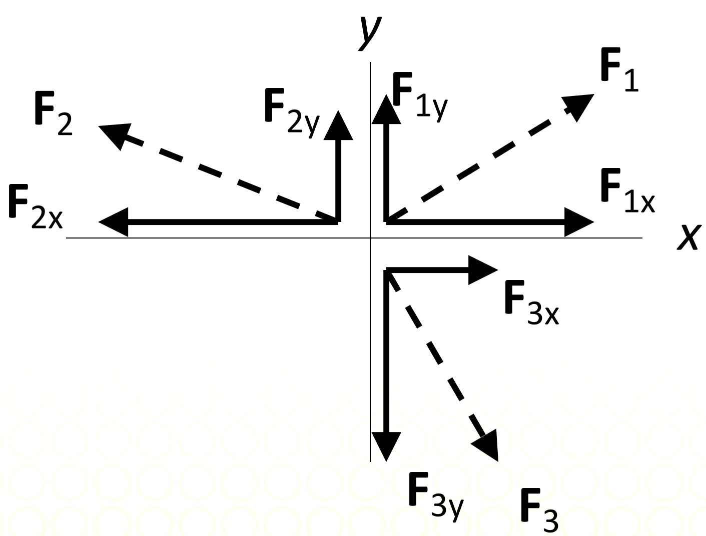
Using Scalar notation

$(\rightarrow +)\quad F_{Rx} = F_{1x} - F_{2x} + F_{3x}$

$(\uparrow +)\quad F_{Ry} = F_{1y} + F_{2y} - F_{3y}$

Good for 2D problems

Using Cartesian vector notation



$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$

$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$

$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$

Good for 3D problems

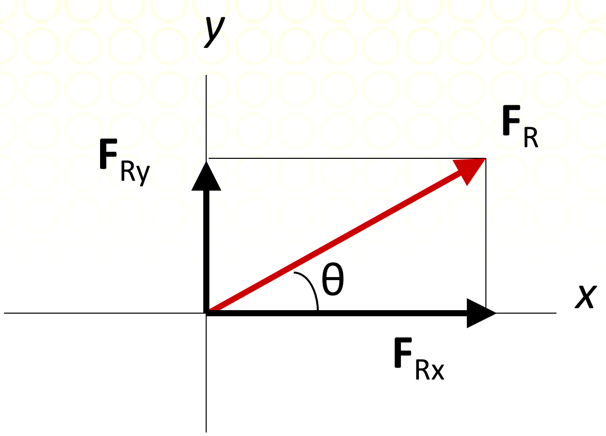
$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= F_{1x}\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\,\mathbf{j}$$

$$= (F_{1x} - F_{2x} + F_{3x})\,\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\,\mathbf{j}$$

$$= F_{Rx}\,\mathbf{i} + F_{Ry}\,\mathbf{j}$$

The resultant can be represented by the algebraic sum of the *x* and *y* components:



$$F_{RX} = \sum F_x$$

$$F_{RY} = \sum F_y$$

The magnitude F_R can be found from Pythagoras theorem;

$$F_R = \sqrt{F_{RX}^2 + F_{RY}^2}$$

The direction angle, θ

$$\theta = \tan^{-1}\frac{F_{RY}}{F_{RX}}$$

Example 7

Express each of the three forces acting the column in Cartesian vector form and compute the magnitude of the resultant force.

[Answer : F_R = 443.5 N , θ = - 84.3^o]